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Radiation of surface waves from a charge rotating around a dielectric cylinder

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Abstract: The radiation of electromagnetic surface waves is investigated for a charged particle rotating around a dielectric cylinder immersed into a homogeneous medium. Formulae are provided for the corresponding electric and magnetic fields and for the radiation intensity. It is shown that the surface waves are radiated on the eigenmodes of the cylindrical waveguide.

Keywords: Instrumentation for synchrotron radiation accelerators; Radiation calculations

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1 Introduction

The interfaces between two media serve as useful tool to control the electromagnetic fields, including the radiation ones. In addition, the presence of separating boundaries gives rise to new types of phenomena in classical electrodynamics. One of the interesting effects is the generation of surface electromagnetic waves propagating along interfaces (for a review see [1]). These waves play an important role in a large number of physical problems and have important applications, in particular, in diagnostics of interfaces, in measurements of various characteristics of materials (for example, the absorption coefficients), in wireless energy transfer.

The problem of the generation of surface electromagnetic waves is exactly solvable for highly symmetric interfaces only. Here we consider the problem of the radiation of surface waves by a point charge rotating around a dielectric cylinder along a circular trajectory. The radiation intensity at large distances from the cylinder is discussed in [2, 3]. In the absence of the cylinder the corresponding results are reduced to those for the synchrotron radiation in a homogeneous medium. The presence of the medium may essentially influence the spectral-angular distribution of the radiation intensity (see, for instance, [4–6] and references therein). It has been shown that, under the Cherenkov condition for the material of the cylinder and the charge velocity, in the corresponding angular distribution strong narrow peaks may appear. The angular density of the radiation intensity at those peaks exceeds the corresponding value in a homogeneous medium by several orders of magnitude. Similar features for a helical motion around a cylindrical waveguide are studied in [7]. In addition to the radiation at large distances from the cylinder, the radiation can be present confined inside the dielectric cylinder. The corresponding energy flux through the cross section of the cylinder has been studied in [8]. Similar investigations for a charge rotating inside a dielectric cylindrical waveguide are presented in [9–12]. In the present paper we consider the third type of radiation corresponding to surface waves propagating along the cylinder surface. As it will be shown below, this type of waves are emitted on the eigenmodes of the dielectric cylinder and exponentially decrease outside the cylinder.

The organization of the paper is as follows. In the next section we consider the electromagnetic fields outside a cylindrical dielectric waveguide. The electromagnetic fields for the surface waves and the corresponding radiation intensity are discussed in section 3. The main results are summarized in section 4.

2 Electromagnetic fields outside a cylinder

Consider a dielectric cylindrical waveguide of radius r_c having dielectric permittivity ε_0 and a point charge q rotating around the waveguide along the circle with radius r_q, $r_q > r_c$ (see figure 1). For generality, we assume that the system is immersed in a homogeneous medium with permittivity ε_1. In a cylindrical coordinate system (r, ϕ, z) with the z-axis along the waveguide axis, the charge coordinates are given by $(r_q, \omega_0 t, z = 0)$, where ω_0 is the angular velocity of the charge rotation. For the charge velocity one has $v = \omega_0 r_q$.

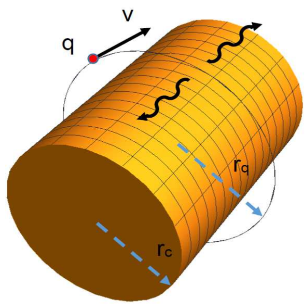

Figure 1. Geometry of the problem.

