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Research paper

Comparison of non-linear, linearized 2nd order and reduced to FOPDT models of CSTR using different tuning methods

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Abstract

Process modelling and design of controller based on the process model is an important step in the process control. In the present study three different mathematical models i.e. non-linear process model, linearized 2nd order model and first order with dead time (FOPDT) model of a CSTR with the concentration of output of product as controlled parameter were developed. Proportional Integral (PI) controllers were designed based on 2nd order and FOPDT models of a CSTR using SIMC (Skogestad internal model control), Hagglund and Astrom, and a computational method with 5% overshoot.

In all the three tuning methods, the nonlinear model provided better results in terms of various time parameters (T_r, T_y, T_s) and in error analysis (IAE, ITAE and ISE).

Keywords: PI controller; FOPDT; SIMC; Hagglund and Astrom

1. Introduction

Industrial separation processes are very important and integral parts of Chemical Industries in which either two or more than two products are separated or impurities are removed from the products. Efficiency and cost of the processes are recent challenges in the separation process. The researchers are working to increase the efficiency as well as lower the cost either by using efficient and effective control methodologies [1–4] or by improving the separation techniques using novel methods such as bio-sorption onto microwave [4], microwave assisted extraction of bioactive compounds [1–3], novel adsorption techniques [1–3,5] and process optimization [1–3].

Continuously stirred tank reactor (CSTR) is an important part of many chemical industries and good control of CSTR plays very important role in the quality of final product. The material balance and chemical equilibria equations provide a highly nonlinear dynamic model of this system which makes it as one of the popular non-linear systems for control studies. Due to nonlinear dynamics and complex behaviour, designing a suitable controller for such CSTR systems is somewhat difficult and need comprehensive effort [6].

The present work is focus on development of efficient and simple control strategy for a non-linear process such as continuously stirred tank reactor (CSTR) which will also be useful for deciding the good control strategy for non-linear separation processes and ultimately resulted in efficient separation and reduction in the cost.

Due to simple configuration and easy implementation, the proportional integral (PI) or proportional integral derivative (PID) controller is still significant and popular among all control loops in process or chemical industries [7]. In the PID controller, the proportional action reduces the maximum amount of error by varying the manipulated variable according to the error signal obtained, the steady-state error or offset is removed by the integral action and this is proportional to the integral of the error signal while the derivative action provides a signal proportional to the derivative of error, and its function is to reduce maximum overshoot. Mathematically, the output from a PID controller is given as:

$$u(t) = k_c\left(e(t) + \frac{1}{\tau_I}\int e(t)\,dt + \tau_D\frac{de(t)}{dt}\right) \quad (1)$$

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Where u(t) is the output control signal, e(t) the error signal defined as the difference between the set-point and the output. k_c = proportional gain, τ_I = integral time and τ_D = derivative time.

Simplicity and optimality are most important aspects of any controller tuning technique and keeping these two important aspects in the mind, a large number of PI/PID tuning rules have been proposed by various researchers in literature. The initial efforts were made by Ziegler and Nichols [8] and Cohen and Coon [9]. They proposed very simple tuning rules and are still the most common and popular for tuning of different processes. These techniques provide quick and effective tuning of the simple process but fail in the case of highly non-linear and complicated processes. The Ziegler–Nichols settings result in a very good disturbance response for integrating processes, but provide poor performance for processes with a dominant delay [7]. A dominant pole placement technique was used by Cohen & Coon in which they fixed three dominant poles, a pair of complex poles and a real pole such that the amplitude decay ratio for load disturbance response is 0.25 and the integrated error $\int_0^\infty e(t)dt$ is minimized. This technique provides good load disturbance rejection and controller-robust PID parameters in the sense of the parametric stability margin when the plant under study satisfies the condition $0 < \theta/\tau < 8.53$. Tyreus and Luyben [10] developed PI controller tuning formula based on the process reaction curve and frequency domain ultimate values which provided better results for processes with a low θ/τ ratio.

If a reasonably accurate dynamics model of the process is available, it is advantageous to use the model-based design techniques for designing of PI/PID controllers because design/tuning parameters can be obtained and response of the process for the different type of disturbances can be calculated without operating the actual process. The controller tuning based on model-based design techniques such as Direct Synthesis [11] and the IMC-PID tuning method of [12] provided very good results for set-point changes. But in the case of input (load) disturbances for lag-dominant (including integrating) processes with τ/θ larger than about 10 gave sluggish response. Astrom and Hagglund [13] developed PI controller tuning relations that maximize performance subject to a constraint on the degree of robustness. Skogestad [14] provides model reduction techniques and proposed a simple analytic tuning rule (SIMC) for PID controller which provided the better result in disturbance rejection. Lee et al. [15] recently proposed a K-SIMC method which includes modification of model reduction techniques and suggestions of new tuning rules and set point filters provided better results for load disturbance rejection. Kumar et al. [16] design the controllers using Ziegler–Nichols (ZN) and relay auto (RA) tuning methods compared the performance of different control schemes like feedback, feedforward, feedback plus feedforward and cascade control for a third order process. The RA method gives better results than ZN tuning method in various time performance.

