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CALCULATION OF STATIONARY ELECTRIC FIELDS IN QUASI-HOMOGENEOUS MEDIA OF MULTI-ELECTRODE COMPOSITE ELECTRIC RADIATORS

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Theoretical bases of calculation of stationary electric fields parameters in quasi-homogeneous media of multi-electrode composite electric radiators on the basis of methods of direct determination of field intensity and conformal transformations have been given. The calculated models, exact and approximate formulas necessary for determining electric conductivity of various electric radiators are offered. Experimental acknowledgement of quasi-homogeneity of resistive layer by specific electrical conductivity is given.

The most efficient way of heating biological and technical objects from the point of view of electric energy conservation is a local surface-distributed electric heating which may be implemented by composite electric radiators.

Multi-electrode composite electric radiators (MCE) are a complex system transforming electric energy into thermal one and supporting specific temperature on the surface of electric radiator according to electro-, thermalphysic parameters of MCE. In this connection these parameters should be accurately calculated in respect to the system of electrode arranged in electrocunductive composite material.

Rigorous solution of the problem may be obtained only as a result of calculation of stationary electric field formed by system of electrodes in quasi-homogeneous medium. The calculated model of electric radiator is accepted subject to the following boundary conditions: MCE may be considered possessing lumped parameters as periodic processes at frequency f=50 Hz are considered to be quasi-stationary; electrode surface along the length may be accepted as equipotential owing to medium low specific conductivity; perimeter boundary of resistance material may be considered as impermeable for electric field lines; taking into account the fact that electrode length coincides with resistive layer size in direction of electrode length, the electric radiator electric field is accepted to be plane-parallel and the same in all z sections directed along electrode length. In this case current density is the same in all points of the field being z situated.

The problem was solved by the method of direct determination of electric field strength [1] in combination with the conformal mapping method. This method is based on introduction of auxiliary function $\gamma^0(x, y)$ stating the value of angle formed by electric-field vector of plane-parallel field in any point of the examined area with one of axes of rectangular coordinate system. Function $\gamma^0(x, y)$ is a harmonic one satisfying two-dimensional Laplace equation and boundary conditions of the first sort arranged subject to orthogonality of force and equipotential lines of the field on the areas where one of the conditions is specified:

$$U_s=\text{const or } \left(\frac{\partial U}{\partial n}\right)_s = 0.$$

Electric field of the systems of axis-symmetric MCE may be described by the Laplace equation in plane meridian system equal in all meridian planes in cylindrical coordinates. (R, φ, z). In the case l/R>1 in cylindrical coordinate system the axis of which coincides with the cylinder axis, the electric field turns out to be plane-parallel and the same for any z.

As a result of introduction of plane-parallel calculated model and conformal mapping of the original plane of complex variable Z to the plane of new complex variable ζ keeping the required conformances of the points of the original and mapped planes the systems of nonlinear transcendental equations were obtained. The system was solved numerically by modification of Newton discrete method. The Jacobian matrix of partial derivative functions of the system was approximated by the first differences; in this case minimal step of argument of function was selected by criterion of significance regret of proper difference. Certain intervals entering the functions of the equation system were calculated at each integration by use of quadrature formula of Newton-Kotess of the eighth order. Simultaneously with calculation of further values of dimensionless mapping parameters the error of their determining is estimated at each integration.

Applying this method for the most abundant MCE systems the calculation of electric conductivity was carried out:

1. between two pairs of coplanar electrodes arranged in conductor of rectangular cross section;
2. between three-electrode systems;
3. multi-electrode systems of low-temperature composite electric radiators;
4. two-electrode axis-symmetric system;
5. three-electrode axis-symmetric system;
6. multi-electrode axis-symmetric systems.

The calculated models and systems in the mapped plane are given in Table 1. The mapping parameters, accurate and approximate design formulas are given in Table 2.

The introduced plane-parallel calculated models allowed reflecting not only qualitative features of operation of multi-electrode composite electric radiators but also obtaining quantitative response.

