**1730** (2021) 012057 doi:10.1088/1742-6596/1730/1/012057

# Dimension reduction of preference profile for aggregation of energy audit data

### S V Muravyov and M A Borisova

National Research Tomsk Polytechnic University Pr. Lenina, 30, Tomsk, 634050, Russian Federation

E-mail: muravyov@tpu.ru

Abstract. The paper aims to improve a technique based on preference aggregation, which allows to process a big amount of data from instrumental examinations of energy losses by substations of electrical energy distribution networks. An approach to transformation of the initial preference profile into the hierarchical one is considered. The absence of the influence of such a transformation on the correctness of processing results is shown. The workability of the proposed method is demonstrated on the basis of real energy surveys using the example of the Khakass enterprise of the Backbone Electric Grids (BEGs), being a part of Siberian BEG of the Unified National Electric Network of Russia. The method makes it possible to identify sources of economically inefficient expenditure of energy resources and unjustified energy losses, and also to provide compression of large volumes of energy survey data without diminution of essential information. The proposed method allows obtaining and representing a great amount of data of instrumental survey of substation energy losses in a form of a compact integral estimate in the ordinal scale. It can be an appropriate and promising tool for organizations engaged in energy consulting.

#### 1. Introduction

Resource conservation is an important factor in maintaining stability of society. An effective resource saving is ensured by systematic energy audits. Traditionally, energy audit outcomes analysis is a work with a large amount of unstructured data that is difficult to fully take into account [1,2].

In such cases, methods of multi-criteria decision making are traditionally used, such as, for example, the method of the generalized quality index or the analytic hierarchy process [3], which realize comparison of the analyzed objects on the basis of a weighted sum of partial indicators. However, these methods suffer from subjectivism, so do not lend themselves to serious axiomatization, and can lead to erroneous decisions [4]. These disadvantages can be avoided with the use of preference aggregation method [5-7], based on the treatment of exclusively binary relations, which does not require the calculation of weights, and gives a compact integral estimation of objects in the ordinal scale.

The paper aims to improve a technique based on preference aggregation, which allows to process a big amount of data from instrumental examinations of energy losses by substations of electrical energy distribution networks. The technique gives compact integral assessment of the data in an ordinal scale convenient for decision-making and visualization.

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#### 1730 (2021) 012057 doi:10.1088/1742-6596/1730/1/012057

### 2. Preference aggregation

As the initial data, we use the table  $V = \{v_{ij}\}$  of values of electric power losses, in MWh, of *i*-th substations by *j*-th attribute.

Let a set  $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_m\}$  of *m* rankings of *n* substations of a set  $A = \{a_1, a_2, ..., a_n\}$  be given. Each ranking is defined by indicating some attribute  $c_k$  from the list  $C = \{c_1, c_2, ..., c_m\}$  according to which each pair of elements from *A* is a strict preference relation,  $a_i \succeq a_j$ , or a tolerance relation,  $a_i \succeq a_j$ , according to

the following rule:

$$v_i \geqq v_j \Leftrightarrow a_i \gneqq a_j, i = 1, \dots, n; j = 1, \dots, m.$$

$$(1)$$

The set  $\Lambda(m, n)$  of rankings constructed by the rule (1) for each of the attributes  $c_k$ , k = 1, ..., m, from the list *C*, we call *preference profile* for given *m* and *n*. The list *C* may, for example, consist of the following attributes: energy losses in the distribution line ( $c_1$ ), in the power transformers ( $c_2$ ), climate-related losses ( $c_3$ ), losses of heating of buildings and of existing equipment ( $c_4$ ), lighting ( $c_5$ ), etc.

To aggregate *m* preferences defined over a set of *n* objects means to determine a unique preference relation  $\beta$  called the *consensus ranking*, which provides the best compromise among the rankings of the initial profile. A meaning of the concept "best compromise" depends on a preference aggregation rule used. In the proposed method, we use the Kemeny rule [5] consisting in determination of such linear order (Kemeny ranking)  $\beta$  of objects that the distance  $D(\beta, \Lambda)$  (defined in terms of the number of pairwise inconsistencies between the rankings) from  $\beta$  to the rankings of the initial profile  $\Lambda$  is minimal for all possible strict orders the objects. To find all possible Kemeny rankings for a given initial preference profile, we use the self-developed *recursive branch-and-bound algorithm* RECURSALL [8].