In accordance with the problem symmetry the strengths for the electric and magnetic fields, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$, are expanded as

$$\mathbf{F}(\mathbf{r}, t) = 2\mathrm{Re}\left[\sum_{n=0}^{\infty}{}' e^{in(\phi-\omega_0 t)} \int_{-\infty}^{\infty} dk_z e^{ik_z z} \mathbf{F}_n(k_z, r) \right], \tag{2.1}$$

where $\mathbf{F} = \mathbf{E}, \mathbf{H}$, and the prime means that the term $n = 0$ should be taken with the weight 1/2. The Fourier components for the fields in the region $r > r_c$ can be decomposed as

$$\mathbf{F}_n(k_z, r) = \mathbf{F}_n^{(0)}(k_z, r) + \mathbf{F}_n^{(c)}(k_z, r) \tag{2.2}$$

where $\mathbf{F}_n^{(0)}(k_z, r)$ are the fields in a homogeneous medium with permittivity ε_1 in the absence of the waveguide and the part $\mathbf{F}_n^{(c)}(k_z, r)$ is induced by the waveguide. In cylindrical coordinate system the components for the magnetic field are given by (for simplicity, in what follows we will omit the arguments for the Fourier components)

$$H_{nl}^{(0)} = \frac{qvk_z i^{2-\sigma_l}}{2\pi c} \sum_{p=\pm 1} p^{\sigma_l - 1} J_{n+p}(\lambda_1 r_<) H_{n+p}(\lambda_1 r_>), \; l = r, \phi, \tag{2.3}$$

and $H_{nz}^{(0)} = 0$ with $\sigma_r = 1$, $\sigma_\phi = 2$, and $\lambda_j^2 = \omega_n^2 \varepsilon_j / c^2 - k_z^2$ with $\omega_n = n\omega_0$, $j = 0, 1$. In (2.3), $J_\nu(x)$ is the Bessel function and $H_\nu(x) \equiv H_\nu^{(1)}(x)$ is the Hankel function of the first kind, $r_< = \min(r_e, r)$, $r_> = \max(r_e, r)$.

By using the electromagnetic field Green tensor from [2], for the part induced by the waveguide in the region $r > r_c$ one gets (see [7] for a helical motion)

$$H_{nl}^{(c)} = \frac{qvk_z i^{2-\sigma_l}}{2\pi c} \sum_{p=\pm 1} p^{\sigma_l - 1} B_n^{(p)} H_{n+p}(\lambda_1 r), \; H_{nz}^{(c)} = -\frac{qv\lambda_1}{2\pi c} \sum_{p=\pm 1} p B_n^{(p)} H_n(\lambda_1 r), \tag{2.4}$$

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with $l = r, \phi$, and

$$B_n^{(p)} = -\frac{\pi}{2i}\frac{V_{n+p}^J}{V_{n+p}^H}H_{n+p}(\lambda_1 r_c) + \frac{p\lambda_0 J_{n+p}(\lambda_0 r_c)J_n(\lambda_0 r_c)}{2r_c\alpha_n(k_z)V_{n+p}^H}\sum_{l=\pm1}\frac{H_{n+l}(\lambda_1 r_q)}{V_{n+l}^H}. \tag{2.5}$$

Here we have introduced the notations

$$\alpha_n(k_z) = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{\lambda_0}{2}J_n(\lambda_0 r_c)\sum_{l=\pm1} l\frac{H_{n+l}(\lambda_1 r_c)}{V_{n+l}^H}, \tag{2.6}$$

and $V_n^F = \lambda_1 J_n(\lambda_0 r_c)F_n'(\lambda_1 r_c) - \lambda_0 F_n(\lambda_1 r_c)J_n'(\lambda_0 r_c)$, with $F = J, H$. The Fourier components for the electric field are found by using the equation $\mathbf{E} = ic\nabla \times \mathbf{H}/(\omega\varepsilon_0)$.

3 Radiation fields and the intensity for surface waves

The formulae given above describe two types of the radiation. The first one corresponds to the radiation propagating at large distances from the waveguide. For this radiation the quantity λ_1 is real and the corresponding spectral-angular distribution has been investigated in [3, 7]. Here we are interested in the second type of the radiation in the region $r > r_c$ that corresponds to purely imaginary values of λ_1. The corresponding fields exponentially decay in the surrounding medium.

