mathematics

Article

Development of an Algorithm for Computing the Force and Stress Parameters of a Cycloid Reducer

Egor A. Efremenkov [1], Sergey A. Shanin [1] and Nikita V. Martyushev [2,*]

[1] Department of Mechanical Engineering, Tomsk Polytechnic University, 30, Lenin Ave., 634050 Tomsk, Russia
[2] Department of Advanced Technologies, Tomsk Polytechnic University, 30, Lenin Ave., 634050 Tomsk, Russia
* Correspondence: martjushev@tpu.ru

Abstract: The paper is devoted to the development of an algorithm for the automated calculation of force characteristics of cycloid toothing when the initial parameters vary widely. The algorithm forms a structured data array that accelerates finding and outputting the necessary parameters and reduces the probability of error in determining these parameters. The algorithm serves the basis for a program that allows for the examination of the change in force and geometrical parameters in various combinations. The study includes the analysis of the dependence of forces and contact stresses in transmission toothing with intermediate rolling elements and a free cage on the initial parameters of this transmission. The obtained results will make it possible to select optimal combinations of initial parameters in order to minimize the force impact on the mechanism parts when designing modern compact mechanisms based on the cycloid with intermediate rolling elements and a free cage.

Keywords: automated calculation; modeling; algorithm; cycloid toothing; transmission toothing; cycloid with intermediate rolling elements

MSC: 65Z05

Citation: Efremenkov, E.A.; Shanin, S.A.; Martyushev, N.V. Development of an Algorithm for Computing the Force and Stress Parameters of a Cycloid Reducer. *Mathematics* **2023**, *11*, 993. https://doi.org/10.3390/math11040993

Academic Editors: Higinio Rubio Alonso, Jesus Meneses Alonso, Alejandro Bustos Caballero and Enrique Soriano-Heras

Received: 28 December 2022
Revised: 12 February 2023
Accepted: 13 February 2023
Published: 15 February 2023

1. Introduction

Currently, it is impossible to develop modern mechanical transmissions in general and transmissions with intermediate rolling elements (IRE) in particular without the automated design of structures and design calculations.

The automation of calculations of cycloids increases the quality of selection of the functional parameters and reduces the time for determining the technical parameters provided by the designed mechanism. In addition, it reduces the probability of design errors in the finished product. Modern mathematical methods allow us to determine stresses and strains in the toothing of various transmissions with high accuracy, and the capabilities of processing a large amount of data allow us to study the cycloid profile in detail.

The transmission with intermediate rolling elements and a free cage (IREFC) is the most promising for use in modern mechatronic and automated systems, since it most fully provides a set of the required technical characteristics. This transmission provides compactness and, at the same time, a high load capacity owing to the toothing multipairing, has a high efficiency due to the replacement of sliding friction in the toothing with rolling friction, and a high toothing accuracy and small angular gaps. The entire set of these characteristics is the most in-demand when designing modern mechanisms.

Currently, the calculation of cycloids with IRE and, especially, transmissions with IREFC, is quite time consuming. The calculation and design of the transmission with IRE often require several iterations of the recalculation of the force characteristics of the mechanism with different combinations of the cycloid initial parameters. This process takes a lot of time, which delays the design stage. Therefore, it is quite relevant to create an algorithm for the automated determination of forces and contact stresses of cycloid toothing of the transmission with IREFC.

Many authors [1–16] are engaged in the study of cycloids using mathematical methods, including the force characteristics of pin gears and transmissions with IRE [17–29]. The determination of forces through deformations in the toothing of the transmission with IRE and a loaded separator is considered in [1]. In [2], the author analyzes the kinematic error and tolerance allocation of cycloidal gear reducers. In another work [3], the author analyzes a cycloidal reducer output mechanism while taking into account machining deviations. In [4], the authors propose a technology for reducing the accuracy of performing cycloid profiles while maintaining the toothing accuracy. The authors in [5,6] investigate the changes in the torque of the drive gearing system. In [7], the dynamic behavior of a two-stage pin cycloid reducer of a new design is analyzed, and the effect of vibrations on the cycloid toothing is considered. In [8,9], the authors analyze the operation of the cycloid. The author [10] describes the design features of the transmission when contacting fixed intermediate links with grooves cut in the torus. He also presents the equations of the spirals used for cutting these grooves. Studies [11,12] analyze contact stresses in a spherical transmission with intermediate rolling elements and propose an algorithm for calculating the number of rolling elements simultaneously transmitting the load. In [13], the authors investigate the forces and torque in a two-stage sine wave gearbox with IRE and analyze the changes in the output torque together with the main parameters. In [14], the authors present methods for generating profiles of plane cycloids with improved geometry; numerical examples are given to illustrate the proposed methods. In [15], the authors propose a generalized dynamic model for the pin gear transmission, which allows for the estimation of the intensity in the contact of the pin with the cycloid profile. Study [16] proposes an approach to optimizing the cycloid drive profile, stress, and efficiency. The authors in [17,18] develop an algorithm for determining contact stresses in the toothing of a pin gear transmission, and the maximum force in the toothing is determined by the method of successive iterations. Works [19–22] consider the contact characteristics of the pins with the cycloid profile. The forces in the pin gear engagement are determined using a single contact stiffness. The forces obtained using the finite element method, including experimental data, are compared. An algorithm for determining the torque, taking into account the friction forces in the engagement of the pin gear toothing, is developed. In [23], the authors analyze the loadings of the cycloid disk using the finite element method. The authors in [24] use an analytical method for determining forces in cycloid toothing and present a program interface for their calculation. The authors of [25] propose a method for calculating the deformation of the cycloid tooth and the gap between the cycloid profile and the rolling element. Study [26] proposes an expression to determine the geometric parameter of a transmission with intermediate rolling elements based on the contact strength condition. Another study, [27], performs the analysis of forces and contact stresses in cycloid toothing of the end transmission with intermediate rolling elements using the numerical modeling of the finite-element model. The author of [28] proposes a new design of a two-stage cycloid with IRE and studies the forces and stresses in toothing. The clearances in the pin toothing are analyzed in [29]. The authors in [30] determine defects in the cycloid toothing of the pin gear based on vibration signals. The authors consider and improve not only the geometry of the cycloid engagement, but also the design of the transmission in [31]. In [32], the authors present a method for diagnosing cycloidal gear damage on a laboratory stand.

The presented analysis of sources confirms various studies of transmissions with cycloid toothing and works on the algorithms to determine stresses and forces in cycloid toothing; however, there was no algorithm for automated calculation and analysis of the connection of force and geometrical parameters of transmissions with IREFC.

Thus, the algorithm for the automated calculation of force parameters of the IRE transmission is relevant. It will expand the field of knowledge on cycloids with intermediate rolling elements and a free cage.