As soon as the Kemeny rule allows the existence of multiple solutions, the number *N* of possible consensus rankings  $\beta$  can exceed 10<sup>7</sup> even for small m = 4 and n = 15 [9]. To transform all the multiple consensus rankings  $B = {\beta_1, \beta_2, ..., \beta_N}$  into a single final consensus ranking  $\beta_{fin}$  we apply the special *convolution rule* [10]. Let B(*N*, *n*) = { $\beta_1, \beta_2, ..., \beta_N$ } be a set of all consensus rankings determined by the Kemeny rule for the profile  $\Lambda(m, n)$  given over some set of alternatives  $A = {a_1, a_2, ..., a_n}$  and *rank*  $r_i^k$  be a position of an alternative  $a_i$  in the consensus ranking  $\beta_k \in B$ , k = 1, ..., N. Let a *total rank*  $r_i$  of the alternative  $a_i$  be defined as  $r_i = \sum_{k=1}^{N} r_i^k$ . Then, for all i < j, i, j = 1, ..., n,

$$r_i < r_j \Longrightarrow a_i \succ a_j \text{ and } r_i = r_j \Longrightarrow a_i \sim a_j,$$
(2)

where both of the relations  $\succ$  and  $\sim$  are in the single final consensus ranking  $\beta_{\text{fin}}$ .

### 3. Conversion of initial profile into hierarchical one

The problem of finding the Kemeny ranking is *NP*-complete, i.e. having an exponential growth of the solution time as a function of the dimension n = |A| of the problem. Notice that, at problem dimension  $n \le 20$  suitable for practical application, it is possible to find all exact solutions within a reasonable time. In situations where n > 20, one should resort to partitioning the set *A* into  $N_p$  disjoint subsets  $A_k$ , i.e.

$$A = A_1 \cup A_2 \cup \dots \cup A_{N_p}; \quad \bigcap_{k=1}^{N_p} A_k = \emptyset; \ |A_k| \le 20, \ k = 1, \dots, N_p; \quad \sum_{k=1}^{N_p} |A_k| = n.$$
(3)

The operation of the set A partition results in *decomposition* of a preference profile  $\Lambda$  into subprofiles, see figure 1. This action transforms the linear profile structure to hierarchical one.

The decomposition process of the initial profile into subprofiles can be described in terms of *partition* of the set *A* into subsets. For this aim, define a *quotient set* over the set of substations *A* generated by some *h*-th equivalence relation  $\varepsilon_h$  on it. The equivalence relation occurs if a pair of elements  $a_i$ ,  $a_j \in A$  has a certain common property. For instance, one of the possible equivalence relations can be produced due to the substations' property "to have a bandwidth that differs by no more than 2.5 times".

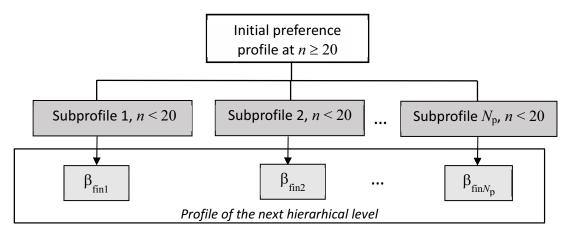


Figure 1. Decomposition of the preference profile.

Let an equivalence class  $A_k^h$ ,  $k = 1, ..., N_p$ , generated by element  $a_i$ , be the set of all such elements  $a_j$  of A that  $(a_j, a_i) \in \varepsilon_h$ ; where  $N_p$  is a number of equivalence classes by relation  $\varepsilon_h$ . Then the set of equivalence classes

$$A/\varepsilon_h = \{A_1^h, A_2^h, ..., A_{N_nh}^h\}$$
(4)

is called quotient set by relation  $\varepsilon_h$ , which, accounting (3), defines corresponding partition of the set *A* into subsets, i.e.  $A = A_1^h \cup A_2^h \cup ... \cup A_{N_{n,h}}^h$ .

Thus, if *H* equivalence relations  $\varepsilon_1, ..., \varepsilon_h, ..., \varepsilon_H$  are considered, then partitions of the set *A* into subsets are possible by these relations, that is

$$A = A_{1}^{l} \cup A_{2}^{l} \cup \dots \cup A_{N_{pl}}^{l} = A_{1}^{2} \cup A_{2}^{2} \cup \dots \cup A_{N_{p2}}^{2} = \dots = A_{1}^{H} \cup A_{2}^{H} \cup \dots \cup A_{N_{pH}}^{H},$$
(5)

and appropriate quotient sets  $A/\varepsilon_1$ ,  $A/\varepsilon_2$ , ...,  $A/\varepsilon_H$  are defined. In its turn, each equivalence class  $A_k^h$  defines the subprofile  $\Lambda_k^h$  consisting of *m* rankings of elements of the set  $A_k^h$  by attributes of the list *C*.

