Measurement error reducing in the ultrasound time-pulse systems

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Abstract. Ultrasound wave propagation in bounded media results in change of the echo-pulse shape, therefore the use of traditional method of determining of the propagation time of the ultrasound pulse based on use of threshold device (comparator) gives a large error of measurement. To reduce this error the new methods of the determining of echo-pulse propagation time are proposed in this paper. The first method is the method based on the use of two comparators, the second one is the method based on second-order polynomial approximation of the echo-pulse envelope. Analysis showed that the steeper the rising edge of the echo-pulse envelope, the less the error of determining of the pulse start time. The more the difference between comparator thresholds the less the error of determining of the pulse start time. Application of the method of two comparators gives two times systematic error reducing. Minimal sampling of input signal is determined for the method based on echo-pulse envelope. Dependence of the error at start echo-pulse determining on adjacent samples amplitude ratio is obtained. Application of echo-pulse envelope gives three times systematic error reducing.

1. Introduction

Ultrasound devices based on time-pulse method are widely used nowadays. Basic error of measurement of such devices is due to inaccuracy of determining of the sound pulse start time. Usually sound pulse start time is determined by the comparator, but due to the complex shape of the sound pulse, the response time of the comparator does not coincide with the pulse start.

Another widely used method to determine echo-pulse start time is zero-crossing method [1, 2]. But it is impossible to determine the number of period in echo-pulse when the algorithm of calculation of zero-crossing instant was applied. Therefore it is impossible to determine the start time of echo-pulse with high accuracy.

One more method uses Hilbert transform. Disadvantage of this method is the complexity of the algorithm that can be implemented only with use of personal computer [3]. Implementation of Gilbert transform is impossible in portable devices, based on microcontrollers.

If the amplitude and shape of a signal does no change, then the error of determining of the time of signal propagation in the medium also remains constant and can be taken into account. However, in practice, signal amplitude is decreases at the signal propagation. This decreasing is due to signal divergence and losses in the medium. Automatic gain control maintains a constant amplitude signal. But if the signal shape changes in process of its propagation, the application of automatic gain control or a comparator with "tracing" threshold in this case does not solve the problem of accuracy in

determining of the pulse start time. The fundamental basis of this phenomenon is the inequality of the phase velocities of different modes (Figure 1).
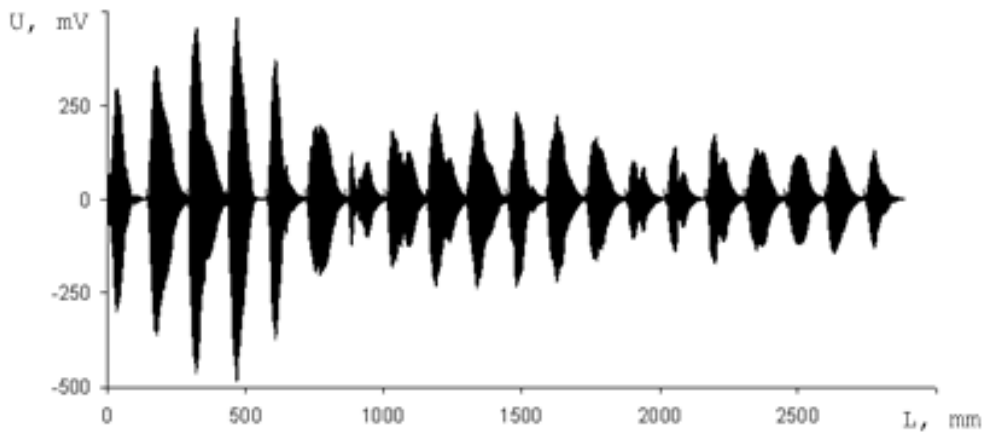

Figure 1. Change of the signal shape when it propagates through the waveguide.

