

Inductive voltage divider modeling in Matlab

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Abstract. Inductive voltage dividers have the most appropriate metrological characteristics on alternative current and are widely used for converting physical signals. The model of a double-decade inductive voltage divider was designed with the help of Matlab/Simulink. The first decade is an inductive voltage divider with balanced winding, the second decade is a single-stage inductive voltage divider. In the paper, a new transfer function algorithm was given. The study shows errors and differences that appeared between the third degree reduced model and a twenty degree unreduced model. The obtained results of amplitude error differ no more than by 7 % between the reduced and unreduced model.

1. Introduction

Inductive voltage dividers (IVD) are widely used in measurement of impedance [1], circuit parameters, gain or attenuation, non-electrical values, physical constant; in realization of the Farad from the DC Quantum Hall Effect [2]; in calibration of high-voltage transformers [3], amplifiers, voltmeters, ADC, DAC; in attenuation measurement systems in the HF and UHF range [4]; in a low-frequency AC power standard based on the programmable Josephson voltage standard [5,6]; in precision AC–DC transfer measurement systems [7,8]. Frequency range extension for IVD is a problem of the present-day interest, because IVD have the most appropriate metrological characteristics in the ultrasonic and sonic frequency ranges. The design concept based on a system approach, mathematical and physical modelling is a solution to frequency range extension for IVD. A perspective method to increase the upper frequency limit is a design of IVD with balanced winding (IVDBW) [9]. Divider concatenation allows extending the dynamic range. Calculation and analysis of these IVDs is complex because of the high order system of equations describing physical processes occurring in them.

In this regard, the complex and correct multistage IVDs design requires new mathematical modeling methods. The most suitable form of the mathematical model is a transfer function. The transfer function must have at most the third order for using in the analysis of the measurement and control systems having the IVD. In [10], V.L. Kim already mentioned the methods of IVD modeling. It is a laborious and time-consuming procedure to carry out experiments. We suggest an automatic and universal method of modelling multidecade IVD, and changes in Simulink models are not needed, only parameters in Matlab code should be involved.

2. Creating the double-decade Simulink model

We consider the methods of taking the double-decade IVD transfer function in Matlab with Simulink (Fig. 1). Figure 1 shows that the first decade is IVDBW (L1-L3) and the second is single-stage IVD (SSIVD) (L4). Both decades are placed on one magnetic core (TV1) [10]. The decade commutation is



performed by switches SW1, SW2.

Matlab Simulink is the suitable software for creating the IVD model. The software tools of Matlab allow obtaining the analytical formulas of the main dynamical characteristics of the electrical model.

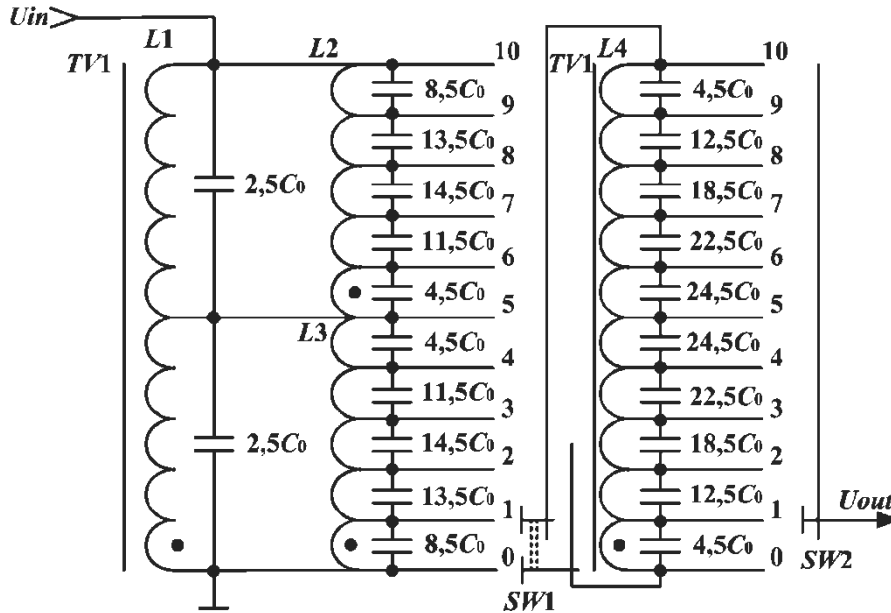


Figure 1. Schematic diagram of IVD with gain 0.01.