Table 1. *The calculated models of multi-electrode composite electric radiators and system in the mapping plane*

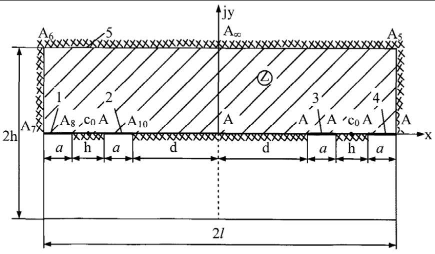
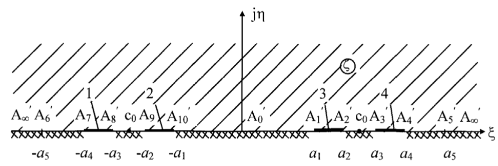
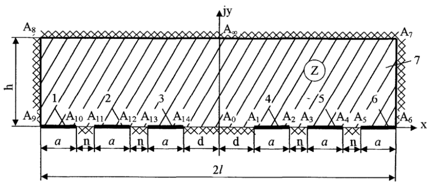
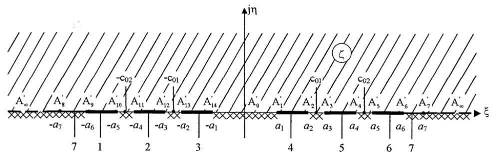
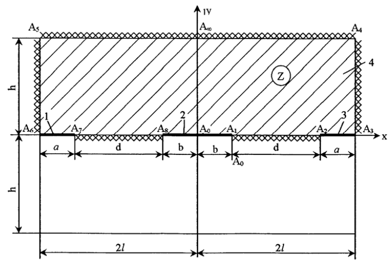
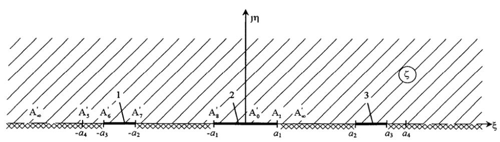
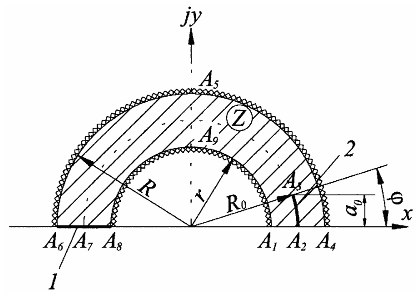
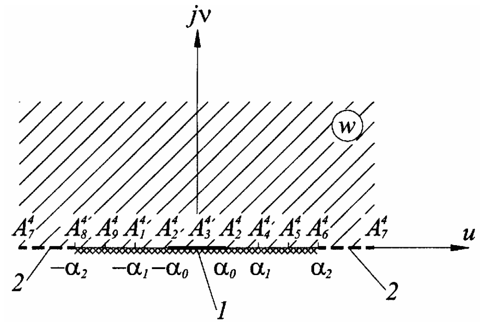
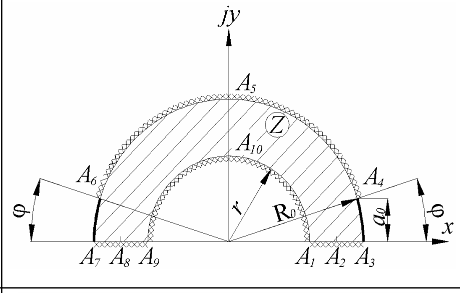
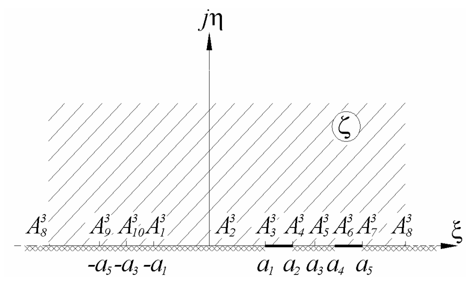
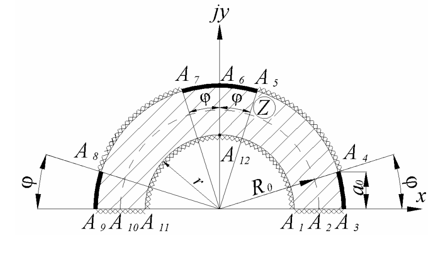
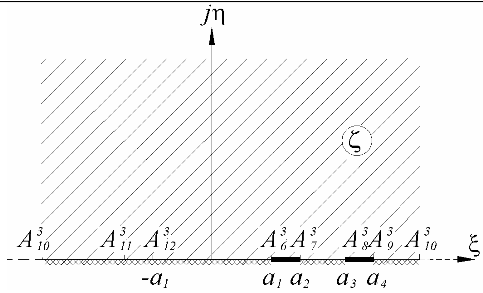
№№	Calculated model	System in the mapped plane
1		
2		
3		
4		
5		
6		

