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THE MODEL OF BEAM OVERLAP FOR BEAMS WITH Q-GAUSSIAN DISTRIBUTION

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МОДЕЛИРОВАНИЕ ПЕРЕКРЫТИЯ ПУЧКОВ С Q-ГАУССОВЫМ РАСПРЕДЕЛЕНИЕМ ЧАСТИЦ

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Аннотация. *В работе исследуется интеграл перекрытия пучков с Q-гауссовым распределением частиц. Получена аналитическая формула для такого интеграла в зависимости от расстояния между орбитами пучков, что позволяет применить ее для моделирования ван-дер-Меер скана. Ван-дер-Меер скан представляет собой метод калибровки светимости современных коллайдеров заряженных частиц. Работа представляет практический интерес в ускорительной физике и физике высоких энергий*.

Introduction. Luminosity is one of the most important parameters of a collider, which characterizes the intensity of the particle collisions at the interaction point. Therefore, its precise calibration is crucial for determining the interaction cross-section. At hadron colliders, a beam separation scan technique, the so-called van-der-Meer scan, is used for luminosity calibration. It is based on sweeping two beams against each other and recording their interaction rate at different separations between the beams orbits; the recorded rates are then fitted and the luminosity is calibrated [1]. The current van-der-Meer scan fit models are based on either Gaussian or double Gaussian models, which follows from the assumption that the colliding bunches have Gaussian or double Gaussian profiles. With the boost evolution of the colliding facilities, it was found that the colliding bunches have a non-Gaussian tail [2]. Thus, for high-precision calibration, the non-Gaussian tails should be considered. It was found that the *q*-Gaussian distribution presents a more realistic description of the bunch profile [3]. It can describe both Gaussian and non-Gaussian tails, from finitely light-tailed to heavy-tailed bunches. In this work, the *q*-Gaussian distribution is used to investigate the effect of the non-Gaussian tails on the overlap integral. An analytical formula for the overlap integral of *q*-Gaussian bunches is obtained. The deviation of the overlap integral of Gaussian and *q*-Gaussian bunches is estimated. A van-der-Meer scan was simulated with *q*-Gaussian bunches. A new fit model is proposed.

Theory. The *q*-Gaussian distribution has been used to investigate emittance evolution and beam profile modeling for the Large Hadron Collider (LHC) [4]. It is known for its remarkable ability to represent a vast array of distributions depending on the value of *q* as shown in Fig. 1a and is defined as:

$$QG\left(u;q,\beta^{qG}\right)=\frac{\sqrt{\beta^{qG}}}{C^{qG}}e_q\left(-\beta^{qG}u^2\right), \tag{1}$$

where β^{qG} is a real positive number, e_q is a q-exponential, which is defined as:

$$e_q(x) = \begin{cases} \exp(x) & q = 1 \\ \left[1+(1-q)x\right]_+^{1/(1-q)} & q \neq 1 \end{cases}$$

and C^{qG} is the normalization constant (see Appendix 1 in [4]). The standard deviation σ of the q-Gaussian distribution is

$$\sigma = \frac{1}{\sqrt{(5-3q)\beta^{qG}}} \tag{2}$$

Figure 1a shows how q controls the tail density of the distribution, at $q < 1$ the distribution becomes finite with light tails with range $u \in \left[-\sqrt{1/(1-q)\beta^{qG}}, \sqrt{1/(1-q)\beta^{qG}}\right]$. For $q > 1$ the tail density increases, and the distribution becomes infinite with heavy tails. At $q = 1$ the standard Gaussian distribution is restored. From equation (2), the tail density q is limited to $q < 5/3$.