The *Simulink-model* “Fivd” of the IVDBW decade is given in Figure 2. The values of the input network elements are $R1 = R2 = 0.01$ Ohm; $R3 = 1.2$ Ohm; $R4 = 10$ kOhm; $L1 = L2 = 0.01$ μ H; $L3 = 11$ μ H; $L4 = 4$ H; $C1 = 500$ pF. The values of the output network elements are calculated from equation

$$r_{k,m} = \sum_{j=k+1}^m r_j, l_{sk,m} = \sum_{j=k+1}^m l_{sj}, c_{k,m} = (k-m)^{-2} \sum_{j=k+1}^m c_j,$$

where k, m ($k \in j, m \in j$) are winding taps, $r_j = 0.12$ Ohm; $l_{sj} = 1.1$ μ H; C_j are represented in Fig. 1, $C_0 = 300$ pF [10].

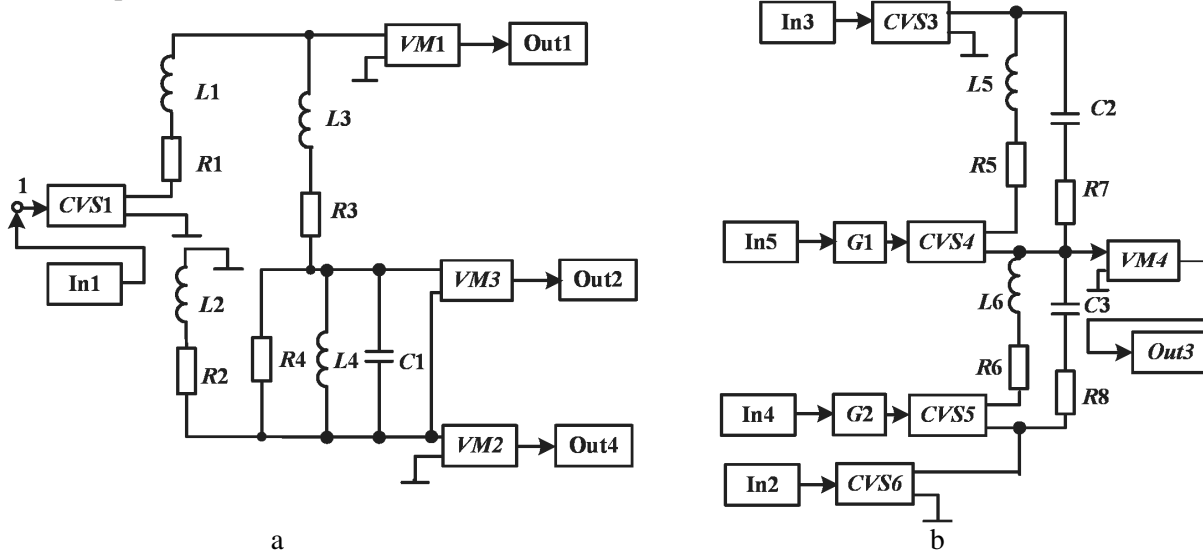


Figure 2. Input (a) and Output (b) parts of the first decade.

For gain $K = U_{out} / U_{in} = 0.1$, we have $R5 = 1.08 \text{ Ohm}$; $L5 = 9.9 \text{ }\mu\text{H}$; $C2 = 357 \text{ pF}$; $R6 = 0.12 \text{ Ohm}$; $L6 = 1.1 \text{ }\mu\text{H}$; $C3 = 2.55 \text{ nF}$; $R7 = R8 = 10^{-5} \text{ Ohm}$. Besides R, L, C elements, the *Simulink-model* has the following blocks: controlled voltage source ($CVS1, CVS3-CVS6$), voltage measurement ($VM1-VM4$), inport ($In1-In5$), outport ($Out1-Out4$), gain ($G1 = 0.9, G2 = 0.1$).

The gain error, calculated with this model and with formulas [10], differ from each other not more than by 7 % at 100 kHz work frequency.

The Simulink-model “Sivd” of the SSIVD decade is given in Figure 3. The values of the output network elements are $r0 = 0.15 \text{ Ohm}$; $ls0 = 0.1 \text{ }\mu\text{H}$; $C0 = 20 \text{ pF}$ from the table [10]. For example, for gain $K_j = 0.1$, we have $R9 = 0.208 \text{ Ohm}$; $L7 = 1.09 \text{ }\mu\text{H}$; $R10 = 1.35 \text{ Ohm}$; $L8 = 0.9 \text{ }\mu\text{H}$; $R11 = 0.15 \text{ Ohm}$; $L9 = L10 = 0.1 \text{ }\mu\text{H}$; $R12 = 0.1 \text{ Ohm}$; $R13 = R14 = 10^{-5} \text{ Ohm}$; $C4 = 39,6 \text{ pF}$; $C5 = 90 \text{ pF}$. Besides R, L, C elements, the Simulink-model have the following blocks: controlled voltage source ($CVS1-CVS4$), voltage measurement ($VM4$), inport ($In1-In4$), outport ($Out1$), gain ($G1=0.9, G2=0.1$). Elements $R7-R8, R13-R14$ are necessary to start the Simulink-model simulation. As the computer experiment shows, such small values of resistance do not influence the simulation results.