The purpose of this work is to develop an algorithm for the automated determination of the maximum contact stress in a transmission with rolling elements and a free cage, which makes it possible to determine the parameters of the transmission IREFC corresponding to the maximum stress. This will require the solution of the following tasks:

1. To identify the dependence of geometric parameters from initial parameters of the transmission with IREFC.
2. To identify the dependence of force characteristics from initial parameters of the transmission with IREFC.
3. To develop an algorithm for calculating the force parameters (contact stresses and forces) in toothing with the specified initial parameters of the transmission with IREFC.
4. To create the software according to the developed algorithm that allows studying the dependence of force parameters on the initial ones in the transmission with IREFC.
5. To analyze the obtained results.

2. Materials and Methods

To solve the set tasks, the authors developed a methodological model of the sequence of determining the force characteristics of the transmission with IREFC and the procedure for their analysis with the initial parameters of this transmission.

When finding the curvature radii of cycloid profiles to determine contact stresses, an analytical expression of the Bobillier geometric construction was used based on the Euler-Savary theorem. The forces in toothing and stresses in the contact of the rolling elements with cycloid profiles of wheels were determined on the basis of the Hertz elasticity theory.

A unique data structure was created for the software implementation of the algorithm for the complex calculation of the force parameters of the transmission with IREFC according to a set of parameters, each of which is varied in its range. The calculation implies that a structure sample is created as a dynamic array element for each combination of input parameters. To determine the range of valid values, the calculation algorithm iteratively selects among all the parameters, each of which has its own step and range of changes.

3. Cycloid with Intermediate Rolling Elements and a Free Cage

The transmission with IREFC [26] having a single-pole engagement (Figure 1) consists of an input link–a planet carrier (1), an internal cycloid wheel–a cam (2) installed on a rolling bearing on the planet carrier (1), rolling elements (3) in a cage (4) and an external cycloid wheel–a crown (5). The cam (2) or the crown (5) may serve as the output link. If the output link is the cam (2), then the crown (5) is rigidly fixed in the mechanism body and is stationary. If the output link is the crown (5), then the cam (2) should be fixed. However, due to the specifics of the transmission with IREFC, the cam (2) has tow motions as a rotation and linear motion; its complete fixation is impossible. In this case, the rotation is locked, and the linear motion remains.

Since two toothings are connected in one transmission with IREFC, there are situations when the pitch points P (Figure 2) of the first and the second toothings do not coincide; then the transmission has two pitch points, and it is named as a double-pole transmission. When the pitch point of both toothings in the transmission coincide, such transmission is called single-pole.

The transmission ratio of the rolling elements and a cam is defined as follows:

$$i_{21} = 1 - \frac{1}{Z_2}. \tag{1}$$

The transmission ratio of the rolling elements and a crown is defined as follows:

$$i_{23} = 1 + \frac{1}{Z_2}. \tag{2}$$

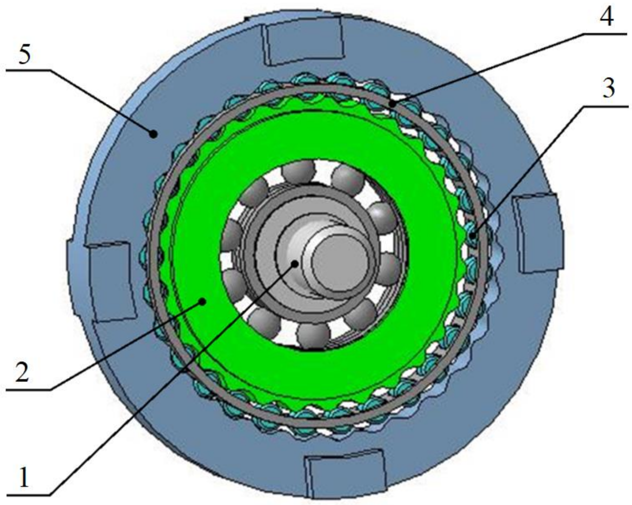

Figure 1. Diagram of the transmission with intermediate rolling elements and a free cage.

The transmission with IREFC has two toothings: rolling elements and a cam (Figure 2a); rolling elements and a crown (Figure 2b) [26].

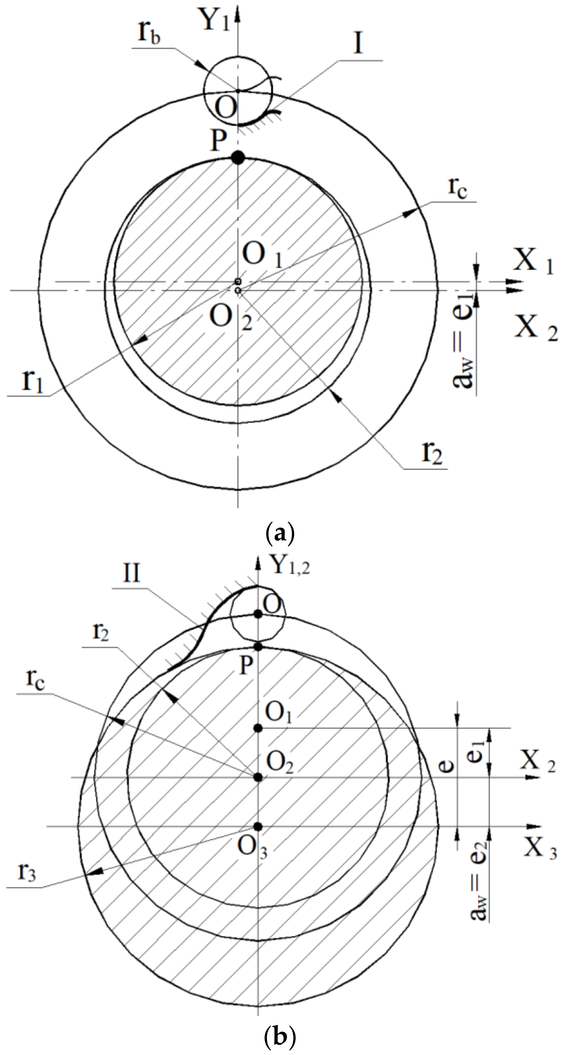

Figure 2. Toothing of the transmission with intermediate rolling elements and a free cage: (**a**)—rolling elements with a cam; (**b**)—rolling elements with a crown.