Figure 2 shows two signals with the same amplitudes, but with different shape of the rising edge. The change of the rising edge of the pulse envelope at 10-15 % results in the error of determining of the pulse position at 2–3 periods of carrier frequency. This error is no longer constant, but varies depending on changes in the duration of the rising edge, and it is impossible take it into account in traditional ways.
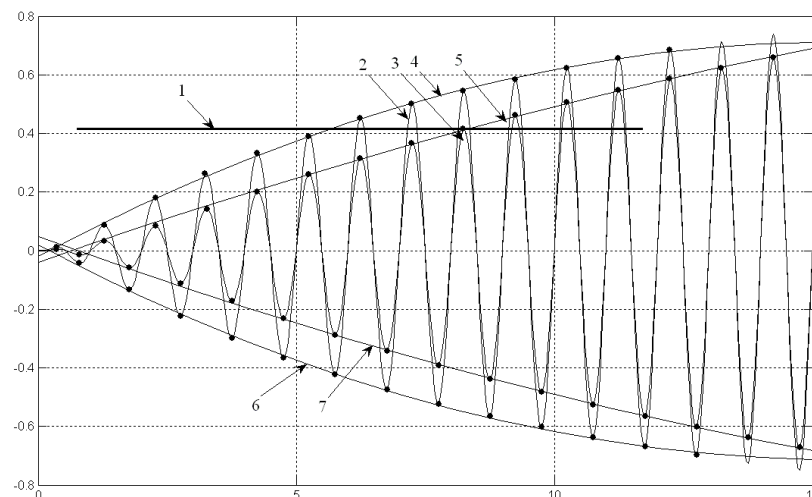

Figure 2. Determining of the coordinates of the pulse start time using the method of approximation of the signal envelope to 2nd power of a polynomial:
1) comparator threshold level; 2) the 1st signal with higher rising edge;
3) the 2nd signal with lower rising edge; 4) envelope of the positive extrema of the 1st signal; 5) envelope of the positive extrema of the 2nd signal; 6) envelope of the negative extrema of the 1st signal; 7) envelope of the negative extrema of the 2nd signal.

2. Description of the first method
It is possible to increase the accuracy of measurement, if we apply modern methods of the echo signals processing. One of such methods is the method of two comparators with different thresholds [4, 5] (Figure 3).
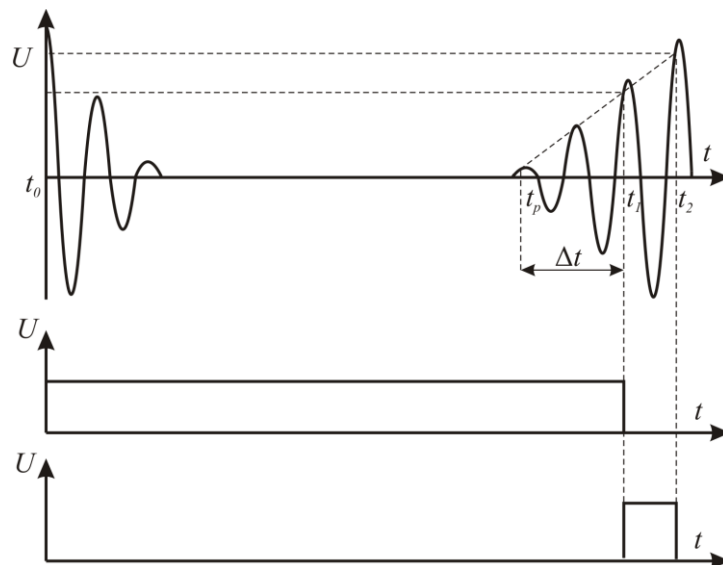
Figure 3. Determining of the echo pulse position using the method of two comparators.