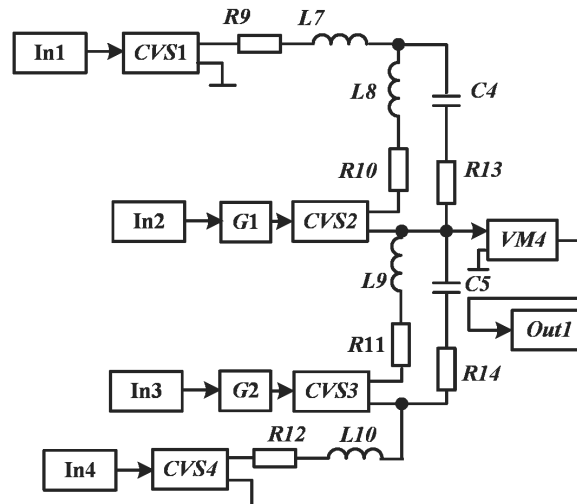


Figure 3. An output part of the second decade.

Figure 4 represents two models “FivdT” – one decade IVD, “SivdT” – two decade IVD. It can be used for higher decade IVDs and other types of decade placement on the cores, because it depends only on input state space models. In the model “FivdT” in blocks LTI System(tf(.)), the argument is a state space model: 1 – ssFivd(1,1); 2 – ssFivd(2,1); 3 – ssFivd (4,1); 4 – ssFivd (3,3); 5 – ssFivd (3,5); 6 – ssFivd(3,4); 7 – ssFivd (3,2). In the model “SivdT” in blocks LTI System(tf(.)), the argument is a state space model: 1 – ssFivdT13; 2 – ssFivd(2,1); 3 – ssFivd (4,1); 4 – ssFivd (1,1); 5 – ssSivd (1,2); 6 – ssSivd (1,3); 7 – ssSivd (1,4).

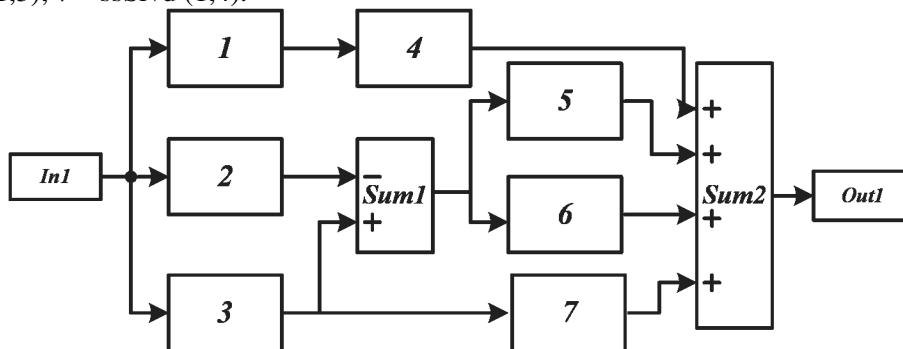


Figure 4. Structural diagram of IVD.

It is also necessary to set the parameters of simulation. In the menu *Simulation/Simulation parameters/Solvers*, the following data are set: *Start time* - 0.0; *Stop time* - 1.0; *Type* - *Variablestep, ode 23 tb [stiff/TR-BDF2]*. It is important to pay attention to the last option (the solution method of the differential equations). As the computer experiments have shown, only *ode 23 tb* allows simulating the design of *IVD Simulink-model*.

3. Transfer function calculation

The transfer function is taken with the *LTI-Viewer* of *Control System Toolbox*. It is necessary to do the following:

- Open the Simulink models: Fivd, FivdT, Sivd, SivdT.
- The *A, B, C, D* matrices of the state space equations from Simulink model may be represented in the following way:

```
[A,B,C,D] = linmod ('Fivd');
```

- Transform matrices to state space model:

```
ssFivd = ss (A,B,C,D);
```

- Get space model of the first decade from Simulink model by the previous way:

```
[A,B,C,D]=linmod ('FivdT');
```

```
ssFivdT =ss (A,B,C,D);
```

- Decrease the model order to 12. Get *g* vector of the resulting gramian:

```
[ssFivdTb,g] = balreal (minreal(tf(ssFivdT)));
```

- Now decrease the model order to 12, removing zero states of gramian:

```
ssFivdT13=modred(ssFivdT13b,13:length(g),'del');
```

- Get space model of the second decade and full IVD from Simulink models:

```
[A,B,C,D]=linmod ('Sivd');
```

```
ssSivd = ss (A,B,C,D);
```

```
[A,B,C,D]=linmod (SivdT');
```

```
ssSivdT =ss (A,B,C,D);
```