In expressions (1) and (2), Z_2 is the number of rolling elements, which is one of the initial transmission parameters. In addition, before the beginning of the design, the following initial parameters are set: the generating circle radius r_2 and the displacement factor χ [26]. After choosing the specified three parameters, the range of possible values of the rolling elements radius r_b is determined based on the dependence [26]:

$$2\frac{r_2}{Z_2} < r_b < r_2 \cdot \chi \cdot \sin\left(\frac{180^\circ}{Z_2}\right). \tag{3}$$

The radius (Figure 2) r_c of the centers of the rolling elements is determined using the following expression:

$$r_c = r_2 \cdot \chi. \tag{4}$$

The displacement coefficient χ in the transmission with IREFC is an important parameter, because it influences the cycloid profile geometry and the transmission assemblability; it cannot be less than 1.25. This parameter shows (4) to what degree the radius of the centers of the rolling elements r_c is greater than the radius of the generating circle r_2, and therefore it has a direct impact on the force values in the cycloid toothing.

One should note that in case of the single-pole transmission with IREFC, the initial parameter r_2 is the same for both toothings (rolling elements with the cam and the crown). In the case of the double-pole transmission, different values of this parameter are used. At the same time, the parameter χ is the same for both single-pole and double-pole transmissions. Further, let us consider the calculations intended for designing a single-pole transmission.

Since the initial parameters are preset arbitrarily at the beginning of designing, the geometry of the cycloid toothing, the power characteristics of the transmission with the IREFC, and the designed mechanism depend on them. The equations presented below to represent the transmission geometry and to determine the mesh forces will be written mainly using the initial parameters of the transmission with IREFC.

The axial distance (eccentricity) is determined according to the following expression:

$$a_w = e_i = \frac{r_2}{Z_2}. \tag{5}$$

The total eccentricity of the transmission with IREFC is calculated using the following formula:

$$e = e_1 + e_2,$$

where e_1 and e_2 are the eccentricities of the engagement of the rolling elements with the cam and the crown, respectively (Figure 2b).

The angle φ_2 is present in all the expressions for calculating and designing the transmission with IREFC, which will be observed further. The cycloid profiles are formed when the rolling elements begin to migrate together with a circle of the radius r_2. The angle of this migration is calculated in the same way as φ_2 is.

The rotation angle φ_2 of the separator with rolling elements is used to determine all geometrical parameters of the transmission with IREFC. It is a basic variable in the calculation of geometry and transmission force parameters. To determine the force parameters of the transmission with IREFC, φ_2 should be taken in the range from 0 to 180°. The rotation angle φ_1 of the cam and the rotation angle φ_3 of the crown also depends on φ_2 [26]:

$$\varphi_1 = \frac{\varphi_2}{i_{21}} = \frac{\varphi_2}{\left(1 - \frac{1}{Z_2}\right)}; \tag{6}$$

$$\varphi_3 = \frac{\varphi_2}{i_{23}} = \frac{\varphi_2}{\left(1 + \frac{1}{Z_2}\right)}. \tag{7}$$

These angles determine the position of the cycloid teeth of the cam and crown, depending on the position of the rolling elements.

The radius of the initial circle of the cam (Figure 2) is determined depending on r_2:

$$r_1 = r_2\left(1 - \frac{1}{Z_2}\right).$$

The geometrical parameters of cycloid wheels are determined according to the following expressions:

- cam tip radius

$$r_{tip}^{cm} = r_2\left(\chi + \frac{1}{Z_2}\right) - r_b; \tag{8}$$

- cam root radius

$$r_{root}^{cm} = r_2\left(\chi - \frac{1}{Z_2}\right) - r_b; \tag{9}$$

- crown tip radius

$$r_{tip}^{cn} = r_2\left(\chi - \frac{1}{Z_2}\right) + r_b; \tag{10}$$

- crown root radius

$$r_{root}^{cn} = r_2\left(\chi + \frac{1}{Z_2}\right) + r_b. \tag{11}$$

The expressions (8)–(11) make it possible to define the range of coordinates of the distribution of contact points of cycloid profiles with rolling elements.

4. Loading Characteristic of the Cycloid

To determine the forces in the transmission toothing with IREFC, let us consider the design model (Figure 3). The torque on the cam through toothing forces is expressed through the formula:

$$T_{cam} = \sum F_i \cdot h_i. \tag{12}$$

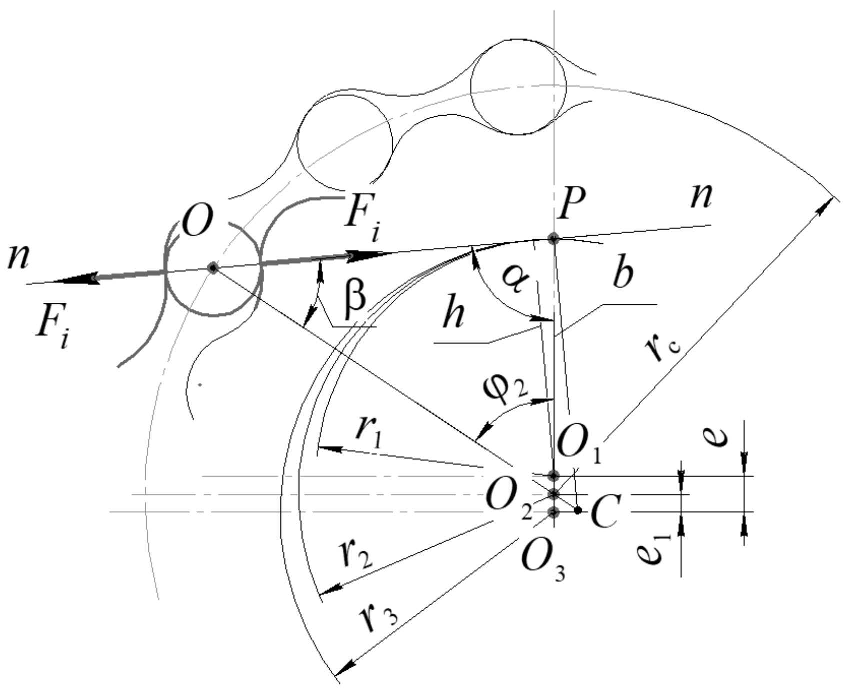

Figure 3. Calculation of forces in the toothing of the transmission with intermediate rolling elements and a free cage.

The arm h is calculated according to the geometry (Figure 3) as follows:

$$h_i = b \cdot sin\alpha_i. \quad (13)$$

where b is the distance from the center of the profile wheel (cam or crown) to the pitch P, which is equal to r_1 for the cam or r_3 for the crown.