At the instant t1 the 1st comparator triggers at the level of $U1$. At the instant t2 the 2nd comparator triggers at the level of $U2$. Constructing the line at the coordinates of these points allows determining the time coordinate t of the echo pulse start time by the expression:

$$t_p = t_1 - \frac{U_1}{U_2 - U_1} \cdot (t_2 - t_1)$$ (1)

Define the maximum error obtained by this method for the linear part of the envelope of the echo pulse rising edge:

$$U_{i+1}^{max} = U_i^{max} + k \cdot U_1,$$ (2)

where $i = 0, 1,..., k < 1$, U_1 is threshold voltage of the first comparator.

2.1 Modeling

The maximum error is obtained, when the levels U_1 and U_2 reach the maxima values of the previous peaks (Figure 4).

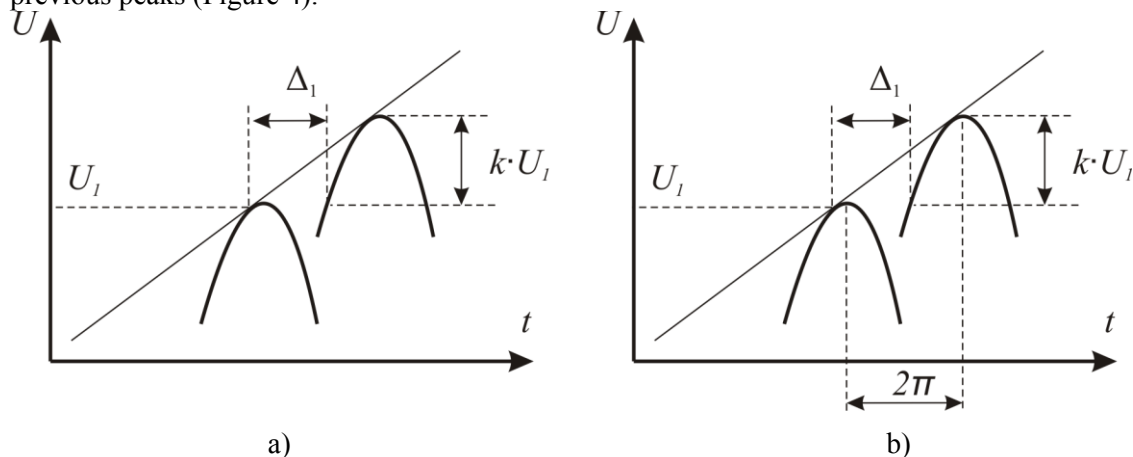a)   b)
Figure 4. Variants of error occurrence: a) due to the 1st comparator, b) due to the 2nd comparator.

Maximum amplitude of i-th period after comparator triggering is obtained using the expression:

$$U_i^{max} = \frac{U_1}{\cos\dfrac{\Delta\varphi_1}{2}} \tag{3}$$

Using the expression (2), we determine the angle:

$$\cos\frac{\Delta\varphi_1}{2} = \frac{U_1}{U_1 + k \cdot U_1}$$

Hence

$$\frac{\Delta\varphi_1}{2} = \arccos\frac{1}{1+k} \tag{4}$$

Thus, the maximum error in determining of the angle by the 1st comparator is equal to:

$$2 \cdot \pi - \frac{\Delta\varphi_1}{2} = 2 \cdot \pi - \arccos\left(\frac{1}{1+k}\right)$$

Therefore, it is possible to define the error when we determine the time of the first sample (Figure4*a*):

$$\Delta_1 = T\left(1 - \frac{1}{2 \cdot \pi} \cdot \arccos\frac{1}{1+k}\right) \tag{5}$$

Similarly, for the second sample we obtain:

$$U_2 + k \cdot U_1 = \frac{U_2}{\cos\dfrac{\Delta\varphi_2}{2}}$$

Denote

$$q = \frac{U_1}{U_2}$$

Using the expression (2), we determine the angle:

$$\frac{\Delta\varphi_2}{2} = 2 \cdot \pi - \arccos\left(\frac{1}{1 + q \cdot k}\right)$$

Therefore, it is possible to find the error when we determine the time of the second sample (Figure 4*a*):

$$\Delta_2 = T\left(1 - \frac{1}{2 \cdot \pi} \cdot \arccos\frac{1}{1 + q \cdot k}\right) \tag{6}$$

