ASYMPTOTIC ASSESSMENT OF DISTRIBUTION MOMENTS OF PRICE INCREMENTS FOR PAIR USD/RUB

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АСИМПТОТИЧЕСКОЕ ОЦЕНИВАНИЕ МОМЕНТОВ РАСПРЕДЕЛЕНИЯ ПРИРАЩЕНИЙ ЦЕН ПАРЫ USD/RUB

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Рассмотрен метод оценивания коэффициентов модели стохастической волатильности при бесконечно возрастающем времени. Найденные зависимости позволяют свести задачу нахождения решения системы стохастических дифференциальных уравнений к отысканию аналитического решения асимптотического уравнения Фоккера–Планка–Колмогорова. Разработанный алгоритм применяется к анализу дневных котировок тиковых значений цены пары USD/RUB.

Ключевые слова: стохастическая волатильность, стохастические дифференциальные уравнения, уравнение Фоккера-Планка-Колмогорова, асимптотический метод.

Introduction. In the last decade there has been substantial growth in the number of studies related to the study of the behavior of complex economic systems and fluctuations in the financial markets. One of the ways of their research is the direct analysis of high-frequency empirical data using the theory of stochastic processes, applied to the price increment of the form:

$$\Delta x(t) = x(t+\Delta t) - x(t) \tag{1}$$

where $x(t)$ – the original stochastic process Дt – time lag.

Determination of the statistical properties of the increments in (1) and simulation of future behavior are central to the dynamics of financial markets. To solve it, a theoretical model of stochastic volatility (SV) [6], including the Heston model [5], is proposed.

The paper gives an asymptotic assessment and determination of the functional dependence of the coefficients μ, σ, p, q of stochastic volatility model of the form:

$$d(\Delta x) = \mu(\Delta x, t)dt + \Delta\sigma(\Delta x, \Delta\sigma, t)dW_1,$$
$$d(\Delta\sigma) = g(\Delta x, \Delta\sigma, t)dt + q(\Delta x, \Delta\sigma, t)dW_2, \tag{2}$$

where Дx – price increments satisfying (1), μ – drift coefficient, Дy=y(t+Дt)-y(t) – increment of volatility, g, q – some continuous functions, dW_i – increment of Wiener processes, $i=1,2$ with correlation $\rho dt = \overline{dW_1, dW_2}$, $t \in [t_0, T]$.

The parameters, found in this way, are used to find of the asymptotic analytical solution of Fokker-Planck-Kolmogorov equation (FPKE).

The algorithm allows to describe the behavior of the price increments and volatility for tick data, recorded during the trading session. In this case it is applied to the analysis of quotations pair USD / RUB. Ten-thirty-minute tick data were used – only 11580 dollar value of ruble prices for the period from September 1, 2014 to February 2, 2015 (data provided by the company Finam, http://finam.ru).

General Provisions. Statistical analysis of empirical data shows the presence of non-zero autocorrelation of the time series Δx_i, that is, as a rule, they are dependent. If $\Delta x(t)$ – Markov stochastic processes, the unconditional density $p(\Delta x_{i+1}, \Delta t_{i+1}, \Delta x_i, \Delta t_i)$ can easily be determined by the conditional:

$$p(\Delta x_{i+1}, \Delta t_{i+1}, \Delta x_i, \Delta t_i) = p(\Delta x_i, \Delta t_i) p(\Delta x_{i+1}, \Delta t_{i+1} | \Delta x_i, \Delta t_i), \quad (3)$$

Knowing $p(\Delta x_{i+1}, \Delta t_{i+1} | \Delta x_i, \Delta t_i)$ and $p(\Delta x_i, \Delta t_i)$, $i = 1, ..., N$, when Δt_i, $\Delta t_{i+1} \to \infty$, the first equation in (3) can be written in the form of the Fokker-Planck-Kolmogorov [8]:

$$\frac{d}{d\tau} p(\Delta x, \tau) = \left[-\frac{\partial}{\partial(\Delta x)} D_1(\Delta x, \tau) + \frac{\partial^2}{\partial(\Delta x)^2} D_2(\Delta x, \tau) \right] p(\Delta x, \tau), \quad (4)$$

where $\tau = T / \Delta t$, $t \in [t_0, T]$, $D_1(\Delta x, \tau)$ and $D_2(\Delta x, \tau)$ – coefficients of drift and volatility of the price increments models (2), defined as moments of the conditional distribution $p(\Delta s, \tau + \Delta\tau, | \Delta x, \tau)$:

$$D_k(\Delta x, \tau) = \frac{1}{k!} \lim_{\Delta\tau \to 0} M^{(k)},$$

$$M^{(k)} = \frac{1}{\Delta\tau} \int_{\Omega} (\Delta x' - \Delta x)^k p(\Delta x', \tau + \Delta\tau, | \Delta x, \tau) d(\Delta x') \quad (5)$$

where $k = 1, 2$, Ω – the range of variation $\Delta x(t)$.