Notice that consensus rankings  $\beta_{\text{fink}}^h$ , found for the sets  $A_k^h$ , do not intersect each other. Therefore, it is necessary to develop a strategy for ascertaining preferences between alternatives included in different consensus rankings. It is impossible to just combine all the alternatives, because the result of such a union would be the original set *A* with unacceptably large *n*.

Therefore, choose  $n_u$  best alternatives (that is alternatives occupying 1-st, 2-nd, ..., and  $n_u$ -th position) in each  $\beta_{\text{fink}}^h$  and compose of them the set

$$A_{\sup}^{h} = \bigcup_{k=1}^{N_{ph}} \left\{ a_{i} \mid r_{i}^{k} \leq n_{u} \right\},$$
(6)

where rank  $r_i^k$  is a position of alternative  $a_i$  in the consensus ranking  $\beta_{\text{fin}k}^h$ . Build a profile  $\Lambda_{\text{sup}}^h$  consisting of rankings of alternatives from  $A_{\text{sup}}^h$  by attributes of *C*. Final consensus relation  $\beta_{\text{fin}\_\text{sup}}^h$  obtained for this profile is the ranking of  $|A_{\text{sup}}^h| = n_u \cdot N_{pH}$  alternatives, where the first alternative is best among the best, and the last alternative is worst among the best.

In the same way, choosing  $n_u$  worst alternatives (that is alternatives occupying  $n_u$  last positions) in each  $\beta_{\text{fink}}^h$ , build a profile  $\Lambda_{\text{inf}}^h$  consisting of rankings of alternatives from the set

$$\mathcal{A}_{\inf}^{h} = \bigcup_{k=1}^{N_{\text{ph}}} \left\{ a_{i} \mid r_{i}^{k} \leq (\mid \beta_{\text{fink}}^{h} \mid \Box n_{\text{u}}) \right\}$$
(7)

#### **1730** (2021) 012057 doi:10.1088/1742-6596/1730/1/012057

by attributes of *C*. Final consensus relation  $\beta_{\text{fin}\_\text{inf}}^{h}$  obtained for this profile is the ranking of  $|A_{\text{inf}}^{h}| = n_{\text{u}} \cdot N_{\text{ph}}$  alternatives, where the first alternative is best among the worst, and the last alternative is worst among the worst.

Recommended value for the number  $n_u$  can be 3, however one should keep in mind that there is an upper bound for it conditioned by expression (3) whence it follows that

$$n_{\rm u} < 20 / N_{\rm ph}$$
 (8)

Clear that, with a large number of alternatives in the original set A, the decomposition process described above can be repeated as many times as necessary.

### 4. Practical example

Let us demonstrate a workability of the proposed decomposition process by an example of the set of 17 substations of the Khakass enterprise of the Backbone Electric Grids (BEGs), being a part of Siberian BEG of the Unified National Electric Network of Russian Federation.

Determine one of the feasible quotient sets (i.e. H = 1) over the substations set  $A = \{s_1, s_2, ..., s_{17}\}$ generated by the equivalence relation  $\varepsilon_1$  over it: "to have a throughput that differs by no more than 2 times". Then the quotient set by relation  $\varepsilon_1$  is

$$A/\varepsilon_1 = \{A_1^1, A_2^1, ..., A_{N_{n_1}}^1\}$$
(9)

and the equation is valid:

$$A = A_{1}^{1} \cup A_{2}^{1} \cup \dots \cup A_{N_{p_{1}}}^{1}.$$
 (10)

In table 1 the initial data of energy losses V by 9 attributes in the Khakass enterprise are shown.