In order to find the parts in the fields corresponding to the radiation with imaginary λ_1, let us consider the limit $z \to \infty$ in the Fourier expansion (2.1) for a fixed value of the radial coordinate r. The phase of the exponent in the integrand has no stationary points. For regular functions $\mathbf{F}_n(k_z, r)$ the corresponding integral will decay exponentially for large z. From here it follows that the radiation will be conditioned by the possible singular points of the integrand. The parts $\mathbf{F}_n^{(0)}(k_z, r)$ in the fields are regular and they do not contribute to the radiation in the region under consideration. The singularities in the waveguide-induced fields $\mathbf{F}_n^{(c)}(k_z, r)$ correspond to the zeros of the function $\alpha_n(k_z)$ in (2.6). They are simple poles and correspond to the eigenmodes of the cylindrical waveguide (see [13]).

For the further consideration it is convenient to write the function $\alpha_n(k_z)$ in the form

$$\alpha_n(k_z) = U_n(k_z)/\left[(\varepsilon_1 - \varepsilon_0)\left(V_n^2 - n^2u^2\right)\right], \tag{3.1}$$

with the notations $V_n = |\lambda_1|r_c J_n'/J_n + \lambda_0 r_c K_n'/K_n$, $u = \lambda_0/|\lambda_1| + |\lambda_1|/\lambda_0$, and

$$U_n = V_n\left(\varepsilon_0|\lambda_1|r_c J_n'/J_n + \varepsilon_1\lambda_0 r_c K_n'/K_n\right) - n^2(\lambda_0^2 + |\lambda_1|^2)\left(\varepsilon_1\lambda_0^2 + \varepsilon_0|\lambda_1|^2\right)\lambda_0^{-2}|\lambda_1|^{-2}, \tag{3.2}$$

where $J_n = J_n(\lambda_0 r_c)$, $K_n = K_n(|\lambda_1|r_c)$, the prime means the differentiation with respect to the argument of the function and $K_\nu(x)$ is the Macdonald function. The equation $U_n = 0$ determines the eigenmodes of the dielectric waveguide. Let $\pm k_{n,s}$, $k_{n,s} > 0$, $s = 1, 2, \ldots$, be the solutions of this equation with respect to k_z for $n \neq 0$. For these solutions we will introduce the notations

$$\lambda_0 r_c = \lambda_{n,s} \equiv r_c\sqrt{\omega_n^2\varepsilon_0/c^2 - k_{n,s}^2},\ |\lambda_1|r_c = \lambda_{n,s}^{(1)} \equiv r_c\sqrt{k_{n,s}^2 - \varepsilon_1\omega_n^2/c^2}. \tag{3.3}$$

For the allowed values of $k_{n,s}$ one has $\omega_n\sqrt{\varepsilon_1}/c \leqslant k_{n,s} \leqslant \omega_n\sqrt{\varepsilon_0}/c$. As a necessary condition for the existence of the eigenmodes we have $\varepsilon_0 > \varepsilon_1$. In the notations (3.3), in (3.2) one has $J_n = J_n(\lambda_{n,s})$, $K_n = K_n(\lambda_{n,s}^{(1)})$.

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For the evaluation of the radiation parts in the fields we need to specify the contour for the integral over k_z in (2.1). This is done by taking into account that in physically realistic situations the dielectric permittivity ε_0 is a complex quantity, $\varepsilon_0 = \varepsilon_0' + i\varepsilon_0''$, with $\varepsilon_0''(n\omega_0) > 0$ for $n > 0$. On the base of this it can be seen that (for details see, for example, [12]) in the integral over k_z the contour avoids the poles $k_z = k_{n,s}$ ($k_z = -k_{n,s}$) from below (above). The condition for $\lambda_{n,s}$ to be real defines the maximum value for s, that will be denoted by s_n. It is determined from the condition $k_{n,s_n} < \omega_n r_c\sqrt{\varepsilon_0}/c < k_{n,s_n+1}$.