Figure 5 shows two cases: in one case the first sample has a maximum error and the second sample has a minimum error; while in the other case the first sample has a minimum error and the second sample has a maximum error. Points A and B are for the ideal envelope, and points C and D are when the errors in samples occur.
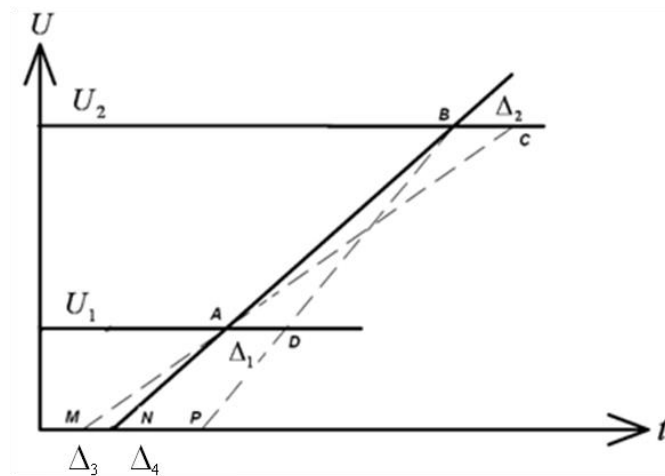
Figure 5. The envelope of the echo pulse rising edge for two samples:
NB is an ideal line,
MC is the line with the error introduced by the 2nd comparator,
BP is the line with the error introduced by the 1st comparator.

Using the similarity of triangles $\triangle$ ADB and $\triangle$ NBP, it is possible to write the expression:

$$\frac{\Delta_4}{\Delta_1} = \frac{U_2}{U_2 - U_1}$$

Hence:

$$\Delta_4 = \Delta_1 \cdot \frac{1}{1-q}$$

Using the similarity of triangles $\triangle$ ABC and $\triangle$ MAN, it follows:

$$\frac{\Delta_3}{\Delta_2} = \frac{U_1}{U_2 - U_1}$$

Thus, the total error is determined as a sum of errors:

$$\Delta_3 + \Delta_4 = \Delta_2 \cdot \frac{q}{1-q} + \Delta_1 \cdot \frac{q}{1-q} \tag{7}$$

From equations (5), (6) and (7):

$$\Delta_3 + \Delta_4 = \frac{T}{1-q} \left[\begin{array}{l} (1+q) - \dfrac{1}{2 \cdot \pi} (\arccos \dfrac{1}{1+k} + \\ +q \cdot \arccos \dfrac{1}{1+q \cdot k}) \end{array} \right]$$

If the result of the experiment is affected by various factors, then their total effect is in accordance with the law of normal distribution (according to the theorem of Lyapunov). Therefore, we can assume that a random variable is normally distributed, and all values of t_0 are in the range:

$$t_0 \in \left(t_0 - \Delta_3, t_0 + \Delta_4 \right)$$

According to the law of three sigma:

$$6 \cdot \sigma = \Delta_3 + \Delta_4$$

That is we obtain σ:

$$\sigma = \frac{1}{6}\left(\Delta_3 + \Delta_4\right)$$

Error confidence interval of 95% in fractions of *T*:

$$\frac{\Delta}{T} = \frac{1}{1-q} \cdot \left[\begin{array}{l} 1+q-\dfrac{1}{2\cdot\pi}\cdot(\arccos\dfrac{1}{1+k}+ \\[2mm] +q\cdot\arccos\dfrac{1}{1+k}) \end{array} \right] \qquad (8)$$