The rotation angle α of the normal profile relative to the vertical transmission axis is found via the sine theorem based on the triangle OPO_2 (Figure 3):

$$sin\alpha_i = \frac{sin\varphi_{2i} \cdot r_c}{L_i} \rightarrow \alpha_i = \arcsin\left(\frac{sin\varphi_{2i} \cdot r_c}{L_i}\right) \quad (14)$$

where the distance L_i is determined through the rotation angle φ_2 of the cage with rolling elements and through the initial parameters:

$$PO = L_i = r_2\sqrt{1 + \chi^2 - 2\chi cos\varphi_{2i}} \quad (15)$$

Assuming the fact that the forces in toothing are connected by the linear dependence with the deformations [33], the correspondence of the maximum force and the maximum deformation can be considered. Then the ratio of the force to the corresponding arm in the toothing of one transmission is equal. We know the position of the line of action of the maximum force, because we know the inclination angle of the normal profile at this point of contact and the arm of this force [26]. Thus, let us determine the force on each rolling element using the proportion:

$$\frac{F_i}{h_i} = \frac{F_{max}}{b},$$

Then the force on the i rolling element equals:

$$F_i = \frac{F_{max} \cdot h_i}{b}. \quad (16)$$

Let us substitute the obtained expression into (13), and after conversion we get an expression to determine the maximum force to engage the transmission with IRE and a free cage through the input torque:

$$F_{max} = \frac{T_{cam} \cdot b}{\sum h_i^2}. \quad (17)$$

Then, according to (17), let us determine the force on each rolling element transmitting the load.

By determining all forces in the transmission toothing with IREFC, i.e., at the points of contact of the rolling elements with the profiles of the cam and the crown, it is possible to determine the stresses in these points. The literature [26] provides the formula for determining the contact strength in the toothing of planetary cycloids:

$$(\sigma_H)_i = \sqrt{\frac{T_{cam} h_i E(\rho_{2i} + \rho_{1i})}{2\pi \cdot l_b \cdot \rho_{1i}\rho_{2i}(1 - \mu^2)\sum h_i^2}}. \quad (18)$$

where T_{cam} is the torque on the cam;

ρ_{1i}, ρ_{2i} are radii of the profile curvature of the first and second contacting bodies at the contact point (cycloid profile of the wheel and the rolling element, respectively);

μ is the Poisson's ratio for the first and second contacting bodies, respectively;

E is the elongation modulus of the first and second bodies, respectively;

h_i is the shortest distance from the axis of the cam center to the normal profile and to the cycloid profiles;

l_b is the large side of the contact spot rectangle, in our case equal to the length of the roller.

To determine the radii of the profile curvature of the cam ρ_{1i} and the crown ρ_{3i}, we use the Euler–Savary theorem [34], which can be expressed through the Bobillier construction [33].

The curvature radius of the cycloid profile of the cam and the crown is then determined using the following relationship:

$$\rho_i = \pm L - r_b \mp l_{MP}, \tag{19}$$

where it should be noted that the upper sign is used to calculate the curvature radius of the cycloid profile of the cam, and the lower sign is used to calculate that of the crown profile. The distance l_{MP} from the pitch to the center of the curvature radius of the cycloid profile is defined based on the expression:

$$l_{MP} = \frac{r_2\left(1 - \frac{1}{Z_2}\right)}{\frac{sin\alpha}{tg\beta} + cos\alpha}. \tag{20}$$

In this case, the tangent of the angle of location of the curvature radius center of the cycloid profile is determined based on the following expression:

$$tg\beta = \frac{l_{O_2C} sin\varphi}{l_{O_2C} cos\varphi + \frac{r_2}{Z_2}}.$$

$$l_{O_2C} = \frac{r_2}{sin\varphi \cdot tg\alpha - cos\varphi}.$$

When determining the curvature radius of the corresponding profile from the expression (20), we get a negative radius for the concave section of the profile and a positive radius for the convex section of the profile.

5. Automatization Calculation Algorithm of Loading Characteristic

Specifying the initial radius of the generating circle (r_2) and the radius of the rolling element (r_b), as well as the number of rolling elements (Z_2) and the displacement coefficient (χ), let us determine the geometrical parameters of the transmission with IREFC. By setting the output torque T_{out} and the length of the rolling body l_b, let us determine the forces in the cycloid toothing and the contact stresses. These calculations are performed using expressions (1)–(20) according to the sequence in the algorithm (Figure 4).

Let us note that we should set the range of changes Z_2 and χ before starting the calculations to analyze the change in the maximum contact stress and the corresponding forces in the toothing of the cycloid with IREFC, as well as other parameters. In this case, the number of rolling elements Z_2 can vary arbitrarily, but only in increments of one, and the displacement coefficient χ is always considered in the range of 1.25–1.67, but in arbitrary increments.

When developing the algorithm, specifying the length of the rolling body is provided: it is equal to the diameter of the rolling body by default and, if required, it is arbitrary. Next, the maximum toothing force is determined by creating a temporary data array.

After determining the maximum force in the cycloid toothing, we generate an array of data for the angle φ_2 of the position of the rolling elements in increments of 0.02°. The study of the cycloid profile is performed in the range of angle φ_2 from 0° to 180°.

Further, using the received ranges Z_2 and χ, as well as the maximum force determined earlier (18), the geometrical parameters of cycloid toothing, forces, and contact stresses at each point of the cycloid profile are calculated. All calculated parameters are entered into the structured array base. After filling the data array, the maximum and corresponding initial data, forces in toothing, and geometrical parameters of the cycloid with IREFC are determined using a sequential comparison according to contact stresses.