- Decrease the model order by deleting high degree until 9:

```
deg=9; inss= ssSivdT;
```

```
[num,den] = tfdata(inss);
```

```
for k = 1:(length(num{1,1}) - (deg+1))
```

```
num{1,1}(k) = 0;
```

```
end
```

```
for k = 1:(length(den{1,1}) - (deg+1))
```

```
den{1,1}(k) = 0;
```

```
end
```

```
outtf = tf(num,den);
```

- Decrease the model order to 3, removing zero states of gramian:

```
[outtfb,g] = balreal (outtf);
```

```
resss=modred(outtfb,4:deg,'del');
```

- Call *tf*-function

```
restf = tf(resss)
```

The transfer functions expression for the first tapping of the second decade appears in the command window:

$$T_{0.01} = \frac{0.01031 \cdot s^3 + 1465 \cdot s^2 + 3.355 \cdot 10^{11} \cdot s + 2.235 \cdot 10^{11}}{s^3 + 6.958 \cdot 10^4 \cdot s^2 + 3.355 \cdot 10^{13} \cdot s + 1.024 \cdot 10^{13}}$$

4. Results

The model error may be estimated with the real IVD gain-frequency characteristic and the IVD model gain-frequency characteristic. But this method is not available, because complex natural experiments must be done. The model accuracy was estimated comparing to the reduced and unreduced models. We attempted to model and describe double-decade IVD, where the first is IVDBW, the second decade is SSIVD. The experiment results are shown in table 1 (work frequency is 100 kHz).

The table shows that the third degree reduced models are acceptable. The model accuracy is enough for practical calculations. This *Simulink-model* allows studying frequency characteristics and calculations of the IVD amplitude error.

Table 1. Calculated gain of the model

Gain Kj	Full model	Reduced model	Relative error of reduced model, %
0.01	0.009994	0.0099955	0.02
0.02	0.0200009	0.0199866	0.07
0.03	0.0300078	0.0299778	0.01
0.04	0.0400148	0.0399689	0.12
0.05	0.0500218	0.0499601	0.12
0.06	0.0600288	0.0599505	0.13
0.07	0.0700358	0.0699426	0.13

5. Conclusion

The effectiveness of analysis and synthesis tasks of IVD depends on the accurate mathematical models. The transfer function is the primary IVD mathematical model, as it allows finding other dynamic characteristics. The suggested method of calculation transfer function in Matlab allows describing one-decade and multi-decade dividers, using the third order transfer functions with acceptable accuracy. It is suitable for functional modelling of measurement systems, having the IVDs, and for solving the task of divider response to external stimuli.

6. Acknowledgments

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References

- [1] Tsutsumi S 2012 Two-stage inductive voltage divider using special winding method *IEEE Transactions on Fundamentals and Materials* **132(10)** 924–9
- [2] Callegaro L, D’Elia V and Trinchera B 2010 Realization of the farad from the DC quantum hall effect with digitally assisted impedance bridges *Metrologia* **47** 464–72
- [3] Jones R G Calibration of high voltage transformers 1998 *CPEM Digest* 207–8
- [4] Iida H, Widarta A, Kawakami T and Komiyama K 2010 Attenuation standard in the frequency range of 5075 GHz *IEEE Transactions on Instrumentation and Measurement* **59(11)** 2921–9

- [5] Burroughs C J, Benz S P and Dresselhaus P D 2007 Development of a 60 Hz Power Standard Using SNS Programmable Josephson Voltage Standards *IEEE Trans. On Instrum. and Meas* **56(2)** 289–94
- [6] Hagen T, Budovsky I, Benz S P and Burroughs C J 2012 Calibration system for AC measurement standards using a pulse-driven Josephson voltage standard and an inductive voltage divider *CPEM* (Gaithersburg: National Institute of Standards and Technology) pp 672–3
- [7] Budovsky I and Hagen T 2009 Precision AC–DC Transfer Measurement System Based on a 1000-V Inductive Voltage Divider *IEEE Transactions on Instrumentation and Measurement* **58(4)** 844–7
- [8] Small G W, Budovsky I F, Gibbes A M and Fiander J R 2005 Precision three-stage 1000 V/50 Hz inductive voltage divider *IEEE Transactions on Instrumentation and Measurement* **54(2)** 600–3
- [9] Kim V L 2004 Calculation of the errors of an inductive voltage divider with a balancing winding *Measurement Techniques* **47(10)** 1005–9
- [10] Kim V L 2009 Means and methods of improving the accuracy of inductive voltage dividers (Tomsk: Tomsk Polytechnic University Press) pp 23–50