With the contour of the integration specified above, the radiation fields are obtained with the help of the residue theorem:

$$\mathbf{F}^{(r)}(\mathbf{r},t) = 4\pi\mathrm{Re}\left\{ i\sum_{n=1}^{\infty} e^{in(\phi-\omega_0 t)}\sum_{s=1}^{s_n}\mathrm{Res}\left[e^{ik_z z}\mathbf{F}_n(k_z,r)\right]\right\}. \tag{3.4}$$

where superscript (r) stands for the radiation parts of the fields. Evaluating the residue, the radiation fields are presented in the form

$$\mathbf{F}^{(r)}(\mathbf{r},t) = \frac{qv}{c}\sum_{n=1}^{\infty}\sum_{s=1}^{s_n}\frac{\mathbf{F}_{n,s}(r)}{\alpha_n'(k_{n,s})}R\left[n(\phi-\omega_0 t)+k_{n,s}z\right], \tag{3.5}$$

where $R(x) = \sin x$ for the components $E_r^{(r)}$, $E_z^{(r)}$, $H_\phi^{(r)}$, and $R(x) = \cos x$ for the components $E_\phi^{(r)}$, $H_r^{(r)}$, $H_z^{(r)}$. For the magnetic field, the components of the vector in the right-hand side of (3.5) are given by the expressions

$$H_{n,s}^l(r) = -k_{n,s}\sum_{p=\pm1}p^{2-\sigma_l}B_{n,s}^{(p)}K_{n+p}(\lambda_{n,s}^{(1)}r/r_c),\ H_{n,s}^z(r) = \frac{\lambda_{n,s}^{(1)}}{r_c}\sum_{p=\pm1}B_{n,s}^{(p)}K_n(\lambda_{n,s}^{(1)}r/r_c), \tag{3.6}$$

where $l = r, \phi$, and

$$B_{n,s}^{(p)} = \frac{\lambda_{n,s}J_{n+p}/J_n}{(V_n - pnu)\,K_n^2}\sum_{l=\pm1}\frac{lK_{n+l}(\lambda_{n,s}^{(1)}r_q/r_c)}{V_n - lnu}. \tag{3.7}$$

For the electric field we get

$$\begin{aligned} E_{n,s}^l(r) &= -\frac{(-1)^{\sigma_l}c}{2\varepsilon_1\omega_n}\sum_{p=\pm1}p^{\sigma_l}\sum_{j=\pm1}\left(\omega_n^2\varepsilon_1/c^2 + jk_z^2\right)B_{n,s}^{(jp)}K_{n+p}(\lambda_{n,s}^{(1)}r/r_c), \\ E_{n,s}^z(r) &= -\frac{c\lambda_{n,s}^{(1)}k_{n,s}}{\varepsilon_1\omega_n r_c}\sum_{p=\pm1}pB_{n,s}^{(p)}K_n(\lambda_{n,s}^{(1)}r/r_c). \end{aligned} \tag{3.8}$$

Note that in (3.5) one can write $\alpha_n'(k_{n,s}) = U_n'(k_{n,s})/\left[(\varepsilon_1-\varepsilon_0)\left(V_n^2 - n^2u^2\right)\right]$. The term with a given n describes the radiation with the frequency ω_n. The corresponding fields are suppressed by the factor $e^{-\lambda_{n,s}^{(1)}r/r_c}$ at distances $r \gg r_c/\lambda_{n,s}^{(1)}$. This allows us to interpret the waves under consideration as surface electromagnetic waves in the exterior region. There is also radiation propagating inside the waveguide. The latter is investigated in [14].

Having the radiation fields we can evaluate the radiation intensity for the surface waves. The corresponding energy flux through the plane z = const perpendicular to the axis of the cylinder is given by the expression

$$I^{(s)} = \frac{c}{4\pi} \int_{r_c}^{\infty} dr \int_{0}^{2\pi} d\phi\, r \left(E_r^{(r)} H_\phi^{(r)} - E_\phi^{(r)} H_r^{(r)}\right). \tag{3.9}$$

