4. Marchetti M., Testa P. and Torrisi F.R., Measurement of Thermal Conductivity and Thermal Contact Resistance in Composite Materials for Space Applications, ESA Journal 1988 Vol 12, Carbon Composite Material, ESA/ESTEC Document No CCM-HTS-SR-001, Coswig, 2001.
5. Componeering Inc.Accessmode:http://www.esacomp.com/componeering/activities,free.

Comparison of Attitude Determination Methods for CubeSat

Matej Straka
Supervisor: Tamara G. Nesterenko, Tomsk Polytechnic University, Associate Professor
Jaroslav Laifr, Czech Technical University, Ing.
Czech Technical University
E-mail: strakmat@fel.cvut.cz

Introduction

Presented satellite is a 1U CubeSat, with dimensions of 10x10x10cm and mass less than 1kg, which is being developed at Czech Technical University (CTU) in Prague, Czech Republic. The main goals of this project are to build and fly the first Czech Republic's CubeSat, and to build reliable CubeSat electronics and mechanics. This is also motivated by the fact, that only approximately 55% CubeSat missions of all were operational [6]. The flight is planned in the first half of 2016 and it should fly in a sun synchronous orbit at an altitude of 600km and an inclination of 98°.

The Attitude Determination System takes measurements from the magnetometer, gyroscopes and sun sensors and process them to get an attitude quaternion which represents an orientation of the satellite. The results will be later used in attitude control. There are many algorithms that can be used for determining the attitude. One of the mostly used is Extended Kalman filter(EKF). Because our aim is to build systems reliable in outer space, the microcontroller chosen for this system was 8-bit one from Silabs, which is a commercial off-the-shelf component and despite having low computation power, is flight proven, and the risk, that it will fail during the mission, is much lower. Although this microcontroller can run on 100MHz, it's power consumption is in this case too high. Thus, also an another algorithm, QUEST (QUaternion ESTimation), was tested, which is less effective than EKF, but has lower power consumption.

Attitude representation

The attitude of the satellite can be expressed as a rotation of a satellite body coordinate frame (body frame) in an outer reference inertial coordinate frame (reference frame). There are several different ways how to express the attitude. Each of them express the rotation differently, and has its advantages and disadvantages. For Kalman filter, which works with dynamical model of the system, it is useful to work with the quaternion attitude representation, where the system model, although it is non-linear, doesn't have any non-linear functions inside, and it doesn't have any singularities. Quaternion can be defined using the axis-angle attitude representation as:

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{q}} \\ q_4 \end{bmatrix} = \begin{bmatrix} \mathbf{e} \sin \frac{1}{2}\theta \\ \cos \frac{1}{2}\theta \end{bmatrix}, \quad (1)$$

where $\mathbf{e}$ is the axis direction vector and θ is the angle of the rotation about the axis. Dynamic model of the satellite rotation with quaternion attitude representation is [3]

$$\begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\boldsymbol{\Omega}\mathbf{q} \\ -\mathbf{J}^{-1}\left(\boldsymbol{\omega} \times (\mathbf{J}\dot{\boldsymbol{\omega}})\right) \end{bmatrix}, \quad (2)$$

where $\boldsymbol{\omega}$ is the angular velocity, $\mathbf{J}$ is the inertia tensor, and

$$\mathbf{\Omega} = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \qquad (3)$$

Methods of attitude determination

Two attitude determination methods are presented in this work. The well-known Extended Kalman filter and the lighter QUEST. If the both algorithms are compared, the biggest advantage of the EKF is, that if one of the sensors is unavailable for a short period of time, the algorithm won't fail and will provide us with results, because it works with dynamic model and has a prediction part. QUEST on the other hand is static, uses only direction vector measurements - magnetic induction vector and sun position vector, and in case of temporary unavailability of one of the sensors, it won't give any results. However, it is less power consuming.

Extended Kalman filter

Kalman filter was introduced in 1960 by R. E. Kalman and is an powerful linear state estimator.[1] A variant of it, an Extended Kalman filter, can be used for non-linear systems like ours. Two types of the EKF are proposed. The first one is a full EKF, it uses all the sensors - magnetometer, gyroscopes and sun sensors. And the second one, a coarse EKF, works without sun sensors and will be used if the sun sensors measurement is unavailable.

