Determination of Radius of Curvature for Teeth With Cycloid Profile

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Abstract. In the article the geometric determination of curvature radius is considered for teeth with cycloid profile. The equations are obtained for the determination of a radius of curvature with point coordinates of a cycloid profile. The conditions of convexo–concavity of a teeth profile are defined for transmission with intermediate rollers.

1. Introduction

Now strict requirements are imposed at balance of wait – dimensions - power for modern mechanisms and technical unit more-and-more. On the base on transmissions with intermediate rollers (TIR) reducers has optimum ratio of technical data. Half of rollers number of TIR hold loading (sometimes all rollers hold loading in some scheme of the transmissions). So, the rollers transfer loading by the angle of 180°. It allows transferring torque more than 5-10 times and repeated shot-time overload in comparison usual toothed gear transmissions. One of representatives of TIR is coaxial flat transmission with intermediate rollers, which is showed on Figure 1.

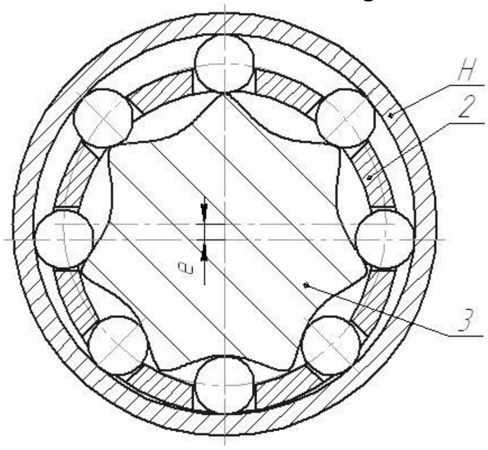

Figure 1. Cross section of flat radial transmission with intermediate roller: *H* – generator (planet carrier); *2* – separator with rollers; *3* – central gear; *e* – eccentricity of the transmission.

The transmission works following way: motor rotates plant carrier *H* and the planet carrier made rollers to move radial in grooves of separator 2, as shows on Figure 1. The separator 2 rigidly fixed in housing of the transmission. The rollers contact with cycloid profile of central gear 3. By the radial movement rollers made to the central gear 3 rotated. The central gear 3 is jointed rigidly with output shaft of the transmission.

It is known that contact durability depends from the specified curvature radius volume of cycloid profile. This shows following equation for calculation of contact stress [2]:

$$\sigma_{\max} = \left(\frac{F\left(R_2 \pm R_1\right)}{\pi L R_1 R_2 \left(\dfrac{1-\varepsilon_1^2}{E_1} + \dfrac{1-\varepsilon_2^2}{E_2} \right)} \right)^{1/2}, \tag{1}$$

where F – normal force to surfaces of contacting bodies, H;

R_1, R_2 – curvature radii of contacting bodies, mm;

L – contact length of roller and profile, mm;

E_1, E_2 – elasticity modulus of first and second contacting bodies respectively, MPa;

ε_1, ε_2 – Poisson's ratios for first and second contacting bodies respectively.

Thus, radii of curvature need for determination of stress-strained state in transmission with intermediate rollers. Besides profile surface radius of curvature needs for technological process of cycloid gear production. Therefor determination of radius of curvature for teeth with cycloid profile is up-to-date.

2. Determination of curvature radius

There are contact between central gear, groove wall of separator, rollers and generator. For roller and generator contact radii of curvature are constant and they equal radius of roller and radius of generator respectively. This contact is convexo-concave.

In roller and groove wall of separator contact is constant. There is contact between radius of roller and flat. Here contact equation will change a little bit.

Let us consider contact between roller and central cycloid gear. Here a radius of curvature of the central gear is variable in difference from both previous cases. Besides, in during this contact roller is moving from convexo-convex contact to convexo-concave contact along cycloid profile of gear and vice versa per one revolution of generator.

Some works are devoted to determination of radiuses of curvature of cycloid profile [3, 4]. There the task be solved by following ways: differential method, Euler-Savari method. Both methods are precise and convenient, but have the difficulties. First method is complex very much, because it demands the capture of the first and second derivative on a generator angle of rotation in the equations of a profile of tooth. These may lead to rather difficult mathematical transformations of the equations. The Bobillier construction is possible at a known a pitch point, but it is unknown for this transmission.