Substituting the expressions (3.5) for the radiation fields we get $I^{(s)} = \sum_{n=1}^{\infty} I_n^{(s)}$, where the radiation intensity on a fixed harmonic n is given by

$$I_n^{(s)} = \frac{q^2 v^2}{8\varepsilon_1} \sum_{s=1}^{s_n} \frac{k_{n,s}\lambda_{n,s}^2 K_n^{-4}}{\omega_n \alpha_n'^2(k_{n,s}) J_n^2} \left(\sum_{l=\pm 1} \frac{l K_{n+l}(\lambda_{n,s}^{(1)} r_q/r_c)}{V_n - lnu}\right)^2 \sum_{p=\pm 1} \frac{J_{n+p}}{V_n - pnu}$$
$$\times \left[\left(\frac{\omega_n^2}{c^2}\varepsilon_1 + k_{n,s}^2\right) \frac{r_c^2 J_{n+p}}{V_n - pnu} - \frac{\lambda_{n,s}^{(1)2} J_{n-p}}{V_n + pnu}\right] \left[K_{n+p}'^2 - \left(1 + \frac{(n+p)^2}{\lambda_{n,s}^{(1)2}}\right) K_{n+p}^2\right]. \tag{3.10}$$

Note that for a given angular velocity of the charge, ω_0, the orbit radius appears in the argument of the function $K_{n+l}(\lambda_{n,s}^{(1)} r_q/r_c)$ and in the coefficient through v^2. For large values of r_q, $\lambda_{n,s}^{(1)} r_q/r_c \gg 1$, the intensity of surface waves is suppressed by the factor $\exp[-2\lambda_{n,s}^{(1)} r_q/r_c]$.

For the number of the radiated quanta at a given harmonic n, per period of the charge rotation, one has $N_n^{(s)} = T I_n^{(s)}/(\hbar\omega_n)$, where $T = 2\pi/\omega_0$. In figure 2, we display the dependence of $N_n^{(s)}$ as a function of the harmonic for the electron energy $E_e = 2\,\mathrm{MeV}$ and for the values of the parameters $\varepsilon_1 = 1$, $\varepsilon_0 = 3.74$ (dielectric permittivity for quartz), $r_c/r_q = 0.99$. For these values of the parameters one has $s_n = 1$ for $1 \leqslant n \leqslant 7$, $s_n = 2$ for $8 \leqslant n \leqslant 12$, and $s_n = 3$ for $13 \leqslant n \leqslant 16$. For the radius of the electron orbit of the order 1 cm and for harmonics $n \sim 10^2$ the corresponding radiation is in the terahertz range. If the circular motion is generated by an external magnetic field H_{ext} then for relativistic electrons $r_q \approx 1.7 \times 10^3 E_e/(m_e c^2 H_{\mathrm{ext}})$ cm, where it is assumed that H_{ext} is measured in Gausses. Note that for neodymium magnets H_{ext} can be of the order 10^4 G.

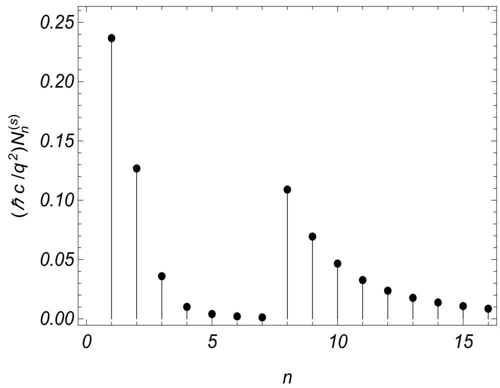

Figure 2. The number of the radiated quanta per period of the rotation on a given mode n for different values of n and for the values of the parameters $E_e = 2\,\mathrm{MeV}$, $\varepsilon_1 = 1$, $\varepsilon_0 = 3.74$, $r_c/r_q = 0.99$.

4 Conclusion

We have investigated the electromagnetic fields and radiation intensity for surface waves emitted by a point charge rotating along a circular trajectory around a dielectric cylinder immersed into a homogeneous medium. These waves are radiated on the eigenmodes of the dielectric cylinder and exponentially decrease in the exterior medium. The radiation fields are expanded as (3.5) where the components of the vector $\mathbf{F}_{n,s}(r)$ for the magnetic and electric fields are given by the expressions (3.6) and (3.8). The energy flux for the surface waves through the plane perpendicular to the cylinder axis is given by the expression (3.10). Note that, in the problem under consideration we have also the radiation at large distances from the cylinder and the radiation propagating inside the cylinder. The geometry considered here is of interest from the point of view of generation and transmitting of waves in waveguides, a subject which is of considerable practical importance in microwave engineering and optical fiber communications.

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