For this algorithm, we need a state model $f(\mathbf{x},w)$ and a measurement model $h(\mathbf{x},\mathbf{y},v)$, where $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the measurement vector and v, w are process and measurement noises, which are not modelled. State model function is expressed in Eq. 2. The two types of EKF differs in the measurement model they are using. Further, for each sensor, there is its own model. The sensors which provide direction vector measurements - the magnetometer and the sun sensors, have similar models

$$\mathbf{s}^b = \mathbf{R}_i^b(\mathbf{q})\mathbf{s}^i, \qquad (4)$$
$$\mathbf{b}^b = \mathbf{R}_i^b(\mathbf{q})\mathbf{b}^i, \qquad (5)$$

where $\mathbf{b}^i$ and $\mathbf{s}^i$ are the sun direction vector and the magnetic induction direction vector in the reference frame and $\mathbf{b}^b$, $\mathbf{s}^b$ are their body frame counterparts. The $\mathbf{R}_i^b(\mathbf{q})$ is a rotation matrix that rotates a vector from the inertial frame to the body frame, and can be construsted using the quaternion $\mathbf{q}$.

Gyroscopes are measuring directly the angular velocity vector used in state model, so their measurement model simply is:

$$\boldsymbol{\omega}_m = \mathbf{I}\boldsymbol{\omega} \qquad (6)$$

where $\boldsymbol{\omega}_m$ is the measured angular velocity and $\mathbf{I}$ is the identity matrix.

The full EKF is using all of these models and the coarse EKF only magnetometer and gyroscopes models.

The equations for the EKF are [2]:

$$\begin{gathered} \mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R})^{-1}, \\ \mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_k^-, \\ \hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k\big(\mathbf{y}_k - h(\hat{\mathbf{x}}_k^-, 0)\big), \\ \hat{\mathbf{x}}_{k+1}^- = f(\hat{\mathbf{x}}_k, 0), \\ \mathbf{P}_{k+1}^- = \mathbf{F}_k \mathbf{P}_k \mathbf{F}_k^T + \mathbf{Q}, \end{gathered} \qquad (7)$$

where $\hat{\mathbf{x}}_k$ is the estimate of the state vector in k-th step, $\mathbf{P}_k$ is the covariance matrix of the estimate of the error of the state, $\mathbf{R}$ is the covariance matrix of the measurement noise, $\mathbf{Q}$ is the covariance matrix of the process noise, $\mathbf{K}_k$ is the Kalman gain in k-th step, and $\mathbf{H}_k$, $\mathbf{F}_k$ are jakobian matrices of functions $h(\mathbf{x},\mathbf{y},v)$ and $f(\mathbf{x},w)$.

QUEST

QUEST (QUaternion ESTimation) is a statistical method of attitude determination based on Wahba's problem [4], that means finding such a rotation matrix $\mathbf{R}_i^b$, that minimises cost function:

$$J(\mathbf{R}_i^b) = \frac{1}{2}\sum_{k=1}^{N} w_k \left|\mathbf{v}_k^b - \mathbf{R}_i^b \mathbf{v}_k^i\right|^2. \qquad (8)$$

QUEST is based on another attitude determination method, the Q-method. The description of the QUEST is showed in [5]. Briefly, it shows, that minimising the cost function $J(\mathbf{R}_i^b)$ is the same as maximizing the gain function

$$g(\mathbf{R}_i^b) = \sum_{k=1}^{N} w_k {\mathbf{v}_k^b}^T \mathbf{R}_i^b \mathbf{v}_k^i, \qquad (9)$$

which can be rewritten into a quaternion form $g(\mathbf{q}) = \mathbf{q}^T\mathbf{K}\mathbf{q}$. For maximizing of the gain function $g(\mathbf{R}_i^b)$, its derivative with respect to quaternion $\mathbf{q}$ should be zero, which reduces the problem to a problem of finding eigenvalues of the matrix $\mathbf{K}$. QUEST assumes [4, 5], that

$$\lambda_{opt} \approx \sum_{k=1}^{N} w_k. \qquad (10)$$

This eliminates the problem of finding the eigenvalues and makes the algorithm much faster. The attitude can be expressed with the help of Rodriguez parameters and transformed to quaternion

$$\mathbf{p} = \left[(\lambda_{opt} + \sigma)\mathbf{I} - \mathbf{S}\right]^{-1}\mathbf{Z}, \qquad (11)$$

$$\mathbf{q} = \frac{1}{\sqrt{1 + \mathbf{p}^T\mathbf{p}}}\begin{bmatrix}\mathbf{p}\\1\end{bmatrix}. \qquad (12)$$