| Substations            | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ | $\lambda_7$ | $\lambda_8$ | λ9   |
|------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------|
| <i>s</i> <sub>1</sub>  | 520.35      | 372.10      | 245.43      | 183.00      | 21.30       | 32.34       | 21.90       | 7.20        | 5.88 |
| <i>s</i> <sub>2</sub>  | 633.25      | 396.11      | 120.69      | 927.54      | 17.33       | 132.12      | 137.25      | 7.10        | 5.88 |
| <i>s</i> <sub>3</sub>  | 788.01      | 423.54      | 327.88      | 1077.31     | 83.11       | 328.09      | 1449.60     | 7.00        | 4.91 |
| <i>s</i> <sub>4</sub>  | 366.54      | 253.69      | 142.30      | 218.30      | 15.30       | 88.20       | 26.16       | 7.10        | 3.75 |
| <i>s</i> <sub>5</sub>  | 823.65      | 387.55      | 241.03      | 831.02      | 38.40       | 317.52      | 828.01      | 7.60        | 6.74 |
| <i>s</i> <sub>6</sub>  | 308.44      | 241.33      | 150.23      | 355.88      | 7.69        | 62.30       | 30.07       | 7.10        | 2.80 |
| \$7                    | 702.47      | 315.87      | 222.55      | 1151.47     | 12.03       | 110.20      | 283.78      | 7.10        | 3.75 |
| <i>s</i> <sub>8</sub>  | 652.88      | 325.44      | 279.66      | 1287.32     | 13.19       | 54.13       | 384.39      | 7.10        | 4.93 |
| <i>S</i> 9             | 1230.54     | 541.23      | 362.10      | 1134.15     | 114.23      | 448.24      | 1449.60     | 7.00        | 8.20 |
| <i>s</i> <sub>10</sub> | 685.03      | 300.55      | 214.30      | 525.32      | 22.41       | 182.31      | 179.52      | 7.10        | 4.91 |
| <i>s</i> <sub>11</sub> | 387.08      | 274.65      | 197.13      | 375.84      | 15.61       | 202.32      | 262.80      | 7.10        | 2.80 |
| <i>s</i> <sub>12</sub> | 358.11      | 202.52      | 182.49      | 273.45      | 11.18       | 100.57      | 5.95        | 7.10        | 1.88 |
| <i>s</i> <sub>13</sub> | 300.28      | 187.54      | 168.22      | 612.35      | 14.16       | 174.70      | 262.80      | 7.10        | 1.87 |
| <i>s</i> <sub>14</sub> | 421.31      | 234.72      | 103.24      | 713.33      | 8.20        | 9.18        | 17.60       | 7.10        | 3.75 |
| <i>s</i> <sub>15</sub> | 203.21      | 111.54      | 86.23       | 44.80       | 3.00        | 8.70        | 3.25        | 3.30        | 0.15 |
| <i>s</i> <sub>16</sub> | 310.23      | 123.01      | 112.38      | 658.96      | 24.53       | 110.20      | 75.09       | 7.10        | 8.20 |
| <i>s</i> <sub>17</sub> | 473.24      | 204.39      | 148.91      | 994.31      | 10.78       | 110.20      | 328.56      | 7.10        | 2.80 |

Table 1. Values v of electrical energy losses by 9 attributes, MWh.

On the base of data of table 1 one can shape the preference profile  $\Lambda$  by the rule (1):

$$\begin{split} \lambda_1 &: s_{15} \succ s_{13} \succ s_6 \succ s_{16} \succ s_{12} \succ s_4 \succ s_{11} \succ s_{14} \succ s_{17} \succ s_1 \succ s_2 \succ s_8 \succ s_{10} \succ s_7 \succ s_3 \succ s_5 \succ s_9 \\ \lambda_2 &: s_{15} \succ s_{16} \succ s_{13} \succ s_{12} \succ s_{17} \succ s_{14} \succ s_6 \succ s_4 \succ s_{11} \succ s_{10} \succ s_7 \succ s_8 \succ s_1 \succ s_5 \succ s_2 \succ s_3 \succ s_9 \\ \lambda_3 &: s_{15} \succ s_{14} \succ s_{16} \succ s_2 \succ s_4 \succ s_{17} \succ s_6 \succ s_{13} \succ s_{12} \succ s_{11} \succ s_{10} \succ s_7 \succ s_5 \succ s_1 \succ s_8 \succ s_3 \succ s_9 \\ \lambda_4 &: s_{15} \succ s_1 \succ s_4 \succ s_{12} \succ s_6 \succ s_{11} \succ s_{10} \succ s_{13} \succ s_{16} \succ s_{14} \succ s_5 \succ s_2 \succ s_7 \succ s_8 \end{cases}$$

**1730** (2021) 012057 doi:10.1088/1742-6596/1730/1/012057

$$\lambda_{5}: s_{15} \succ s_{6} \succ s_{14} \succ s_{17} \succ s_{12} \succ s_{7} \succ s_{8} \succ s_{13} \succ s_{4} \succ s_{11} \succ s_{2} \succ s_{10} \succ s_{10} \succ s_{16} \succ s_{5} \succ s_{3} \succ s_{9}$$

$$\lambda_{6}: s_{15} \succ s_{14} \succ s_{1} \succ s_{8} \succ s_{6} \succ s_{4} \succ s_{12} \succ s_{7} \sim s_{16} \sim s_{17} \succ s_{2} \succ s_{13} \succ s_{10} \succ s_{11} \succ s_{5} \succ s_{3} \succ s_{9}$$

$$\lambda_{7}: s_{15} \succ s_{12} \succ s_{14} \succ s_{1} \succ s_{4} \succ s_{6} \succ s_{16} \succ s_{2} \succ s_{10} \succ s_{11} \sim s_{13} \succ s_{7} \succ s_{7} \succ s_{8} \succ s_{5} \succ s_{3} \sim s_{9}$$