Figure 6 shows calculation results of expression (8) for different values of q.
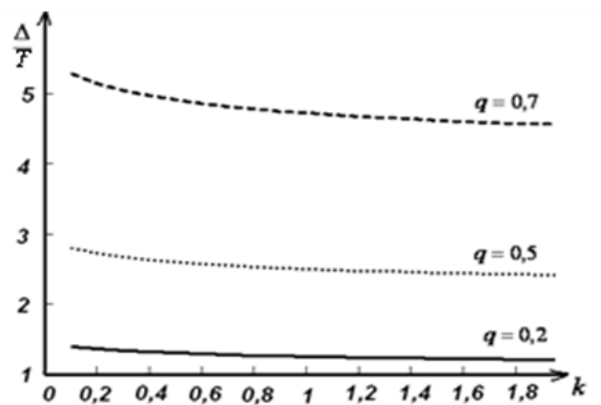

Figure 6. Value of time determining error vs. steepness of the envelope rising edge.

The steeper the rising edge of envelope (the larger k) the smaller the error in determining of the start time of the echo pulse. The error in determining of the echo pulse start time decreases as the difference in the thresholds of comparators U_1 and U_2 increases (the smaller q). Effect of the difference of thresholds is stronger than the effect of the steepness of the rising edge of the envelope. To estimate the accuracy of the theoretical results obtained using given method, laboratory tests were carried out. The tests showed that the accuracy depends on the threshold values. When performing the experiments we used a waveguide with a diameter of 52 mm. The frequency of ultrasonic vibrations was 40 kHz and step size was 20 ns.

2.2 Experimental study
Figure 7 shows the dependence between the error of measurement and distance for the ratios of comparator thresholds of $U_1 = 0.21 \cdot U_m$; $U_2 = 0.85 \cdot U_m$. At other ratios of comparator thresholds the error of measurement increases and at similar thresholds the errors closes to the error of measurement for the method of one comparator.
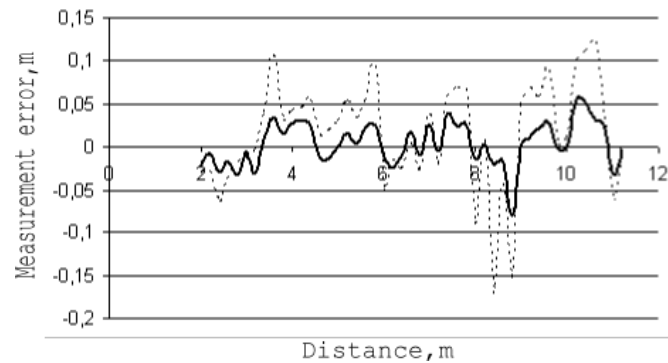
Figure 7. Error of measurement vs. distance:
dotted line – single comparator method,
full line – method of two comparators ($U_1 = 0.21 \cdot U_m$; $U_2 = 0.85 \cdot U_m$)

3. Description of the second method

Application of the method of two comparators is limited only by the case of linear rise of the echo pulse rising edge of the envelope.

$$s = a \cdot t^2 + b \cdot t + c \tag{9}$$

where s is the amplitude of the envelope, t is a time, a, b, c are the coefficients of the polynomial.

Since the shape of the echo pulse is asymmetric with respect to the t axis, two curves are used to approximate the rising edge of the pulse. One curve is the envelope for positive pulses, and the other one is the envelope for negative pulses (Figure 8). These curves have two common points, and one of them is taken as the start time of the echo pulse [6, 7].

3.1 Modeling

To define *a*, *b*, *c* coefficients the amplitude values of the ultrasonic pulse in three points were considered. These three points correspond to the peaks of a sinusoidal signal (the extrema) in three neighboring periods. Using these data, a system of equations is considered:

$$\begin{cases} s_1 = a \cdot t_1^2 + b \cdot t_1 + c \\ s_2 = a \cdot t_2^2 + b \cdot t_2 + c \, , \\ s_3 = a \cdot t_3^2 + b \cdot t_3 + c \end{cases} \tag{10}$$

where *s*1, *s*2, *s*3 are the amplitudes of ultrasound pulse in peaks at *t*1, *t*2, *t*3 instances. Using this system of equations (10) coefficients *a, b, c* can be calculated.