Table 2. *Formulas for determining parameters of conformal mapping and dimensionless electric conductivities for multi-electrode systems given in Table 1*

№№	Mapping parameters	Accurate and approximate calculated expressions
1	$\zeta = a_4 \mathrm{sn}(K_0 \frac{z}{l}, k_0), \frac{a_1}{a_4} = \mathrm{sn}(K_0 \frac{d}{l}, k_0), \frac{a_1}{a_4} = \mathrm{sn}(K_0 \frac{d}{l}, k_0),$ $\frac{a_1}{a_4} = \mathrm{sn}(K_0 \frac{d}{l}, k_0), \frac{h}{l} = \frac{K_0'}{K_0}, k_0 = \frac{a_4}{a_5}, k_0' = \sqrt{1-k_0^2},$ where K_0 and K_0 are the complete elliptic integrals of the first sort with modules k_0 and k_0'	$\frac{G_l}{\gamma} = \frac{I_l}{\Delta U} = \frac{I_7 - I_5 + \frac{I_1}{I_2}(I_6 - I_8)}{\frac{I_1 I_4}{I_2} - I_3},$ where I_1–I_8 are the hyperelliptic integrals determined numerically according to [2]
2	$\frac{a_1}{a_6} = \mathrm{sn}(K_0 \frac{d}{l}, k_0), \frac{a_2}{a_6} = \mathrm{sn}(K_0 \frac{d+a}{l}, k_0),$ $\frac{a_3}{a_6} = \mathrm{sn}(K_0 \frac{d+a+n}{l}, k_0), \frac{a_4}{a_6} = \mathrm{sn}(K_0 \frac{d+2a+n}{l}, k_0),$ $\frac{a_5}{a_6} = \mathrm{sn}(K_0 \frac{d+2a+2n}{l}, k_0), \frac{a_7}{a_6} = \frac{1}{k_0}.$	$\frac{G_l}{\gamma} = \frac{D_1 - D_2 + D_3}{I_7 - (c_{01}^2 + c_{02}^2) I_8 + c_{01}^2 c_{02}^2 I_9},$ where $D_1 - I_{10} - I_{13} + I_{16}, D_2 - (c_{01}^2 + c_{02}^2)(I_{11} - I_{14} + I_{17}),$ $D_3 - c_{01}^2 c_{02}^2 (I_{12} - I_{15} + I_{18}),$ I_1–I_{16} are the hyperelliptic integrals determined numerically according to [3]
3	$\zeta = a_3 \mathrm{sn}(K_{01} \frac{Z}{2l}, k_{01}), k_{01} = a_3 / a_4, \frac{a_1}{a_3} = \mathrm{sn}(K_{01} \frac{b}{2l}, k_{01});$ $\frac{a_2}{a_3} = \mathrm{sn}(K_{01} \frac{2l-a}{2l}, k_{01}),$ $k_{01} = 4\sqrt{q_1} \left[\frac{1 + q_1^{1\cdot2} + q_1^{2\cdot3} + q_1^{3\cdot4} + \cdots + q_1^{n(n+1)}}{1 + 2q_1 + 2q_1^4 + 2q_1^9 + \cdots + 2q_1^{n^2}} \right]^2,$ where $q_1 = e^{-\frac{\pi h}{2l}}.$	$\frac{G_3}{\gamma} = 4 \frac{K'(k)}{K(k)},$ $k = \sqrt{\frac{1 - a_1^2 / a_2^2}{1 - a_1^2 / a_3^2}},$ where $K'(k)$ and $K(k)$ are the complete elliptic integrals with modules k and $\sqrt{k = 1-k^2}$ [4]