Thus, we need to define method of determination of radius of curvature simpler, but the same effective, as well as considered above. In our case, the method is more suitable for determination of curvature radius by geometric method (Figure 2). For chords are choosing, we need now a coordinates of points around contact point of cycloid profile. The points can be choose by special computer program. Then we create the massif from the points of a profile is found and we define a angle between the two chords.

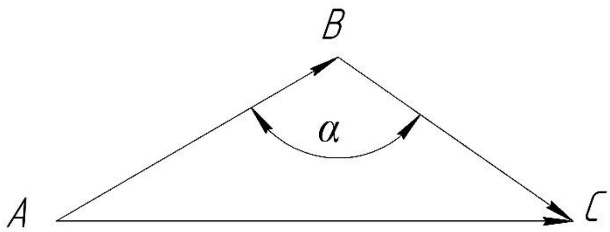

Figure 2. For curvature radius determination of gear profile

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Let us consider the points (Figure 2) of the cycloid profile $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$. We connected these points by the lines and constructed through them three vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{AC}$.

Let us determine the coordinates of vectors through the coordinates of pointes A, B, C:

$$\overrightarrow{AB} = \{x_2 - x_1, y_2 - y_1\}$$
$$\overrightarrow{BC} = \{x_3 - x_2, y_3 - y_2\} \qquad (2)$$
$$\overrightarrow{AC} = \{x_3 - x_1, y_3 - y_1\}$$

Scalar product of vectors $\overrightarrow{AB}$ и $\overrightarrow{BC}$:

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (x_2 - x_1) \cdot (x_3 - x_2) + (y_2 - y_1) \cdot (y_3 - y_2) \qquad (3)$$

Lengths of vectors:

$$\left|\overrightarrow{AB}\right| = \left((x_2 - x_1)^2 + (y_2 - y_1)^2\right)^{1/2}$$
$$\left|\overrightarrow{BC}\right| = \left((x_3 - x_2)^2 + (y_3 - y_2)^2\right)^{1/2} \qquad (4)$$
$$\left|\overrightarrow{AC}\right| = \left((x_3 - x_1)^2 + (y_3 - y_1)^2\right)^{1/2}$$

The Cosine of the angle between vectors $\overrightarrow{AB}$ и $\overrightarrow{BC}$ are determined by a formula:

$$\cos(\alpha) = \frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{BC}\right|} \qquad (5)$$

Define radius of a circumscribed circle according to the theorem of sine [5]:

$$2R = \frac{\left|\overrightarrow{AC}\right|}{\sin(\alpha)}, \qquad (6)$$

Transform (6) taking into account that:

$$\sin(\alpha) = \left(1 - \cos^2(\alpha)\right)^{1/2} \qquad (7)$$

Then

$$R = \frac{\left|\overrightarrow{AC}\right|}{2 \cdot \sin(\alpha)} = \frac{\left|\overrightarrow{AC}\right|}{2 \cdot \left(1 - \cos^2(\alpha)\right)^{1/2}} \qquad (8)$$

Substituting (5) in (8) received:

$$R = \frac{\left|\overrightarrow{AC}\right|}{2 \cdot \left(1 - \left(\frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{BC}\right|}\right)^2\right)^{1/2}} \qquad (9)$$

3. Determination concave – convex contact

The offered option allows determining curvature radius any curve in engagement. The equation (9) gives a radius which always positive, but curvature of a profile changes from convex to the concave. As the profile of tooth located in the first or fourth quarter of the Cartesian coordinate system and the roller is above this profile, it is possible to define convexo-concave considered Figure 3.

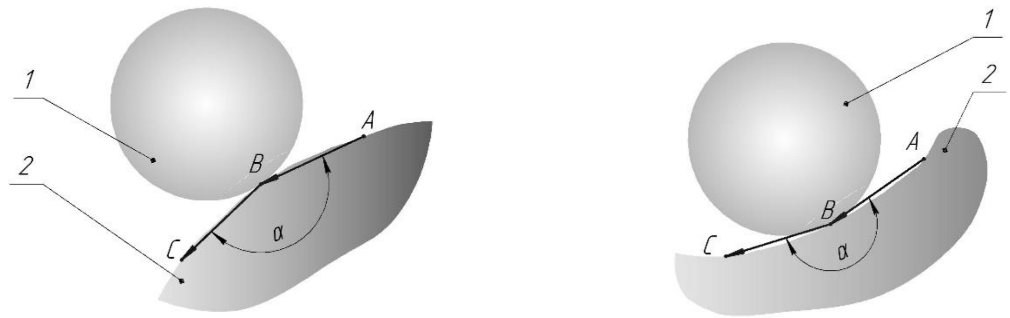

Figure 3. For convexo-concave determination of gear profile: *1* – roller; *2* – profile of the teeth.