Although non-linear models of any real system are closer to the real system yet in the process control mostly linear equivalent models of non-nonlinear systems are used for close loop performance studies. The linear equivalent of non-linear systems was taken due to their simplicity and ability to convert into the form of transfer function using Laplace Transform. Transfer function form of the model is very simple and extremely useful from control applications point of view. However by linearization, the model behaviour may deviate significantly from real-time behaviour as compared to a non-linear model which is closer to the real system. Keeping the above points in mind the objective of the present study is to design the controller for a CSTR which is a non-linear system using available controller design techniques and compare the performance of designed controller in feedback mode on linear and non-linear models which is closer to the real behaviour. Three different process models of CSTR i.e. non-linear, 2nd order linear and FOPDT were taken for control study with PI controller. The PI controller was tuned using SIMC method proposed by Skogestad [14], Astrom and Hagglund [13] and a computational optimization approach with 5% overshoot criteria and output concentration of CSTR is compared to load as well as set point changes.

2. Process description, modelling and designing of controller

Fig. 1 shows a CSTR in which first-order chemical reaction $A \rightarrow B$ is occurring.

The mathematical model of the above process is given by Roffel and Betlem [17]. The mass balance for component A can be given as:

$$V\frac{dC_A}{dt} = F(C_{Ain} - C_A) - Vke^{-E/RT}C_A \tag{2}$$

and the energy balance is

$$\rho V c_p \frac{dT}{dt} = F\rho c_p(T_{in} - T) + Vke^{-E/RT}C_A\Delta H + Q \tag{3}$$

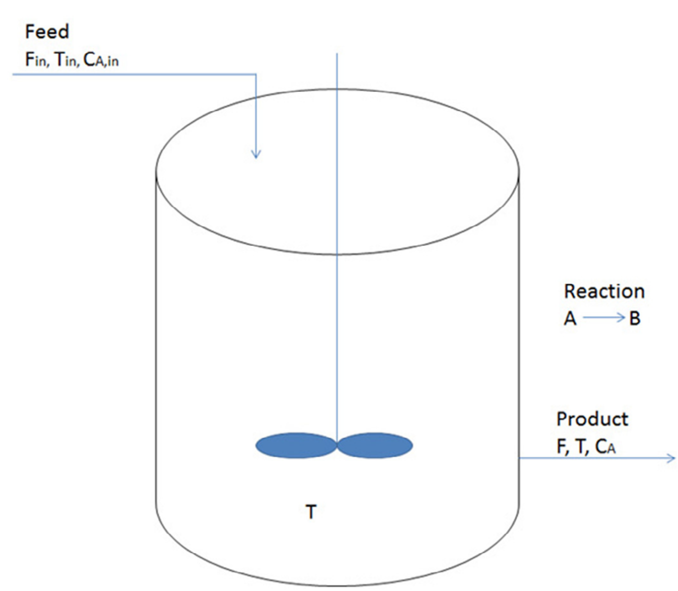

Fig. 1. Chemical reactor with first-order chemical reaction.

Table 1
The Steady-state parameter of CSTR [17].

Parameter	Value
Reactor volume, V	$5m^3$
Outlet concentration of component A, C_A	200.13 kg/m^3
Inlet concentration of component A, C_{Ain}	800 kg/m^3
Total volumetric flow, F	$0.005m^3/s$
Pre-exponential constant, k	18.75 s^{-1}
Activation energy for the reaction, E	30 kJ/mol
Reactor temperature, T	413 K
Temperature of inlet flow, Tin	353
Density, ρ	800 kg/m^3
Specific heat, c_p	1.0 KJ/kg.K
Heat of reaction (exothermic), ΔH	5.3 KJ/kg
Heat supplied to the reactor, Q	224.1 kJ/sec
Gas constant, R	0.0083 kJ/mol.k

By using Taylor's series expansion, the non-linear equations (1) and (2) are linearized around steady state values and the linear equivalent of a non-linear model as given below is obtained using the parameters give in Table 1.