4	$Z_1 = \ln Z, Z_2 = Z_1 - (d + l), \zeta = a_1 \mathrm{sn}\left(K_0 \frac{Z_2}{l}, k_0 \right),$ $\omega = \sqrt{\zeta^2 + b^2}, \frac{\alpha_0}{\alpha_1} = sn(K_0 \frac{a}{l}, k_0'), \frac{\alpha_1}{\alpha_2} = \frac{k_0}{dn(K_0 \frac{a}{l}, k_0)},$ where $k_0' = \sqrt{1-k_0^2}$, $dn(K_0 \frac{a}{l}, k_0')$ is the delta amplitudinis, $\frac{a}{l} = \frac{2a_0}{\sqrt{Rr} \ln \frac{R}{r}}, k_0 = 4\sqrt{q} \left[\frac{1 + q^{1\cdot2} + q^{2\cdot3} + \cdots + q^{n\cdot(n+1)}}{1 + 2q + 2q^4 + \cdots + 2q^{n^2}} \right]^2,$ $q = e^{-\rho}, \rho = \frac{2\pi^2}{\ln \frac{R}{r}}, R, k_0, a_0$ are the design values of electric radiator	$G_l = \gamma \frac{K(k)}{K(k')}, k = \frac{a_0}{a_1} \cdot \frac{a_1}{a_2} = k_0 \frac{sn(u, k_0')}{dn(u, k_0')},$ where $u = K_0 \frac{2a_0}{R\sqrt{\frac{r}{R}} \ln \frac{R}{r}},$ $\tilde{G}_l \approx \gamma \frac{R - r}{2\pi r \left(1 + \frac{R-r}{\pi^2 r} \ln \frac{R-r}{\pi a_0} \right)}.$
5	$k_0 = \frac{a_1}{a_5}; \frac{a_1}{a_2} = \frac{a_4}{a_5} = dn(K_0', \frac{h_1}{h}, k_0');$ $\frac{a_1}{a_4} = \frac{a_2}{a_5} = \frac{k_0'}{dn(K_0', \frac{h_1}{h}, k_0')}, \frac{h_1}{h} = \frac{a_0}{\pi R_0},$ $k_0 = 4l^{\frac{-\pi^2}{\ln R_0 / r}}.$	$G_l = 4\gamma \frac{K(k)}{K(k')}; k = \sqrt{\frac{(1 - {}^{a_4}/_{a_5})(1 - {}^{a_1}/_{a_2})}{(1 - {}^{a_2}/_{a_5})({}^{a_4}/_{a_2} - {}^{a_1}/_{a_2})}} = \sqrt{k_0} \frac{1 - dnu}{dnu - k_0},$ where $u = K_0 \frac{2a_0}{R_0 \ln \frac{R_0}{r}}, G_l \approx \frac{4(R_0 - r)}{\pi r \left(1 - \frac{4(R_0 - r)}{\pi^2 r} \ln sh \frac{\pi a_0 r}{2R_0 (R_0 - r)} \right)}.$
6	$\zeta = a_1 \mathrm{sn}(K_0 \frac{Z_3}{l}, k_0)$, where $k_0 = a_1 / a_4; \frac{h_3}{l} = \frac{K_0'}{K_0};$ $\xi = \frac{a_1}{dn(K_0' \frac{y}{h_3}, k_0')}; \xi = a_2 = \frac{a_1}{dn(K_0^I \frac{h_4}{h_3}, k_0^I)};$ $\xi = a_3 = \frac{a_1}{dn(K_0' \frac{h_3 - h_5}{h_3}, k_0')}; k_0 = 4e^{\frac{-\pi h_3}{2l}} = 4e^{\frac{-\pi^2}{2\ln R_0 / r}};$ $k = \sqrt{\frac{\left(1 - \frac{a_3}{a_4}\right)\left(\frac{a_2}{a_4} - \frac{a_1}{a_4}\right)}{\left(1 - \frac{a_2}{a_4}\right)\left(\frac{a_3}{a_4} - \frac{a_1}{a_4}\right)}} = \frac{1 - dn\,u}{dn\,u - k_0} \sqrt{k_0};$ $u = K_0' \frac{a_0}{R_0 h_3} = K_0 \frac{2a_0}{R_0 \ln \frac{R_0}{r}} \approx \pi \frac{a_0}{R_0 \ln \frac{R_0}{r}}.$	$G_l = 8\gamma \frac{K(k)}{K(k')};$ $\tilde{G}_l \approx \frac{8\ln(R_0 / r)}{\pi} \left[1 - \frac{8\ln R_0 / r)}{\pi^2} \ln \mathrm{sh} \frac{u}{2} \right]^{-1} \approx$ $\approx \frac{8(R_0 - r)}{\pi r} \left[1 - \frac{8(R_0 - r)}{\pi^2 r} \ln \mathrm{sh} \frac{\pi a_0}{2R_0} \frac{r}{(R_0 - r)} \right]^{-1}.$