Numerical integration in (5) can be carried out by any quadrature formula of high order, for example, by Simpson.

Substitute them in Fokker-Planck-Kolmogorov equation (4). Then, if $m \geq n$, the solution of Fokker-Planck Kolmogorov equation when $t \to \infty$ reaches a steady state solution of the homogeneous differential equation

$$\frac{\partial}{\partial \Delta x} \left(P_1(\Delta x) p(\Delta x, \tau) + \frac{1}{2} \frac{\partial}{\partial \Delta x} P_2(\Delta x) p(\Delta x, \tau) \right) = 0,$$

which has view

$$p(\Delta x, \tau) = \frac{C}{P_2(\Delta x)} \exp\left\{ -\int \frac{P_1(\Delta x)}{P_2(\Delta x)} d\Delta x \right\}, \quad (6)$$

where *C* – constant, determined from the reference conditions

$$C = \int_{-\infty}^{\infty} p(\Delta x, \tau) d\Delta x = 1.$$

Equation (6) describes the asymptotic behavior of the density distribution Δx of a random process or the behavior of tails of the density distribution.

Data Analysis. Asymptotic assessment of stochastic volatility model was carried out. To do this, one analytically solved UFPE (4) with determined numerical coefficients (5) relative to increments in prices and volatility at time lag $\Delta\tau = 2$ and lags in the calculation of volatility $\Delta t_1 = 1$, $\Delta t_2 = 3$. To simplify the functional dependence of the parameters of the model, the non-linear polynomial regression is applied to them and their polynomial approximation is found. The proposed method of parameter assessment was used to find the functional dependence of the coefficients of the model (2) for the ten-minute and thirty-minute tick data – only 11580 values ruble price of the dollar in the period from September 1, 2014 to February 2, 2015 (data provided by the company Finam, http: //finam.ru).

As a result of the calculations in the implementation of the proposed method, arrays of coefficient values and functional dependences of coefficients of models (2) and (6) were found.

It should be noted that the proposed assessment algorithm of the model parameters in the form of a polynomial dependence gives a unique solution of FPKE [8] and does not restrict the choice of the probability distribution law.

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MODERNIZATION OF STREAT ROAD NETWORK IN ANYLOGIC 7

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Abstract. This article discusses a problem that today in large cities like Tomsk has a problem with traffic jams. The goal of this article is to show the best solution may be to optimize the current road network by changing the operation of traffic lights, cars traffic patterns. This method will minimize costs, thanks to a preliminary modeling options and assessing their impact. AnyLogic is simulation tool that supports all the most common simulation methodologies that allows you to build such model, and to evaluate it.

Keywords: AnyLogic, traffic jam, modeling, simulation, system of Automatic Traffic Control.

Introduction. In our days large cities like a Tomsk has a problem with transport planning (traffic jam, congestion) associated with a sharply increasing number of vehicles. Also do not forget about the historical heritage. Architects, who are planed the city 400 years ago, could not imagine that the city will face with transport difficulties.

The one of the main solution for this problem is a full reconstruction of current road network at the macro level. It means that this solution needed the great amount of resources like money and time. Money is the main problem of this solution, because their amount which is needed for this solution is more than budget of the Tomsk region. Second solution is integration of Automatic Traffic Control Systems on existing road network. These systems consist of cameras, radars and road weather station which are connected in one network and work under one program. This program collecting data and analyze current situation on the road network. Special modules can optimize the work of traffic lights for solving problem with traffic jams or congestions. [3]

These two solutions needed grate amount of money and time, but what can we do now, today?

Modeling traffic network in AnyLogic 7. Simulation modeling is the third solution, which can help to reduce the load of traffic nodes without rebuilding existing traffic network.

AnyLogic is the only simulation tool that supports all the most common simulation methodologies in place today: System Dynamics, Process-centric (AKA Discrete Event), and Agent Based modeling. [2] The unique flexibility of the modeling language enables the user to capture the complexity and