$$\frac{C_A(s)}{F(s)} = 6.69\times 10^4 \frac{457.5s+1}{2.55\times 10^5 s^2 + 1255.5s+1} \tag{4}$$

The second order linear model (equation 3) is again simplified to first order plus dead time (FOPDT) model using its dynamic open loop response for a step change of 5% in the reactor input flow rate F. The response is generated using SIMULINK and FOPDT parameters were calculated to develop the FOPDT model as shown below.

$$\frac{C_A(S)}{F(S)} = \frac{60000}{706s+1}e^{-1s} \tag{5}$$

The controller parameters were obtained using 2nd order linear model (eq. 3) and FOPDT models of the system (eq. 4). SIMC and Astrom and Hagglund [13] methods for FOPDT model and computational method with 5% overshoot criteria for 2nd order linear system were used to calculate the PI parameters (Table 2).

3. Simulation results

SIMULINK based closed loop feedback diagram of CSTR used to get the response for load and setpoint change are shown in Fig. 2. The PI parameters obtained in the previous section (Table 2) were used to control the output concentration of reactant A in the CSTR in close loop feedback mode using three different process models (Nonlinear, 2nd order linear and FOPDT models) for change in load as well as setpoint and results are shown in Figs 3, 4 and 5. The response in terms of speed and time to reach final steady state was found best in the case of nonlinear model followed by a 2nd order linear and FOPDT models. Table 3 shows the comparative analysis in terms of different performance parameters such as rise time (T_r), settling time (T_s) and maximum overshoot Y_p and the simulation results show that the nonlinear model has better

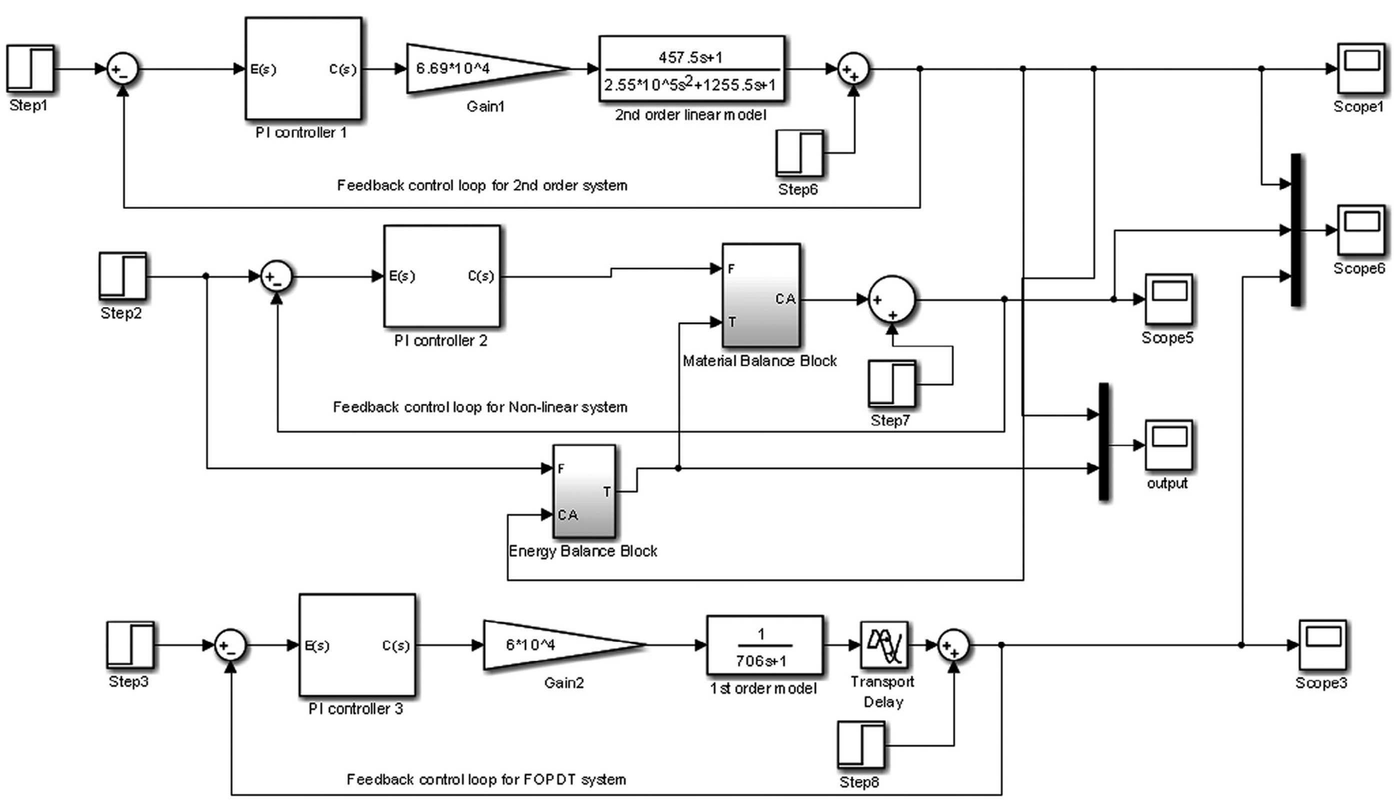

Fig. 2. Simulink model of different process models.