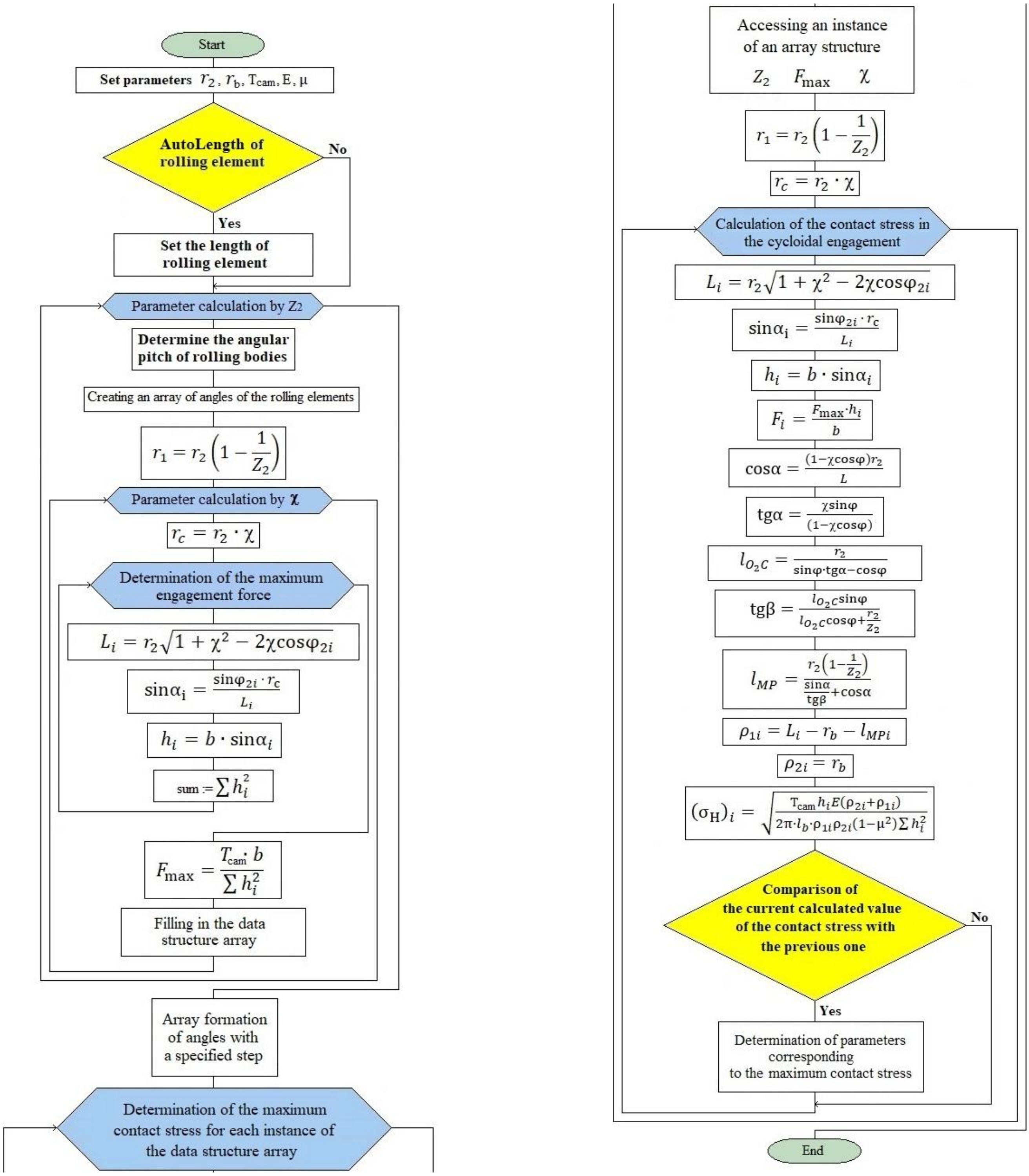

Figure 4. Algorithm for determining the maximum contact stresses in a cycloid.

Having determined the maximum contact stress in the cycloid toothing, let us determine the dependencies between the force and geometrical parameters in the form of graphs.

Thus, a data structure was created for the software implementation of a complex transmission calculation with intermediate rolling elements based on a set of parameters, each of which varies in its range. The calculation implies that a structure sample is created as a dynamic array element for each combination of the input parameters. To determine the range of valid values, the calculation algorithm iteratively selects among all the parameters, each of which has its own step and range of changes. The calculation results are recorded in the corresponding copy of the dynamic array. After calculating all the combinations of the parameters, the results are displayed on the screen as a text and a graph.

6. Results and Discussions

A calculation program was written in the Lazarus cross-platform development environment according to the developed algorithm. The average calculation and plotting time is 0.2 s for the parameters taken as examples. However, the calculation time directly depends on the specified ranges and the steps of the calculated parameters, such as the number of rolling elements Z_2 and the displacement coefficient χ. That is, the wider the ranges and the smaller the calculation step, the longer the calculation time. However, in the case of our calculations, having different ranges and calculation steps up to 0.002, the calculation time was not more than 0.3 s. A personal computer with an Intel(R) Core(TM) i5-9400F 2.9 GHz processor and random-access memory of 8 GB was used for the calculations.

According to the algorithm of the automated calculation of the force parameters of the cycloid with IREFC, the major result is the multiple of force characteristic values of the transmission with IRE and the geometrical parameters corresponding to them. In contrast to [35], where a computational program with a user-friendly interface is proposed, this algorithm allows us to comprehensively study the changes in the power characteristics in a wide range of several initial parameters of the transmission with IREFC.

The algorithm makes it possible to obtain a large amount of data on the values of forces and contact stresses in the toothing of the cycloid with IREFC by determining these data at high resolution (small step). This allows for a more accurate analysis of the cycloid profile loading. The use of a structured data array allows us to reduce the time taken to find the necessary information and the time taken for designing the entire mechanism.

Figure 5 shows the interface of the program developed according to the above algorithm, which allows us to quickly change the initial parameters of the transmission with IRE and their limits of testing (on the left). The calculated values of the force parameters and geometrical parameters corresponding to them (on the right) are immediately displayed with high accuracy. The use of the algorithm makes it possible to obtain an exact value of the contact stress, since the cycloid profile is analyzed in increments of 0.02° over the angle.

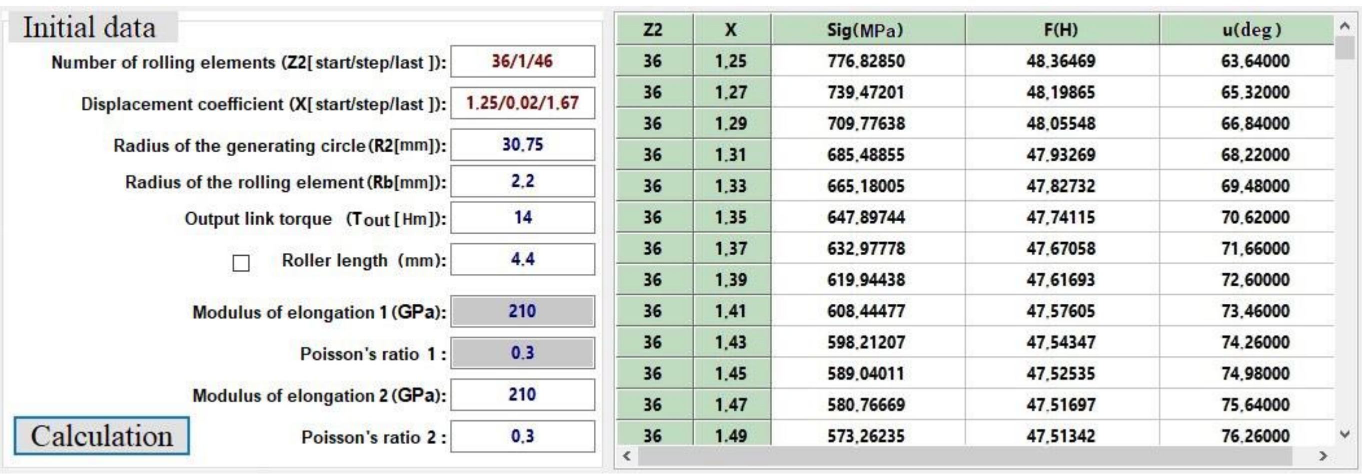

Figure 5. Interface of the program for automated calculation of the transmission force parameters with intermediate rolling elements and a free cage according to the developed algorithm.

