SPACE FRACTIONAL FISHER-KOLMOGOROV-PETROVSKII-PISKUNOV EQUATION WITH ANOMALOUS DIFFUSION[1]

A.A. Prozorov

Tomsk, Tomsk Polytechnic University
e-mail: prozorov_a_a@mail.ru

Analytical solutions are constructed for the nonlocal space fractional Fisher-Kolmogorov-Petrovskii-Piskunov equation with anomalous diffusion.

$$u_t(x,t) = Du_\alpha(x,t) + au(x,t) - \chi u(x,t)\int_{-l}^{l} b_\gamma(x-y)u(y,t)dy,$$

Such solutions allow us to describe quasi-steady state patterns. Special attention is given to the role of fractional derivative [1]. The resulting solutions spread faster than the classical solutions and may exhibit asymmetry, depending on the fractional derivative used. The phenomenon of pattern formation was studied using a number of models based on generalized Fisher-Kolmogorov-Petrovskii-Piskunov equations taking into account nonlocal interaction effects [2]. We have focused on a special type of pattern formation with anomalous diffusion. Main conclusion of the work: the lower the order of the fractional derivative, the greater the displacement of the center and stronger the deviation from the steady state.

This solution is spatially homogeneous and monotonically depending on time. By analogy with previous studies, we have assumed that the patterns above can be described as large time perturbations of this exact solution. The large time asymptotics are constructed explicitly, to within $O(1/T^2)$, in the class of functions which tend to the above exact solution as $T \to \infty$. Thereby, the exact solution can be regarded as an attractor of the constructed class of asymptotic solutions and, hence, of the corresponding concentrated patterns. As the patterns evolve monotonically without qualitative changes to some steady-state, these asymptotic solutions describe approximately the quasi-steady-state patterns.

The approach used allows to gain information on the most essential characteristics of patterns and to apply the methods developed for 1D problems to multidimensional problems. The resulting solution:

$$u(x,t) = \frac{\beta_{00}e^{at}}{1+\frac{\chi b_0\beta_{00}}{a}(e^{at}-1)} + \frac{1}{T}\sum_{j=-\infty}^{\infty}\frac{\beta_{1l}e^{\tilde{a}_j t}e^{ij\pi x/l}}{\left[1+\frac{\chi\lambda_0\beta_{00}}{a}(e^{at}-1)\right]^{(b_j+b_0)/b_0}}, \quad \tilde{a}_j = \left(D\left(\frac{ij\pi}{l}\right)^\alpha + a\right).$$

REFERENCES

1 Samko, S.; Kilbas, A.A.; and Marichev Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach Science, 1993.

2 Fisher R. A. The wave of advance of advantageous genes // Annual Eugenics. V. 7. P. 255 (1937)

[1] The work is perfomed as a part of state task “Nauka”.