Quasi-homogeneity of the medium by the conditions of specific electric conductivity was confirmed by the direct researches of composite material (CM) of MCE electroconductive layer by the methods of electron scanning, transmission diffraction and optical microscopy.

Metallographic examination on proper scale levels showed that there are isotropic and anisotropic particles with the size from 0,4 to 100 mkm in CM. Isotropic particles have the form of globe and polyhedron. Smaller particles are combined in separate groups. Sizes and density of propagation of anisotropic and isotropic particles as well as small particle groups indicate the fact that particles visible on scale level supported by metallographic microscopes are not basic elements of regular structure of electrocunductive filler implementing the mechanism of MCE electrocunductivity. Besides, on this level it is possible to estimate the consistency of CM production technology by the methods of plane geometry.

To study electroconductive structure technologies and methods allowing studying CM with higher increase and resolution are required. The technique of scanning electronic microscopy is such method. The investigations were carried out at microscope Tesla BS 301 in the mode of reflected and secondary electrons. The surface of fresh cleavage obtained by sample destruction at liquid nitrogen temperature was irradiated. Structure radiated in wide range of scale of magnification from 50 to 20000 times with maximum resolution 10 nm. Particle sizes were determined by the method of randomly thrown secant studying sample micrographs obtained in scanning electronic microscope and digitized by personal computer.

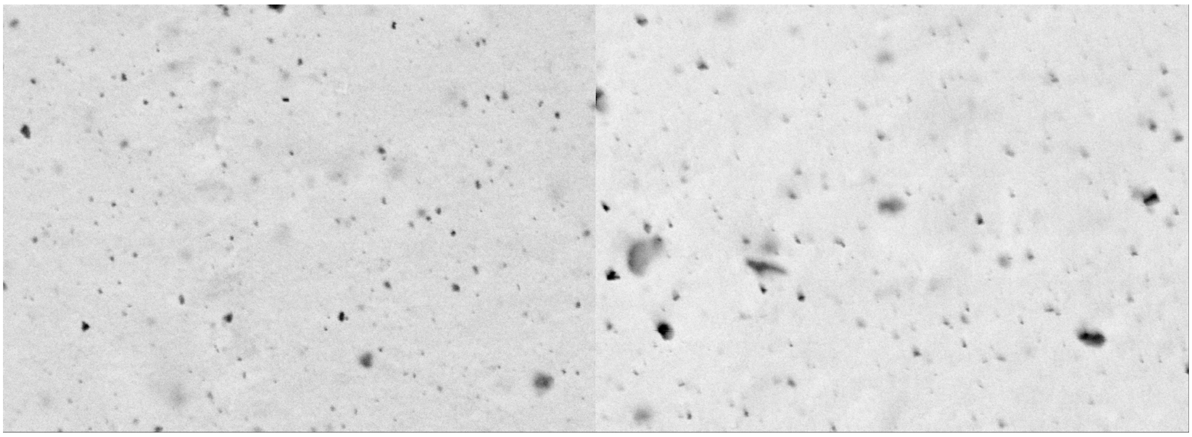
The investigations were carried out at the samples of butyl rubber (BR) with filler in the form of industrial sorts of carbon black (CB) П-234, П-324 of four different concentrations (45, 58, 75 and 145 m.f. per 100 m.f. of polymer). Studying cleavage surface structure by the technique of scanning electronic microscopy at increases from 500 to 5000 times the high density of separate particles is observed. Identification of these particles is possible only when applying complex methods of optical, scanning and transmission microscopy at one sample [6, 7]. But on the basis of the fact that at the given increases the average sizes of separate particles amount to units of micrometers and relying on the data of investigations of the original state of various ingredients one can state that visible particles are not separate particles of CB. Taking into account the fact that CB particles may be coagulated when obtaining CM one can state with considerable degree of confidence that visible particles are the largest agglomerates of CB or separate particles of other ingredients of CM of proper size.