According to the obtained data, the structured array element allows us to build and study the dependencies of various parameters of the transmission with IREFC. For example, the maximum contact stress for different values of χ and Z_2 corresponds to the graph of studying the change in the position angle of rolling elements (Figure 6). The graph (Figure 6) shows that for different values Z_2, the angle of occurrence of the maximum contact stress varies within different limits in the specified range of the variation of the displacement coefficient χ. In this way, in the presence of definite parameters Z_2 and χ, one angle φ_2 corresponds to the maximum contact stress. Thus, the use of a structured array allows us to quickly obtain a change in the force parameters depending on the initial ones for the transmission with IREFC, which is also illustrated below.

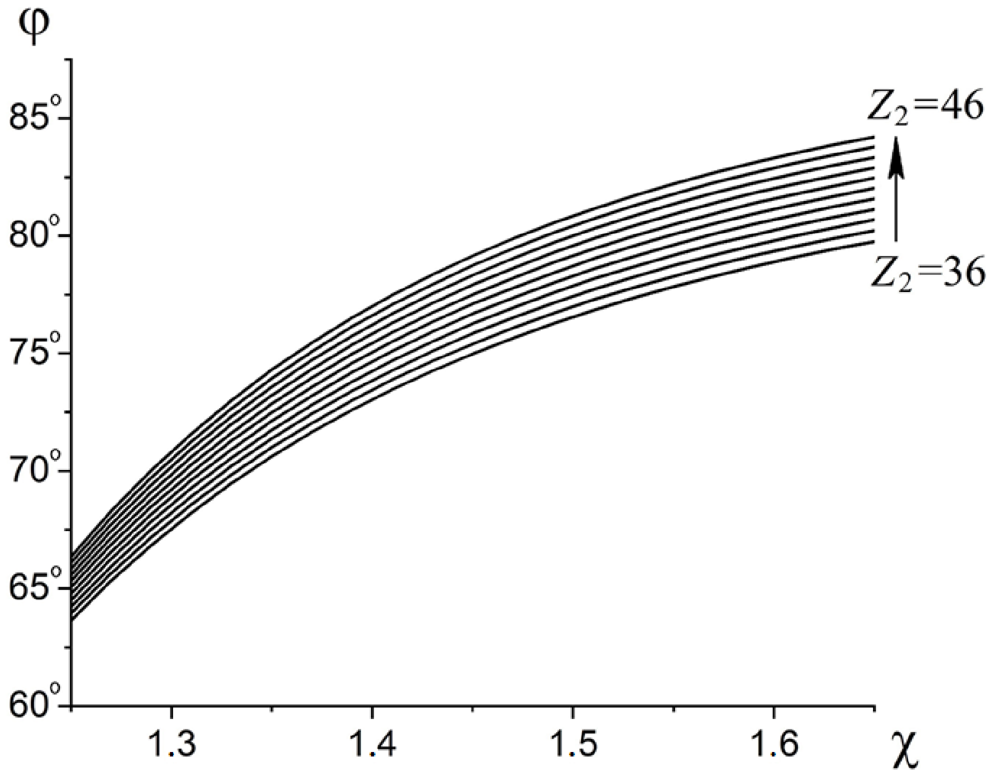

Figure 6. Dependence of the maximum contact stress angle on the displacement coefficient χ at different values of the rolling elements number Z_2.

Figure 7 shows the multiple of the graphs of the change in the maximum contact stress in the cycloid toothing with IREFC for the same range χ and a different number of transmission rolling elements. The algorithm makes it possible to clearly see what displacement factor should be selected for a particular number of rolling elements in order to obtain the minimum value of the contact stress in the designed mechanism. Or otherwise, it allows us to select the number of rolling elements for a particular displacement factor in order to minimize loads in the designed product.

It is also possible to study the change in the maximum force in the toothing of the cycloid with IREFC (Figure 8). The use of the algorithm makes it possible to see that the maximum force in the cycloid toothing for a certain number of rolling elements almost does not depend on the displacement coefficient χ.

The results of the algorithm operation determine the dependence of the force characteristics on the initial geometric parameters of the cycloid with IREFC; the composite parameters, such as eccentricity, circle radius of centers, etc., are not analyzed at this stage.

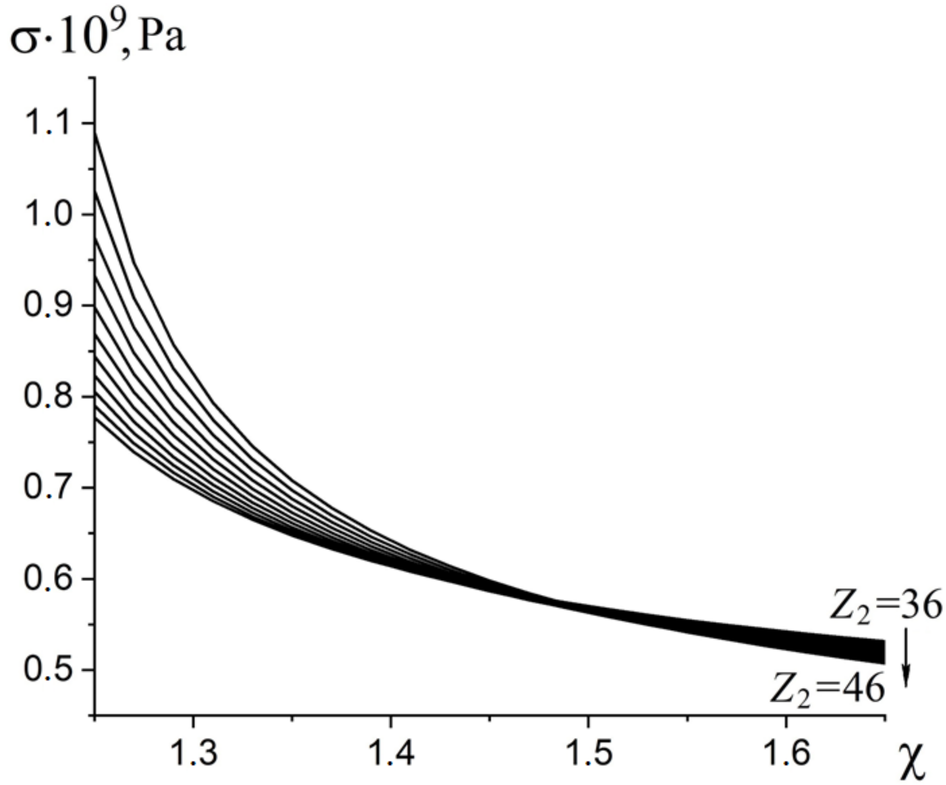

Figure 7. Dependence of the maximum contact stress on the displacement coefficient χ at different values of the rolling elements number Z_2.