$$\lambda_{8}: s_{15} \succ s_{3} \sim s_{9} \succ s_{2} \sim s_{4} \sim s_{6} \sim s_{7} \sim s_{8} \sim s_{10} \sim s_{11} \sim s_{12} \sim s_{13} \sim s_{14} \sim s_{16} \sim s_{17} \succ s_{1} \sim s_{5}$$

$$\lambda_{9}: s_{15} \succ s_{12} \sim s_{13} \succ s_{6} \sim s_{11} \sim s_{17} \sim s_{4} \sim s_{7} \sim s_{14} \succ s_{3} \sim s_{8} \sim s_{10} \succ s_{1} \sim s_{2} \succ s_{5} \succ s_{9} \sim s_{16}$$

For the obtained profile, the algorithm RECURSALL has found the following consensus ranking:

$$\beta_{\text{fin}}: s_{15} \succ s_6 \succ s_{12} \succ s_4 \sim s_{13} \sim s_{14} \succ s_{16} \succ s_{11} \sim s_{17} \succ s_1 \succ s_2 \succ s_{10} \succ s_7 \succ s_8 \succ s_5 \succ s_3 \succ s_9.$$
(11)

It is clear from (11) that the most problematic substations that require special attention and corrective measures to reduce electricity losses are  $s_5$  (Aluminievaya),  $s_3$  (Abakanskaya), and  $s_9$  (Oznachennoe).

Now we are going to show that the decomposition of the original profile when applying preference aggregation does not lead to a significant change in the result of processing energy audit data. For this aim, the set of substations of the Khakass enterprise is decomposed by the throughput in correspondence with expression (9).

For each subprofile, the substations lists obtained as a result of the decomposition are shown in table 2.

| Table 2. Substation set A decomposition by the throughput, <i>i</i> is substation <i>a</i> number in a subprofile, |
|--|
| <i>h</i> is number of equivalence relation.  |

| Subprofile                   | Substation name       | Throughput, MWh | $s / a_i^h$      |
|------------------------------|-----------------------|-----------------|------------------|
|                              | Ak-Dovurak            | 13371.961       | $s_4 / a_1^1$    |
|                              | Askiz                 | 15356.933       | $s_6 / a_2^1$    |
| Subprofile $A_1$             | Alyuminievaya         | 27831.932       | $s_5/a_3^1$      |
| (Throughput up to            | Teya                  | 32215.143       | $s_{12} / a_4^1$ |
| 40000 MWh)                   | Tuim                  | 32290.467       | $s_{13}/a_5^1$   |
|                              | Khandagayty           | 33199.701       | $s_{15}/a_6^1$   |
|                              | Chadan                | 37371.961       | $s_{16} / a_7^1$ |
|                              | Abaza                 | 54967.813       | $s_1 / a_1^1$    |
| Subprofile $A_2$             | Kyzylskaya            | 56010.371       | $s_7/a_2^1$      |
| (Throughput up to 80000 MWh) | Oznachennoe           | 52405.760       | $s_9/a_3^1$      |
|                              | Turan                 | 63078.134       | $s_{14}/a_4^1$   |
| Subprofile $A_3$             | Abakan-rayonnaya      | 15258.769       | $s_2 / a_1^1$    |
| (Throughput up to            | Minusinsk-opornaya    | 147648.053      | $s_8/a_2^1$      |
| 160000 MWh)                  | Shushenskaya-opornaya | 120798.929      | $s_{17}/a_3^1$   |
| Subprofile A <sub>4</sub>    | Abakanskaya           | 313093.425      | $s_3 / a_1^1$    |
| (Throughput up to            | Sora                  | 271848.496      | $s_{11}/a_2^1$   |
| 320000 MWh)                  | Oznacheno-rayonnaya   | 237257.394      | $s_{10}/a_3^1$   |

For each subprofile, consensus rankings  $\beta_{\text{fin}}$  were obtained (see table 3 and figure 2). Rankings in the table are represented in a vertical form, in which the more preferred element is located higher than the less preferred; tolerant elements occupy the same position (for example, elements  $a_2^1$  and  $a_4^1$  in the ranking  $\lambda_8$  of subprofile  $A_2$ ).