Similarly, we approximate the shape of rising edge of the pulse:

$$s = a \cdot t^3 + b \cdot t^2 + c \cdot t + d$$.

In some cases, the 3rd power of a polynomial approximates rising edge of the envelope of the echo pulse with better accuracy, which gives the best result. To obtain the values of *a*, *b*, *c*, *d* coefficients, a system of four equations is considered:

$$\begin{cases} s_1 = a \cdot t_1^3 + b \cdot t_1^2 + c \cdot t_1 + d \\ s_2 = a \cdot t_2^3 + b \cdot t_2^2 + c \cdot t_2 + d \\ s_3 = a \cdot t_3^3 + b \cdot t_3^2 + c \cdot t_3 + d \\ s_4 = a \cdot t_4^3 + b \cdot t_4^2 + c \cdot t_4 + d \end{cases} \tag{11}$$

When solving system of equation 9 and 10, the Cramer method is of a big help [8]. Since it can be no intersection of the envelopes, the point of shortest distance between the envelopes is taken as the echo pulse start time.

Contrary to the method of the comparator with fixed threshold, the error of method of the 2nd and the 3rd power of a polynomial does not depend on the amplitude of the echo pulse, while it depends on its shape.

In practice, the shape of the echo-pulse is affected by the noises, what results in errors at the calculation of envelope equations. To reduce the noise influence the least-square method has been applied for determining of the equation coefficients. When using this method we can consider the number of points that exceeds the number of unknown variables, and we construct an average envelope by these points. In general case, the least-squares method is used to estimate the unknown values according to measurements, having random errors [9].

The coefficients of approximation are calculated by:

$$\begin{cases} \sum_{i=1}^{n} t_i^2 \cdot s_i = a \cdot \sum_{i=1}^{n} t_i^4 + b \cdot \sum_{i=1}^{n} t_i^3 + c \cdot \sum_{i=1}^{n} t_i^2 \\ \sum_{i=1}^{n} t \cdot s_i = a \cdot \sum_{i=1}^{n} t_i^3 + b \cdot \sum_{i=1}^{n} t_i^2 + c \cdot \sum_{i=1}^{n} t_i \; , \\ \sum_{i=1}^{n} s_i = a \cdot \sum_{i=1}^{n} t_i^2 + b \cdot \sum_{i=1}^{n} t_i + c \cdot n \end{cases}$$

where n is the number of bend points taken out for polynomial approximation and the current number of an experimental point.

3.2 Experimental study
The proposed methods have been tested by constructing the envelopes of the 2nd and the 3rd order for the echo-pulse (Figure 8).

Figure 8 shows the ultrasound echo-pulse, whose shape was approximated by the envelopes of the 2nd order. The vertical line stands for the estimated time of ultrasound pulse start time.
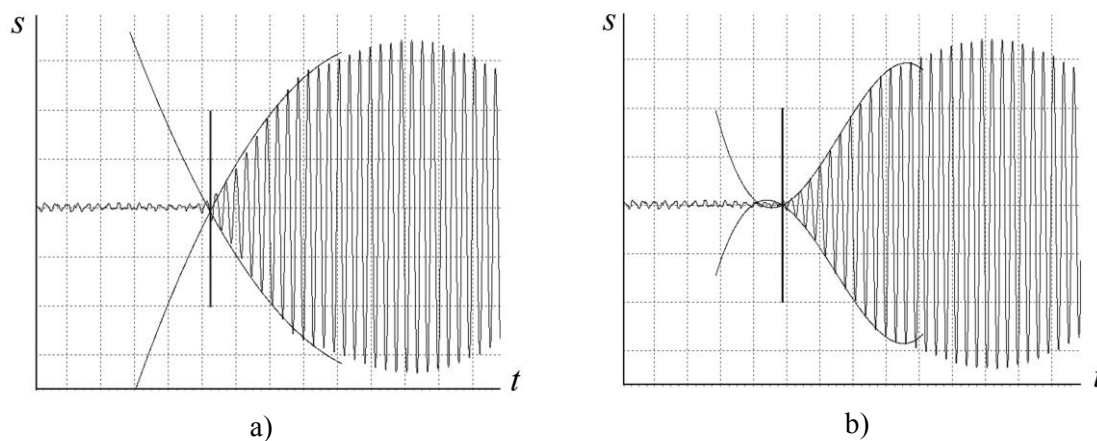