The overlap integral Ω of two arbitrary bunches with the separation Δ_u between their orbits is defined as:

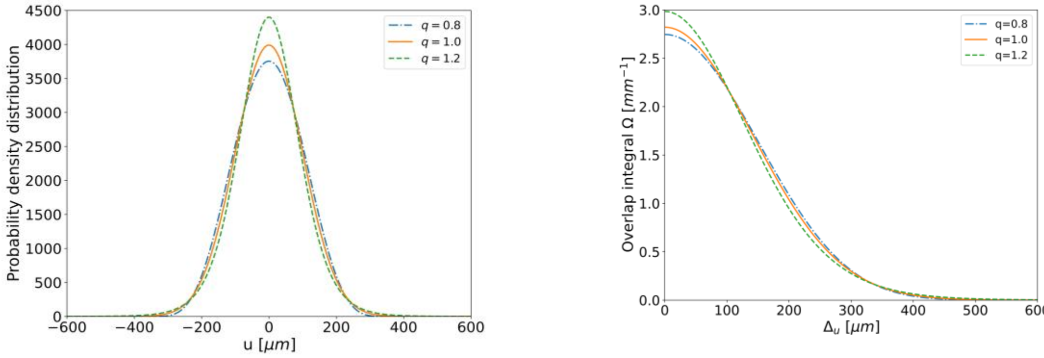
$$\Omega(\Delta_u) = \int \rho_1\left(u - \frac{\Delta_u}{2}\right)\rho_2\left(u + \frac{\Delta_u}{2}\right)du \tag{3}$$

By substituting q-Gaussian distribution (1) into equation (3), the analytical formula for the overlap integral of q-Gaussian bunches with equal dimensions and tail densities in their respective direction (i.e. $\sigma_{1u} = \sigma_{2u} = \sigma_u$ and $q_{1u} = q_{2u} = q_u$) is obtained as:

$$\Omega^{qG}(\Delta_u) = \begin{cases} \dfrac{\sqrt{\beta^{qG}}}{\sqrt{1-q}\,C^{qG2}}\left(1-\sqrt{(1-q)\beta^{qG}\dfrac{\Delta_u^2}{4}}\right)\left(1-(1-q)\beta^{qG}\dfrac{\Delta_u^2}{4}\right)^{2/(1-q)} & \\ \times\, Beta\left(\dfrac{1}{2},\dfrac{2-q}{1-q}\right){}_2F_1\left(\dfrac{-1}{1-q},\dfrac{1}{2};\dfrac{5-3q}{2-2q};\dfrac{\left(1-\sqrt{(1-q)\beta^{qG}\dfrac{\Delta_u^2}{4}}\right)}{\left(1+\sqrt{(1-q)\beta^{qG}\dfrac{\Delta_u^2}{4}}\right)}\right) & q < 1 \\ \dfrac{\sqrt{\beta^{qG}}}{\sqrt{2}C^{qG}}\exp\left(-\beta^{qG}\dfrac{\Delta_u^2}{2}\right) & q = 1 \\ \dfrac{\sqrt{\beta^{qG}}}{\sqrt{q-1}\,C^{qG2}}Beta\left(\dfrac{1}{2},\dfrac{5-q}{2q-2}\right){}_2F_1\left(\dfrac{1}{q-1},\dfrac{5-q}{2q-2};\dfrac{q+1}{2-2q};(1-q)\beta^{qG}\dfrac{\Delta_u^2}{4}\right) & q > 1 \end{cases} \tag{4}$$

To study the effect of the non-Gaussian tails on the luminosity calibration, the analytical formula of the overlap integral of q-Gaussian bunches (4) is used to simulate a ‘toy’ van-der-Meer scan. A data set of 25 pairs of (Δ_u, Ω^{qG}) is obtained for different separations Δ_u and their respective overlap integral Ω for bunches with equal dimensions $\sigma = 100$ μm and for two different cases of tail densities q: light-tailed ‘0.8’ and heavy-tailed ‘1.2’. Then the resultant data sets are fitted by Gaussian, double Gaussian and q-Gaussian fit models, where the double Gaussian model was applied only for heavy-tailed bunches, as its concept of application does not coincide with the light-tailed bunches.