|                           |             |             |             | ueco        | ompo        | Jseu        | Uyı         | nrougnput.                            |                                 |                      |
|---------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|---------------------------------------|---------------------------------|----------------------|
| G 1 (71                   | Rankings    |             |             |             |             |             |             |                                       |                                 |                      |
| Subprofiles               | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ | $\lambda_7$ | $\lambda_8$                           | λ9                              | $\beta_{\text{fin}}$ |
|                           | $a_6^1$     | $a_6^1$     | $a_6^1$     | $a_6^1$     | $a_{6}^{1}$ | $a_6^1$     | $a_6^1$     | $a_6^1$                               | $a_6^1$                         | $a_{6}^{1}$          |
|                           | $a_5^1$     | $a_{7}^{1}$ | $a_{7}^{1}$ | $a_1^1$     | $a_2^1$     | $a_2^1$     | $a_4^1$     | $a_1^1 a_3^1 a_2^1 a_2^1 a_4^1 a_7^1$ | $a_1^1 a_2^1 a_4^1 a_5^1 a_7^1$ | $a_2^1$              |
|                           | $a_2^1$     | $a_5^1$     | $a_1^1$     | $a_4^1$     | $a_4^1$     | $a_1^1$     | $a_1^1$     |                                       |                                 | $a_5^1$              |
| Subprofile A <sub>1</sub> | $a_{7}^{1}$ | $a_4^1$     | $a_2^1$     | $a_5^1$     | $a_5^1$     | $a_4^1$     | $a_2^1$     |                                       |                                 | $a_1^1$              |
|                           | $a_4^1$     | $a_2^1$     | $a_5^1$     | $a_2^1$     | $a_1^1$     | $a_{7}^{1}$ | $a_{7}^{1}$ |                                       |                                 | $a_4^1$              |
|                           | $a_1^1$     | $a_1^1$     | $a_4^1$     | $a_{7}^{1}$ | $a_{7}^{1}$ | $a_5^1$     | $a_5^1$     |                                       |                                 | $a_{7}^{1}$          |
|                           | $a_{3}^{1}$ | $a_3^1$     | $a_3^1$     | $a_3^1$     | $a_{3}^{1}$ | $a_3^1$     | $a_{3}^{1}$ | $a_5^1$                               | $a_3^1$                         | $a_{3}^{1}$          |
|                           | $a_4^1$     | $a_4^1$     | $a_4^1$     | $a_1^1$     | $a_4^1$     | $a_4^1$     | $a_4^1$     | $a_{2}^{1} a_{4}^{1}$                 | $a_2^1$                         | $a_4^1$              |
| Subprofile $A_2$          | $a_1^1$     | $a_2^1$     | $a_2^1$     | $a_2^1$     | $a_2^1$     | $a_1^1$     | $a_1^1$     | $u_2 u_4$                             | $a_4^1$                         | $a_1^1$              |
| Subprome A <sub>2</sub>   | $a_2^1$     | $a_1^1$     | $a_1^1$     | $a_4^1$     | $a_1^1$     | $a_2^1$     | $a_2^1$     | $a_1^1$                               | $a_1^1$                         | $a_2^1$              |
|                           | $a_{3}^{1}$ | $a_3^1$                               | $a_3^1$                         | $a_{3}^{1}$          |
|                           | $a_1^1$     | $a_{3}^{1}$ | $a_1^1$     | $a_1^1$     | $a_{3}^{1}$ | $a_2^1$     | $a_1^1$     |                                       | $a_3^1$                         | $a_{3}^{1}$          |
| Subprofile A <sub>3</sub> | $a_2^1$     | $a_2^1$     | $a_{3}^{1}$ | $a_{3}^{1}$ | $a_2^1$     | $a_{3}^{1}$ | $a_{3}^{1}$ | $a_1^1 a_2^1 a_3^1$                   | $a_2^1$                         | $a_1^1$              |
|                           | $a_{3}^{1}$ | $a_1^1$     | $a_2^1$     | $a_2^1$     | $a_1^1$     | $a_1^1$     | $a_2^1$     |                                       | $a_1^1$                         | $a_2^1$              |
| Subprofile $A_4$          | $a_2^1$     | $a_{3}^{1}$ | $a_2^1$     | $a_2^1$     | $a_2^1$     | $a_{3}^{1}$ | $a_{3}^{1}$ | $a_{3}^{1} a_{2}^{1}$                 | $a_2^1$                         | $a_2^1$              |
|                           | $a_{3}^{1}$ | $a_2^1$     | $a_{3}^{1}$ | $a_{3}^{1}$ | $a_{3}^{1}$ | $a_2^1$     | $a_2^1$     | $a_3 a_2$                             | $a_3^1$                         | $a_{3}^{1}$          |
|                           | $a_1^1$                               | $a_1^1$                         | $a_1^1$              |

| Table 3. Subprofiles and their consensus rankings for the substation set of Khakass enterprise |
|--|
| decomposed by throughput.  |