a)   b)
Figure 8. Approximation of the ultrasound echo-pulse by:
a) the envelope of the 2nd order; b) the envelopes of the 3rd order.

To estimate the accuracy of the proposed method we derived the dependence of the measurement error of the level by the comparator method envelopes of the 2nd and 3r order and the distance L between a transmitter and a receiver. A round metal wave guide of 50mm was used in the experimental study. Ultrasound converters MA40V of 16 mm diameter were taken as a transmitter and

receiver. They were placed in the center of the waveguide. The frequency of ultrasound oscillations was 40 kHz.

From Figure 9 it can be seen that systematic inaccuracy was reduced more than two times, and random error was 1.5 times lower. When using the methods of the envelopes of the 2nd and 3rd order the measurement errors do not sufficiently differ. Although the calculation of the echo pulse position takes much more time when using the method of envelope of the 3rd order.
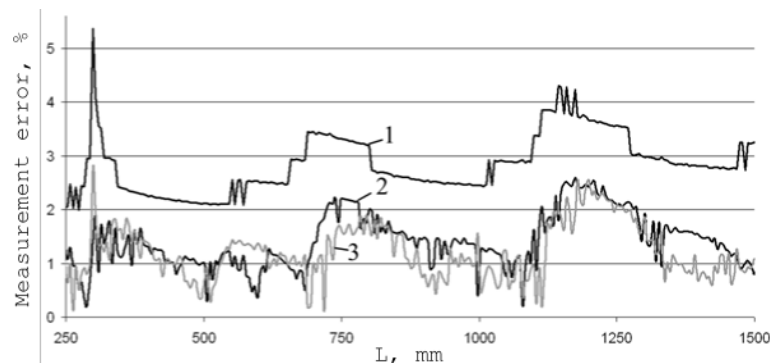

Figure 9. Measurement error vs. distance *L* between
transmitter and receiver when using:
1) method of comparator; 2) method of envelope of the 2nd order; 3) method of
envelope of the 3rd order.

From Figure 9 it can be seen that systematic inaccuracy was reduced more than two times, and random error was 1.5 times lower. When using the methods of the envelopes of the 2nd and 3rd order the measurement errors do not sufficiently differ. Although the calculation of the echo pulse position takes much more time when using the method of envelope of the 3rd order.

4. Conclusion

The use of new methods of echo-pulse processing allows sufficiently increasing of the accuracy of ultrasound devises based on pulse-time methods. Application of new methods of processing of echo-pulses can significantly reduce the measurement error of ultrasonic devices using time-pulse method.

Using a first-degree polynomial approximation for the envelope of the rising edge of the echo-signal allows a twofold increase of the measurement accuracy of ultrasonic devices. With the increase in the steepness of the rising edge of envelope of the echo-signal the error in determining of the start time of the pulse-echo decreases. Increase of the difference of comparators thresholds U_1 and U_2 also reduces an error in determining of the start time of the echo-pulse. However, the choice of optimum thresholds requires a detailed analysis of the acoustic channel measuring instrument. In addition, the calculation result will affect the accuracy of measuring the time interval between the insertion of the magnitude comparator and the dc component in the echo-signal.

Using a second-degree polynomial approximation for the envelope of the rising edge of the echo-signal is more preferable. However, to obtain high metrological characteristics it is necessary to provide at least 10 samples per period. Therefore, the application area of this method is limited to frequencies up to 10 MHz.

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