At increases up to 500 times inclusive only particles with the size from 3...4 mkm to 20...30 mkm are distinctly singled out; such particles are coarse grain of such ingredients of CM as: phenol-formaldehyde resin, hexole and barite that is conditioned by imperfection of technological production process. However, it should be noted that such particle concentration in the examined material is negligibly low.

At increases 1000 and 2000 times a great number of separate particles which are rather homogeneous in size may be seen; they should be referred to the largest CB agglomerates which participate in formation of electrocunductive structure of he examined material. Typical micrographs of BR vulcanizates with CB of different sorts of concentration are given in Fig. 1.

Absolute majority of particles in Fig. 1 may be divided into two groups by morphological characters. Particles with correct surface faceting and sharp image of fission-fragment form as a rule are referred to the first group and particles with round shape the images of which are amorphous and have no clearly defined boundaries are referred to the second one. Average distances, diameter and surface density of particles are determined for each group. To estimate homogeneity of CB propagation in rubber matrix the dispersion and root-mean-square deviation of distance of particles of proper groups from each other were calculated (Table 3).

In addition, to estimate homogeneity, the obtained micrographs (Fig. 1) were divided into four fragments. Then average distances and surface density of determined types of particles were determined in each fragment



a *b*

Fig. 1. *Micrograph of vulcanizate on the basis of butyl rubber BR-1675 with content of CB of the type: a) П-324 58 m.f. and b) П-234 45 m.f. per 100 m.f. of polymer at increase of 200 times*

(Fig. 4). The obtained data indicate statistically disordered arrangement of particles that allows considering the proposed technology of producing CM to be rather successful supporting generation of the system with random distribution of dispersed filler particles.

Table 3. *Particle size, their surface density, distance between particles, dispersion and root-mean-square deviation by the results of scanning microscopy*

Particle group	The calculated parameter	CB П-234 with concentration 45 m.f. per 100 m.f. of polymer	CB П-324 with concentration 58 m.f. per 100 m.f. of polymer
1	Average diameter of particle d_{cp}, mkm	2,158	0,943
	Average distance L_{cp}, mkm	10,299	8,890
	Particle surface density, mm^{-2}	9426	12651
	Dispersion	21,498	12,628
	Root-mean-square deviation	4,637	3,554
2	Average diameter of particle d_{cp}, mkm	1,411	1,388
	Average distance L_{cp}, mkm	9,224	9,963
	Particle surface density, mm^{-2}	11753	10074
	Dispersion	15,692	28,714
	Root-mean-square deviation	3,961	5,359

Table 4. *Average distance between particles and their surface density corresponding to different fragments (F) of micrographs*

Calculated parameter	CB П-234, concentration 45 m.f. per 100 m.f. of polymer				CB П-324, concentrtaion 58 m.f. per 100 m.f. of polymer			
	Ф 1	Ф 2	Ф 3	Ф 4	Ф 1	Ф 2	Ф 3	Ф 4
Average distance L_{cp}, mkm	10,286	10,381	10,346	8,746	10,138	10,524	11,500	9,399
Particle surface density, mm^{-2}	9452	9279	9343	13072	9730	9029	7561	11318

At higher increase the structure of agglomerates of carbon black in butyl rubber matrix may be estimated (Fig. 2).