Results. Figure 1 shows the profiles of q-Gaussian bunches and their overlap integrals at different separations. The heavy-tailed bunches have the highest overlap integral at small separations with $\Delta_u < 1.05\ \sigma$ and large separations $\Delta_u > 3.345\ \sigma$, while for $1.05 < \Delta_u < 3.345\ \sigma$, the overlap integral of light-tailed bunches is higher. Table 1 summarizes the deviations of the overlap integral of *q*-Gaussian from that of Gaussian.



(a) *(b)*

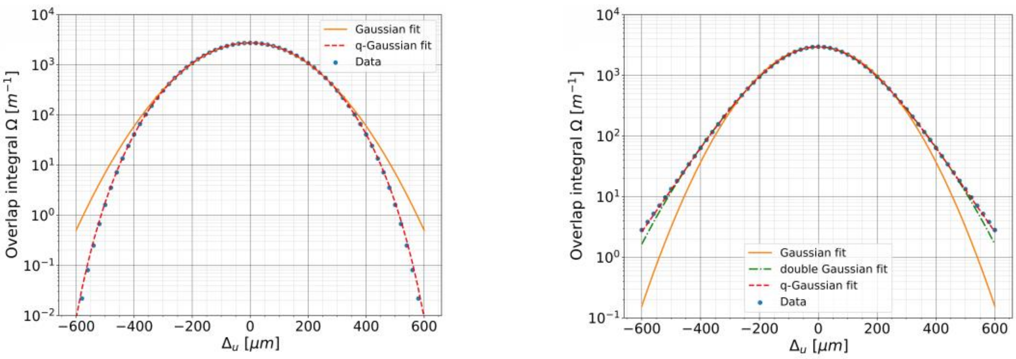
Fig. 1. The bunch profile of q-Gaussian bunches with dimension σ = 100 μm and tail densities q = 0.8, 1.0 and 1.2 (1a) and their overlap integral during the separation scan over separation Δ_u ranges from 0 to 6 σ (1b)

Table 1

The deviations of the overlap integral of q-Gaussian bunch from that of Gaussian

Tail population density	Δ_u <1.05 σ	1.05 < Δ_u < 3.345 σ	Δ_u > 3.345 σ
Light-tailed $q < 1$	−2.64 %	5.75 %	−20 %
Heavy-tailed $q > 1$	5.79 %	−10.46 %	24 %

Figure 2 shows the fitting of the van-der-Meer scan data. For light-tailed bunches with $q = 0.8$, the *q*-Gaussian fit model describes the scan data with high precision, while Gaussian fit model could not describe the data. For heavy-tailed bunches with $q = 1.2$, double Gaussian and *q*-Gaussian fit models could describe the scan data, where the *q*-Gaussian fit model has a higher accuracy than double Gaussian. Table 2 summarizes the deviations of the overlap integral predicted by the fit models from that obtained by the analytical formula (4).



(a) *(b)*

Fig. 2. The fitting of the 'toy' van-der-Meer scan data for light-tailed 'q = 0.8' (a) and heavy-tailed 'q = 1.2' (b) q-Gaussian bunches

Table 2

The deviations of the overlap integral predicted by different fit models from the analytical formula (4)

Tail population density	Gaussian	double Gaussian	*q*-Gaussian
Light-tailed $q < 1$	0.461 %	———	– 0.022 %
Heavy-tailed $q < 1$	– 0.771 %	0.037 %	0.018 %

Conclusion. It was shown that in the case of non-Gaussian beams, a more precise model is needed for the van-der-Meer scan since using Gaussian or double Gaussian models might result in an overestimation of the overlap integral and, therefore, the calibrated luminosity at the same beam size (RMS). The *q*-Gaussian model fits not only the heavy-tailed bunches but also the light-tailed ones more precisely than the current models based on Gaussian or double Gaussian assumptions, and it represents a good approximation of the analytical model with a simpler form. The models based on *q*-Gaussian are advised to be further studied and applied for the HL-LHC upgrade.

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