## Set of substations A

| Subprofile A <sub>1</sub>   | Subprofile A <sub>2</sub>                              | Subprofile A <sub>3</sub>  | Subprofile A <sub>4</sub> |  |
|---|--|--|---------------------------|--|
|   | 1. 1. 1. 1   |  | 1                         |  |
| $\boldsymbol{\beta_{fin1}}: a_6^1 \succ a_2^1 \succ a_5^1 \succ \\ s_{15} \succ s_6 \succ s_{13} \succ s_5$ | , .  | $\boldsymbol{\beta_{\text{fin3}:}}  \boldsymbol{a}_3^1 \succ \boldsymbol{a}_1^1 \succ \boldsymbol{a}_2^1 = \\ \boldsymbol{s}_{17} \succ \boldsymbol{s}_2 \succ \boldsymbol{s}_8$ | Final<br>rankings         |  |
| $\mathbf{R} \cdot a^{1} \subseteq a^{1} \subseteq a^{1} \subseteq$  | $\bullet a_3^1 = s_{14} \succ s_1 \succ s_7 \succ s_9$ | $\begin{bmatrix} \mathbf{R} & \mathbf{a}^{\mathrm{I}} \leq \mathbf{a}^{\mathrm{I}} \leq \end{bmatrix}$   | $a^1 = a + a + a$         |  |
| <b>Pfin2</b> • $u_4 \neq u_1 \neq u_2 \neq$   | $u_3 - s_{14} - s_1 - s_7 - s_9$                       | $\boldsymbol{\beta}_{fin4}: a_2^1 \succ a_3^1 \succ a_1^1 = s_{11} \succ s_{10} \succ s_3$   |                           |  |

Figure 2. Consensus rankings found for subprofiles 1-4.

On can see from table 3 and figure 2 that the consensus rankings  $\beta_{\text{fin1}}$ , ...,  $\beta_{\text{fin4}}$  do not intersect each other. Having taken  $n_{\text{u}} = 3$  we compose the set of the best alternatives  $A_{\text{sup}}^1$  by formula (5), see table 4.

**Table 4.** Set  $A_{sup}^1$  of the best alternatives of subprofiles.

| Subprofile       | Subprofile $A_1$      | Subprofile A <sub>2</sub> | Subprofile A <sub>3</sub> | Subprofile A <sub>4</sub> |
|------------------|-----------------------|---------------------------|---------------------------|---------------------------|
| Best substations | $S_{15}, S_6, S_{13}$ | $S_{14}, S_1, S_7$        | $S_{17}, S_2, S_8$        | $s_{11}, s_{10}, s_3$     |

The profile  $\Lambda_{sup}^1$ , shaped of the best alternatives of the subprofiles on the base of Table 1 data, is as follows:

$$\begin{split} \lambda_1 &: s_{15} \succ s_{13} \succ s_6 \succ s_{11} \succ s_{14} \succ s_{17} \succ s_1 \succ s_2 \succ s_8 \succ s_{10} \succ s_7 \succ s_3 \\ \lambda_2 &: s_{15} \succ s_{13} \succ s_{17} \succ s_{14} \succ s_6 \succ s_{11} \succ s_{10} \succ s_7 \succ s_8 \succ s_1 \succ s_2 \succ s_3 \\ \lambda_3 &: s_{15} \succ s_{14} \succ s_2 \succ s_{17} \succ s_6 \succ s_{13} \succ s_{11} \succ s_{10} \succ s_7 \succ s_1 \succ s_8 \succ s_3 \\ \lambda_4 &: s_{15} \succ s_1 \succ s_6 \succ s_{11} \succ s_{10} \succ s_{13} \succ s_{14} \succ s_2 \succ s_{17} \succ s_3 \succ s_{77} \succ s_8 \\ \lambda_5 &: s_{15} \succ s_6 \succ s_{14} \succ s_{17} \succ s_7 \succ s_8 \succ s_{13} \succ s_{11} \succ s_2 \succ s_1 \succ s_{10} \succ s_3 \\ \lambda_6 &: s_{15} \succ s_{14} \succ s_1 \succ s_8 \succ s_6 \succ s_{77} \sim s_{17} \succ s_{18} \succ s_{13} \\ \lambda_7 &: s_{15} \succ s_{14} \succ s_1 \succ s_6 \succ s_{2} \succ s_{10} \succ s_{11} \sim s_{13} \succ s_{17} \succ s_{17} \succ s_8 \succ s_3 \\ \lambda_8 &: s_{15} \succ s_3 \succ s_2 \sim s_6 \sim s_{77} \sim s_8 \sim s_{10} \sim s_{11} \sim s_{13} \sim s_{17} \succ s_{18} \succ s_{13} \\ \lambda_9 &: s_{15} \succ s_{13} \succ s_6 \sim s_{11} \sim s_{17} \succ s_{77} \sim s_{14} \sim s_{10} \sim s_{17} \end{cases}$$

Obtained for this profile consensus ranking has the view:

$$\beta_{\text{fin}\_\text{sup}}^{1} = \{ s_{15} \succ s_{6} \succ s_{14} \succ s_{13} \succ s_{11} \succ s_{17} \succ s_{1} \succ s_{2} \succ s_{10} \succ s_{7} \succ s_{8} \succ s_{3} \}.$$
(12)

Compose set  $A_{inf}^{l}$  of the worst alternatives by formula (6), see table 5.