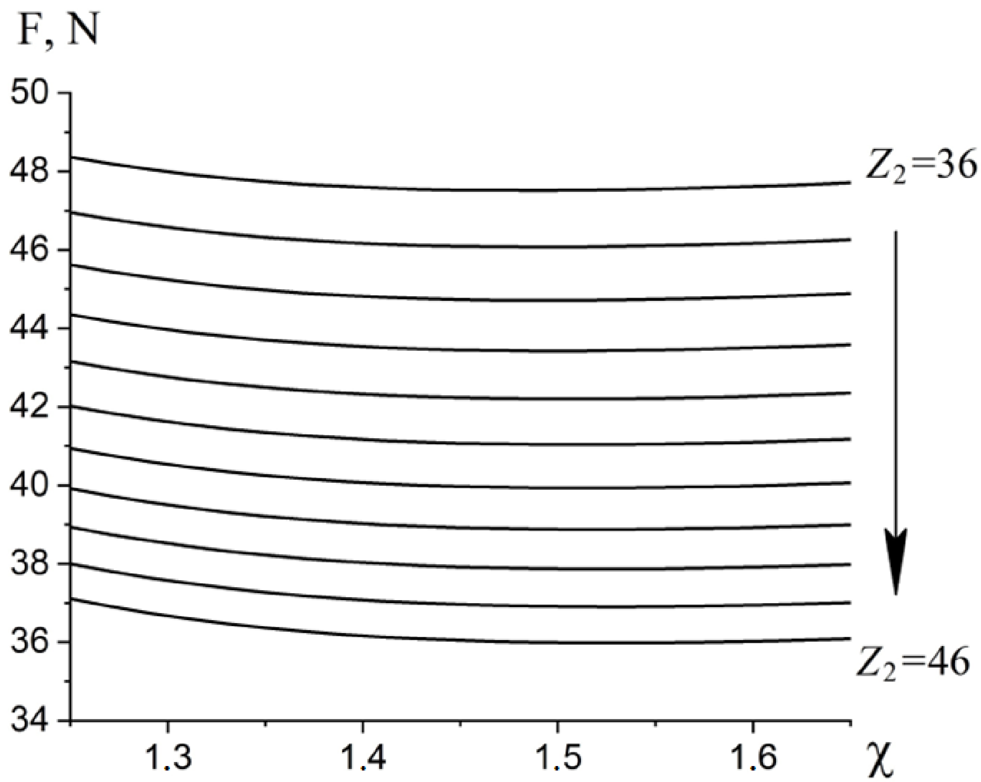

Figure 8. Dependence of the maximum force in the toothing on the displacement coefficient χ at different values of the rolling elements number Z_2.

7. Conclusions

Thus, the developed algorithm of the automated determination of force characteristics depending on the initial and geometrical parameters of the cycloid with intermediate rolling elements and a free cage allows us to reduce the time required to provide data for the design of the information mechanism. The proposed algorithm made it possible to build a program into the Lazarus cross-platform development environment. The program allows us to study various combinations of the force and geometrical parameters, such as:

- the maximum contact stress in toothing;
- angles of rolling element positions corresponding to the maximum contact stress;
- toothing forces corresponding to the maximum contact stress;
- number of rolling elements varying over a wide range; and
- the displacement factor varying over a wide range.

These studies can also be carried out for various materials of contacting links: both cycloid wheels and rolling elements.

The algorithm made it possible to create a structured array of force and geometric parameters of the transmission with IREFC, which facilitates the access to the necessary parameters and eliminates errors in providing the incorrect combination of the requested parameters.

The work of the algorithm and the developed program, as compared to the design time without using the algorithm, allowed us to reduce the time allotted for selecting the correct parameters by 30% and the time for designing the mechanism based on the transmission with IREFC by 15%. Without the proposed algorithm, the selection of operable transmission parameters and the process of designing the mechanism on its base took more time.

The algorithm allows us, with a new degree of accuracy, to analyze the cycloid profile, which, in turn, makes it possible to determine most accurately both the values of the maximum force and contact stress in the cycloid toothing, and the accuracy of the values themselves.

The algorithm allows us to analyze both the dependence of the mesh force on the displacement coefficient and the forces arising due to the change in the radius of the generating circle or the center circle of the rolling elements. It is also possible to analyze changes in contact stresses arising due to the eccentricity of the transmission with IREFC and the number of rolling elements, the change in the number of rolling elements owing to the displacement coefficient, and the number of rolling elements due to the mesh force. Obviously, not all of these graphs have been presented in this work. The radius of the centers of the rolling elements and the eccentricity of the transmission are complex composite parameters, and the analysis of changing the forces owing to these parameters is the next stage of the work. The analysis of the influence of clearances in the toothing on the distribution of forces, when rolling elements contact with the cycloid profile, is also planned to be included in the next stage of the work.

The developed algorithm can be used when designing one-, two-stage, etc., reducers, but it is necessary to design each stage separately. In the long term, the end-to-end design of multi-stage cycloids is planned to be studied thoroughly. The algorithm may also be extended to determine the gap in the toothing of the designed cycloid.