Table 2
Different tuning technique of PI controller and their Parameters.

Process	Tuning methods	Kc	$\tau_I(s)$	$K_I = \frac{Kc}{\tau_I}$
FOPDT $G(s) = k\frac{e^{-\theta s}}{\tau s + 1}$	SIMC (2003)	$\frac{1}{k}\frac{\tau}{\tau_c + \theta}$	$min\{\tau_I, 4(\tau_c + \theta)\}$	
		0.00588	8	0.00073
	Astrom and Hagglund [13]	$\frac{0.14}{k} + \frac{0.28\tau}{\theta k}$	$0.33 + \frac{6.8\theta\tau}{10\theta + \tau}$	
		0.0033	7.035	0.00047
$\frac{C_A(s)}{F(s)} = 6.6910^4 \times \frac{457.5s + 1}{2.55 \times 10^5 s^2 + 1255.5s + 1}$	Computational	0.009		0.0007

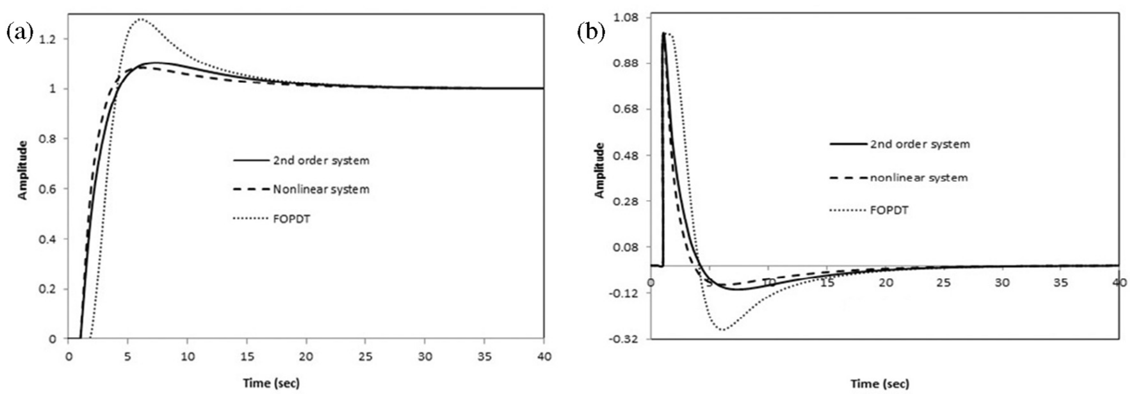

Fig. 3. Unit step response using SIMC tuning method. (a) Servo problem. (b) Regulatory problem.

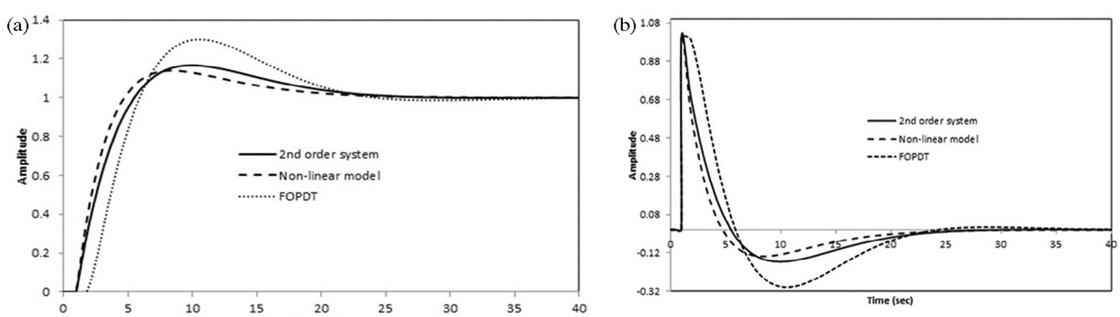

Fig. 4. Unit step response using Astrom and Hagglund [13] tuning method. (a) Servo problem. (b) Regulatory problem.

performances in terms of T_r, T_s and Y_p. Furthermore, a comparative analysis has also been made in terms of time integral error indices such as IAE, ISE and ITAE and the results are presented in Table 4. The time integral error indices IAE, ISE, and ITAE are minimum for the nonlinear system in all the three tuning techniques. Generally, ISE is used for a response that has large errors and continues for a long time because the square of error. However, ITAE reduces response that has error persist for a long time and IAE is not important for large error.