**Table 5.** Set  $A_{inf}^{1}$  of the worst alternatives of subprofiles.

| Subprofile        | Subprofile $A_1$      | Subprofile $A_2$ | Subprofile A <sub>3</sub> | Subprofile A <sub>4</sub> |
|-------------------|-----------------------|------------------|---------------------------|---------------------------|
| Worst substations | $s_{12}, s_{16}, s_5$ | $S_1, S_7, S_9$  | $S_{17}, S_2, S_8$        | $s_{11}, s_{10}, s_3$     |

The profile  $\Lambda_{inf}^{1}$ , shaped of the best alternatives of the subprofiles on the base of table 1 data, is as follows:

$$\begin{split} \lambda_1 &: s_{16} \succ s_{12} \succ s_{11} \succ s_{17} \succ s_{1} \succ s_{2} \succ s_{8} \succ s_{10} \succ s_{7} \succ s_{3} \succ s_{5} \succ s_{9} \\ \lambda_2 &: s_{16} \succ s_{12} \succ s_{17} \succ s_{11} \succ s_{10} \succ s_{7} \succ s_{8} \succ s_{1} \succ s_{5} \succ s_{2} \succ s_{3} \succ s_{9} \\ \lambda_3 &: s_{16} \succ s_{2} \succ s_{17} \succ s_{12} \succ s_{11} \succ s_{10} \succ s_{7} \succ s_{5} \succ s_{1} \succ s_{8} \nvDash s_{3} \succ s_{9} \\ \lambda_4 &: s_1 \succ s_{12} \succ s_{11} \succ s_{10} \succ s_{16} \succ s_{5} \succ s_{2} \succ s_{17} \succ s_{8} \succ s_{3} \succ s_{9} \\ \lambda_5 &: s_{17} \succ s_{12} \succ s_{7} \succ s_{8} \succ s_{11} \succ s_{2} \succ s_{1} \succ s_{10} \succ s_{16} \succ s_{5} \succ s_{3} \succ s_{9} \\ \lambda_6 &: s_1 \succ s_{8} \succ s_{12} \succ s_{7} \sim s_{16} \sim s_{17} \sim s_{10} \succ s_{10} \succ s_{10} \succ s_{15} \succ s_{3} \succ s_{9} \\ \lambda_7 &: s_{12} \succ s_{1} \succ s_{16} \succ s_{2} \succ s_{10} \succ s_{11} \succ s_{7} \succ s_{17} \succ s_{8} \succcurlyeq s_{5} \succ s_{3} \sim s_{9} \\ \lambda_8 &: s_{3} \sim s_{9} \succ s_{2} \sim s_{7} \sim s_{8} \sim s_{10} \sim s_{11} \sim s_{12} \sim s_{16} \sim s_{17} \succ s_{15} \\ \lambda_9 &: s_{12} \succ s_{11} \sim s_{17} \succ s_{10} \sim s_{10} \sim s_{11} \sim s_{2} \succ s_{17} \sim s_{17} \succ s_{17} \succ s_{17} \gg s$$

Obtained for this profile consensus ranking has the view:

$$\beta_{\text{fin}\_\text{inf}}^{1} = \{ s_{12} \succ s_{16} \succ s_{11} \succ s_{17} \succ s_{1} \succ s_{2} \succ s_{7} \succ s_{10} \succ s_{8} \succ s_{5} \succ s_{3} \succ s_{9} \}.$$
(13)

One can see that the final consensus rankings for the best (11) and the worst (12) alternatives of the decomposed preference profile substantially coincide with the consensus ranking (10) for original undecomposed ranking.

# 5. Conclusion

The results of the developed method application to real energy audit data have shown, that the decomposition of the profile does not lead to a significant change in the outcomes of the energy audit data processing. The validity of this statement is demonstrated by comparison of the consensus rankings obtained for the set of best substations and the set of worst substations composed of subprofiles consensus rankings with the consensus ranking obtained for the initial (undecomposed) profile. This means that the decomposition is allowable and justified way of reducing the dimension n when it is necessary.

# Acknowledgment

This work was supported by the Russian Science Foundation, project 18-19-00203.

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