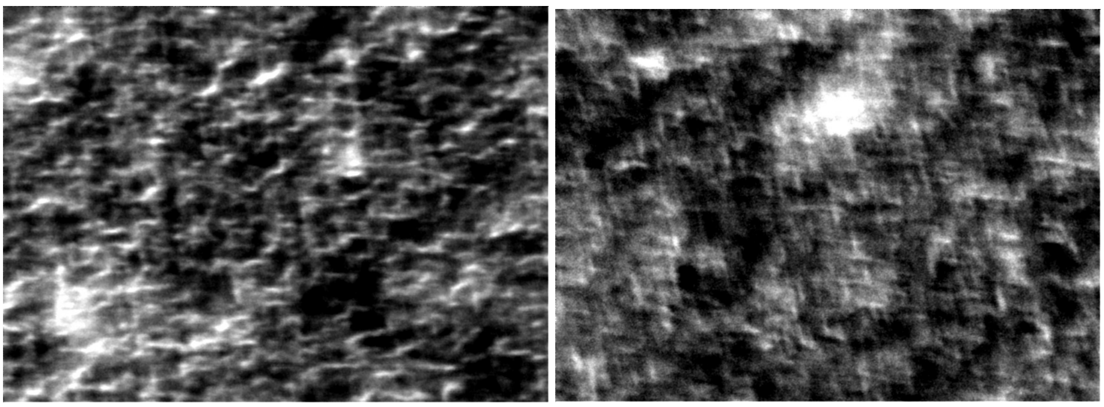
To identify the particles observed at increase of 2000 times which form electrocunductive structure of CM the material was studied by transmission diffraction microscopy by coal replica technique with extraction.

Objects of investigation in the form of samples of electroconductive layer of MCE for transmission microscopy were prepared in the following way: sample fragment was frozen in liquid nitrogen and damaged. The extracted finest grain was arranged on a slide and placed in exhaust cart where thin coal layer was sprayed. The formed film was segregated from glass by gelatin water solution and placed on special copper meshes. Copper meshes with extracted replicas were placed in the column of electronic microscope which was used for studying phase composition, morphology, granulometry and impurity defect structure. Method of planimetry was used for determining average size of particles [6]. Micro electron-diffraction patterns were indicated by standard method.

In spite of the fact that the obtained coal replica is the replica with extraction and does not bear information on morphology of macroobject surface it confirms identification of particles forming electroconductive structure of CM as the main quantity of particles extracted to the replica were the CB particles with hexagonal crystal lattice that is indicated by typical structure of micro electron-diffraction patterns.

Typical electron microscope image of CB particle with hexagonal crystal lattice is given in Fig. 3, *a*; and micro electron-diffraction pattern of CB particle is given in Fig. 3, *b*.

Inhomogeneity of CB distribution determined by micrographs with increase of 20000 times and calculated by the level of grey of different areas amounted to ≈7 %. The degree of dispersion determined at macrolevel by standard method of Li-Dagmor, as well as by the method of comparison with sample micrographs in the range from 93,0 to 97,5 %.



a *b*

Fig. 2. *Micrograph of vulcanizate on the basis of butyl rubber BR-1675 with content of CB of the type: a) П-324 45 m.f. and b) П-234 58 m.f. per 100 m.f. of polymer at increase of 20000 times*

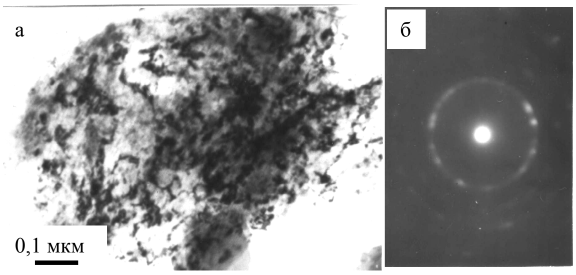


Fig. 3. *Electron microscope image (a) and micro electron-diffraction pattern of CB particle with hexagonal crystal lattice (b)*

The obtained indices of homogeneity and dispersion degree confirm definitely quasi-homogeneity of the examined medium by conditions of specific electroconductivity.

Thus, parameters of stationary electric field (Table 2, 3) were determined by specific structural dimensions of multi-electrode electric radiators by the parameters of conformal transformation and selection of accurate or approximate formulas. The given expressions show that in spite of methodological identity the calculations are individual for each type of systems.

The obtained formulas determine, in particular, that dependence of electric conductivity of axis-symmetric electric radiator on electrode width is of logarithmic character, increase of ratio of inner radius of electrocunductive layer to outer one results in decreasing values of dimensionless conductivity at constant magnitudes of ratios of electrode width to the difference of the above mentioned radii; the increase of the last ratio at constant ratios of radii of electroconductive layer increases electric radiator conductivity.

The developed models of complex CE systems in conjunction with the obtained complex of accurate and approximate expressions of parameters of stationary electric fields in quasi-homogeneous media are theoretical basis of engineering design procedure of constructive parameters of wide range of composite electric radiators.

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