Author Contributions: Conceptualization, E.A.E.; methodology, S.A.S.; software, S.A.S.; validation, E.A.E.; formal analysis, N.V.M.; investigation, E.A.E.; resources, N.V.M.; data curation, S.A.S.; writing—original draft preparation, E.A.E.; writing—review and editing, N.V.M.; visualization, S.A.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available from the corresponding authors upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kan, A.; Il'In, A.S.; Lazurkevich, A. Load analysis of the planetary gear train with intermediate rollers. Part 2. *IOP Conf. Ser. Mater. Sci. Eng.* **2016**, *124*, 012004. [CrossRef]
2. Lin, K.-S.; Chan, K.-Y.; Lee, J.-J. Kinematic error analysis and tolerance allocation of cycloidal gear reducers. *Mech. Mach. Theory* **2018**, *124*, 73–91. [CrossRef]
3. Bednarczyk, S. Analysis of the cycloidal reducer output mechanism while taking into account machining deviations. *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **2021**, *235*, 7299–7313. [CrossRef]
4. Efremenkov, E.A.; Martyushev, N.V.; Skeeba, V.Y.; Grechneva, M.V.; Olisov, A.V.; Ens, A.D. Research on the Possibility of Lowering the Manufacturing Accuracy of Cycloid Transmission Wheels with Intermediate Rolling Elements and a Free Cage. *Appl. Sci.* **2021**, *12*, 5. [CrossRef]
5. Ren, Y.; Bian, W.; Yan, R. Optimization analysis of air gap magnetic field in electromagnetic harmonic movable tooth drive system. *Int. J. Comput. Eng.* **2019**, *4*, 132–135.
6. Ren, Y.; Ma, Y.; Wan, R. Parametric design of inner teeth of center wheel in electromagnetic harmonic movable teeth transmission. *J. Phys. Conf. Ser.* **2021**, *1965*, 012009.
7. Blagojević, M.; Matejic, M.; Kostić, N. Dynamic Behaviour of a Two-Stage Cycloidal Speed Reducer of a New Design Concept. *Teh. Vjesn.—Technol. Gaz.* **2018**, *25*, 291–298. [CrossRef]
8. Bechhoefer, E. Automated Condition Monitoring of a Cycloid Gearbox. In *Maintenance Management-Current Challenges, New Developments, and Future Directions*; IntechOpen: London, UK, 2022. [CrossRef]
9. Grosso, P.; Massaccesi, G.; Cavalaglio Camargo Molano, J.; Mottola, G.; Borghi, D. Signal model of a cycloidal drive for diagnostic purposes. In Proceedings of the ISMA-USD Conference, Leuven, Belgium, 12–14 September 2022; pp. 750–764.
10. Prudnikov, A.P. Planetary Torus Helical Transmission. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *795*, 012022. [CrossRef]
11. Lustenkov, M.E.; Lustenkova, E.S. Load Capacity of Spherical Roller Transmission with Double-Row Pinion. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *795*, 012020. [CrossRef]
12. E Lustenkov, M.; Moiseenko, A.N. Analysis of contact strength of spherical roller transmission with double-row pinion. *IOP Conf. Ser. Mater. Sci. Eng.* **2021**, *1118*, 012006. [CrossRef]
13. Xu, L.; Song, W. Two-step sine movable tooth drive. *Adv. Mech. Eng.* **2017**, *9*, 1–9.
14. Litvin, F.L.; Feng, P.-H. Computerized Design and Generation of Cycloidal Gearing. *Mech. Mach. Theory* **1996**, *31*, 891–911. [CrossRef]
15. Xu, L.X.; Chen, B.K.; Li, C.Y. Dynamic modelling and contact analysis of bearing-cycloid-pinwheel transmission mecha-nisms used in joint rotate vector reducers. *Mech. Mach. Theory* **2019**, *137*, 432–458.
16. Sensinger, J.W. Unified Approach to Cycloid Drive Profile, Stress, and Efficiency Optimization. *J. Mech. Des.* **2010**, *132*, 024503. [CrossRef]
17. Hu, Y.H.; Li, G.; Zhu, W.D.; Cui, J.K. An Elastic Transmission Error Compensation Method for Rotary Vector Speed Re-ducers Based on Error Sensitivity Analysis. *Appl. Sci.* **2020**, *10*, 481.
18. Li, X.; Tang, L.; He, H.; Sun, L. Design and Load Distribution Analysis of the Mismatched Cycloid-Pin Gear Pair in RV Speed Reducers. *Machines* **2022**, *10*, 672. [CrossRef]
19. Malhotra, S.; Parameswaran, M. Analysis of a cycloid speed reducer. *Mech. Mach. Theory* **1983**, *18*, 491–499. [CrossRef]
20. Blanche, J.G.; Yang, D.C.H. Cycloid Drives with Machining Tolerances. *J. Mech. Des.* **1989**, *111*, 337–344. [CrossRef]
21. Gorla, C.; Davoli, P.; Rosa, F.; Longoni, C.; Chiozzi, F.; Samarani, A. Theoretical and Experimental Analysis of a Cycloidal Speed Reducer. *J. Mech. Des.* **2008**, *130*, 112604. [CrossRef]
22. Wang, H.; Shi, Z.-Y.; Yu, B.; Xu, H. Transmission Performance Analysis of RV Reducers Influenced by Profile Modification and Load. *Appl. Sci.* **2019**, *9*, 4099. [CrossRef]
23. Thube, S.V.; Bobak, T.R. *Dynamic Analysis of a Cycloidal Gearbox Using Finite Element Method*; American Gear Manufacturers Association: Alexandria, Virginia, 2012; 13p.
24. Bao, J.; He, W. Parametric Design and Efficiency Analysis of the Output-Pin-Wheel Cycloid Transmission. *Int. J. Control Autom.* **2015**, *8*, 349–362. [CrossRef]
25. Blagojevic, M. Analysis of clearances and deformations at cycloid disc. *Mach. Des.* **2014**, *6*, 79–84.
26. Efremenkov, E.A.; Efremenkova, S.K. Design of a heavily loaded cycloidal transmission with a free cage based on the contact strength condition. Bulletin of Tomsk Polytechnic University. *Eng. Georesour.* **2021**, *332*, 182–188.
27. Nam, W.K.; Shin, J.W.; Oh, S.H. Design of thin plate-type speed reducers using balls for robots. *J. Mech. Sci. Technol.* **2013**, *27*, 519–524. [CrossRef]
28. Wu, S.; Yan, H.-Z.; Bi, R.; Wang, Z.; Zhu, P. Nonlinear Optimization Method for Transmission Error of Hypoid Gear Machined by the Duplex Helical Method. *Math. Probl. Eng.* **2020**, *2020*, 626089. [CrossRef]
29. Król, R. Analysis of the backlash in the single stage cycloidal gearbox. *Arch. Mech. Eng.* **2022**, *69*, 693–711.

30. Al Hajj, M.A.; Quaglia, G.; Schulz, I. Condition-Based Monitoring on High-Precision Gearbox for Robotic Applications. *Shock. Vib.* **2022**, *2022*, 6653723. [CrossRef]
31. Maccioni, L.; Concli, F.; Blagojevic, M. A new three-stage gearbox concept for high reduction ratios: Use of a nested-cycloidal architecture to increase the power density. *Mech. Mach. Theory* **2023**, *181*, 105203. [CrossRef]
32. Komorska, I.; Olejarczyk, K.; Puchalski, A.; Wikło, M.; Wołczyński, Z. Fault Diagnosing of Cycloidal Gear Reducer Using Statistical Features of Vibration Signal and Multifractal Spectra. *Sensors* **2023**, *23*, 1645. [CrossRef]
33. Kudryavtsev, V.N. *Planetary Gears*; M/S: Moscow, Russia, 1966.
34. Litvin, F.L.; Fuentes, A.; Curti, G. *Gear Geometry and Applied Theory*; Cambridge University Press: Cambridge, UK, 2004; pp. 267–286.
35. A Efremenkov, E.; Bonnard, E. Power Parameters Automated Calculation for Transmission with Intermediate Rolling Bodies and Free Cage. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *795*, 012025. [CrossRef]