Tables 3 and 4 also show that the SIMC provided better results in the terms of T_r, T_s, Y_p and integral errors as compared to other two tuning techniques.

4. Conclusions

The nonlinear model has better results in the terms of performance parameter T_r, T_s and Y_p also in terms of performance error indices IAE ISE and ITAE as compared to the 2nd order linear and FOPDT models. Among all the tuning techniques used to design controller, the SIMC provided better values of PI parameters. The controller design based on FOPDT and 2nd

Table 3
Quantitative analysis between different process models.

Tuning method	Models	T_r (s)	T_s (s)	Y_p (%)
SIMC	Linear	4.19	14.75	10
	Nonlinear	3.62	11.55	8
	FOPDT	4.07	15.5	28
Astrom & Hagglund [13]	Linear	5.50	19	16
	Nonlinear	4.72	16	14
	FOPDT	5.91	21	30
Computational	Linear	4.02	8	5
	Nonlinear	4.02	12	8
	FOPDT	3.5	13	35

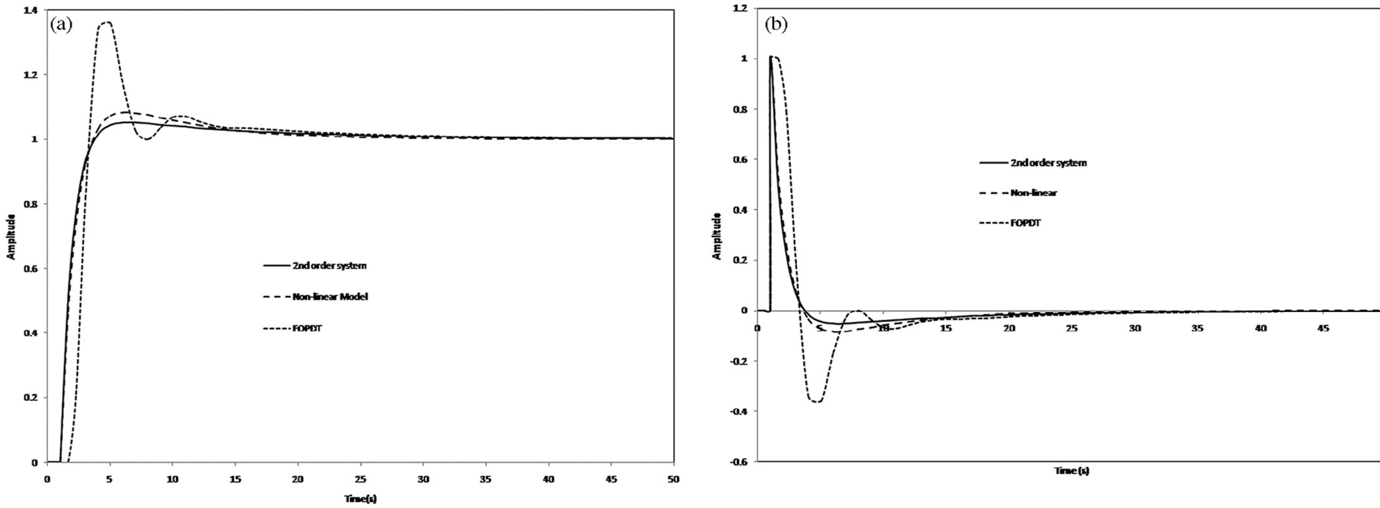

Fig. 5. Unit step response using Computational tuning method. (a) Servo problem. (b) Regulatory problem.

Table 4
Time integral performance indices comparison with different process models.

Tuning method	Models	IAE	ISE	ITAE
SIMC	Linear	2.37	0.78	16.77
	Nonlinear	1.85	0.58	12.26
	FOPDT	4.27	2.13	26.29
Astrom & Hagglund (2001)	Linear	3.47	1.25	26.24
	Nonlinear	2.72	0.94	18.66
	FOPDT	5.73	2.77	46.77
Computational	Linear	1.56	0.46	13.1
	Nonlinear	1.714	0.53	11.33
	FOPDT	3.22	1.68	21.28

order linear system also work well on non linear model of the process which is closer to the real system.

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