Results

Both of the algorithms were tested using the Matlab/Simulink environment. The coarse EKF was not tested, as it is only a backup algorithm for the full EKF. In the simulations, various sources of errors were simulated, white noises for each sensor that were modelled to have similar properties as the sensors that will be used on the final platform, and an instability of a bias for the MEMS gyroscopes. In the test, the satellite was rotated 45° about each axis there and back. The results are displayed in Euler angles *yaw*, *pitch*, *roll*, as they are more understandable then quaternions. The results of these tests are depicted in figures 1 and 2.

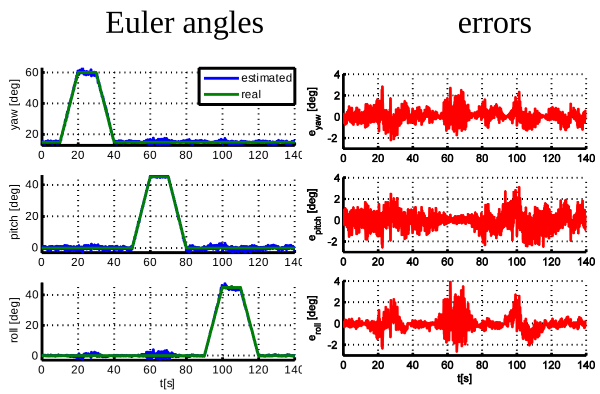

Figure 1 - Attitude determination by the Quest

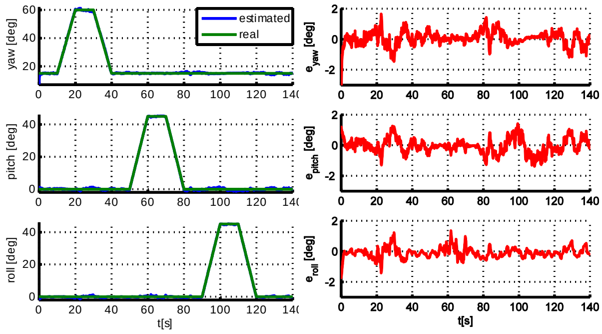

Figure 2 - Attitude determination by the full EKF

Then the algorithms were implemented on the satellite platform with 8-bit microcontroller, and the processing time required for each algorithms was tested. The results are in table 1.

TABLE 1 - Processing time needed for algorithms

Algorithm	QUEST	EKF
Time	1.15	30.02

Quest was in this test much faster than EKF, approximately 26 times. But the test also showed, that in current state of our project, we have enough power to afford the EKF.

Conclusion

This paper presented the Attitude Determination System algorithm selection for the CubeSat mission. There are shown two algorithms, the first one is static, statistical QUEST - QUaternion ESTimation, which requires less power, and the second one, more robust, is Extended Kalman Filter, recursive state estimator, in which the dynamical model of the system is implemented. As the tests showed, the results of EKF had less noise but consumed more power. Mostly because of its robustness, the EKF was implemented to the satellite platform.

References:

1. Kalman, R. E., A New Approach to Linear Filtering and Prediction Problems, Transaction of the ASME-Journal of Basic Engineering, pp. 35-45, 1960.

2. Welch, G. and Bishop, G., An Introduction to the Kalman Filter, Chapel Hill: University of North Carolina at Chapel Hill, 2006.

3. Tewari, A., Atmospheric and Space Flight Dynamics: Modeling and Simulation with MATLAB and Simulink, Boston: Birkhäuser, 2007.
ISBN 978-0-8176-4437-6.

4. Markley, F. L., Attitude Determination Using Two Vector Measurements, NASA Goddard Space Flight Center, 1998.

5. Hall, C. D., Spacecraft Attitude Dynamics and Control, Lecture Notes posted on Handouts page [online]. 12.1.2003. http://www.dept.aoe.vt.edu/~cdhall/courses/aoe4140.

6. Swartwout, M., *CubeSat Database*, [online]. February, 2015. https://sites.google.com/a/slu.edu/swartwout/home/cubesat-database.