It is possible to define profile convexity by angle α:
if α<180˚ – the profile is convex;
if α>180˚ – the profile is concave.

It is possible to determine the angle α through the directing cosines of vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$, which is showed on Figure 4:

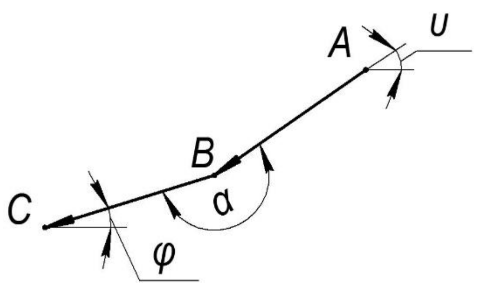

Figure 4. The directing cosines of vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$

For calculation of an angle α the equation is following:

$$\alpha = 180 - \varphi + \upsilon \tag{10}$$

For determine of an angles υ and φ equations is using [4]:

$$\upsilon = \arccos\left(\frac{x_2 - x_1}{|\overrightarrow{AB}|}\right) \tag{11}$$

$$\varphi = \arccos\left(\frac{x_3 - x_2}{|\overrightarrow{BC}|}\right) \tag{12}$$

So, necessary angles having defined from expressions (10) – (12) it is possible to know radius is convex or concave, it found on the equation (9).

4. Example

Let us consider example of calculation by the technique for flat radial transmission with intermediate rollers. We will use equations (4), (9) – (12) to determine the radius of curvature of cycloid profile for a transmission with the following initial parameters: r_2=22 mm – centrode radius; z_2=10 – number of rollers; r_{tk}=3 mm – roller radius; i=11 – transmission ratio.

Profile equation is taken from [1] and calculate a profile points by special program for Matlab-system.

After calculation we obtain the diagram:

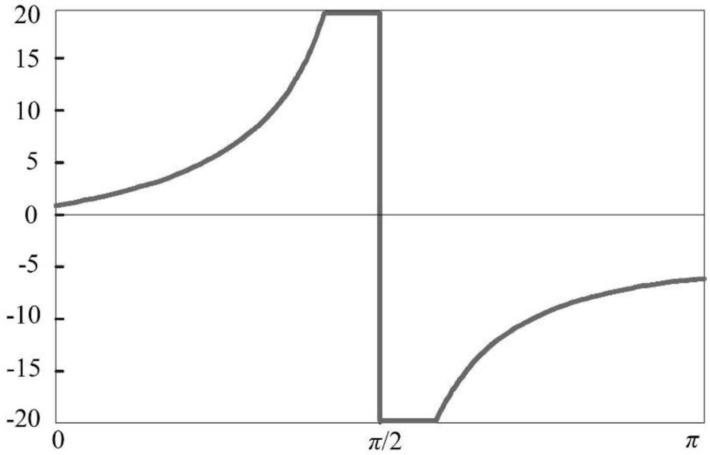

Figure 5. Dependence of curvature radius of a tooth profile on a rotation angle of an input link

On abscissa axis, (Figure 5) the angle of input link is presented. On ordinate axis, values of curvature radius are presented. On graphics the straight section is shown (near an angle π/2) for which points weren't set. The technique of dependence of the directing cosines and curvature radius was used [5] by preparation of the program for Matlab. On graphics, we can see in positive area radiuses of a convex part of the profile, and in negative area radiuses of a concave part of the profile.

Thus, this dependence can be used in definition of a sign of radius of curvature.

It does not matter to consider any position of the tooth profile in space, but it is better to start calculation from top point of profile and move along profile to tooth space.

Conclusions

So, we considered geometric determination of curvature radius for teeth with cycloid profile. The equation was obtained for the determination of a radius of curvature. The conditions of convexo–concavity of a teeth profile were defined for transmission with intermediate rollers.

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[5] Privalov I I 2003 Analytical geometry: textbook (SPb.: Lanj)