UDC 621.039.51

ON COMPUTATION OF FISSION NEUTRON AGE IN METAL-WATER MIXTURES

A.V. Alekseev, A.V. Kuzmin

Tomsk Polytechnic University
E-mail: kuzminav@tpu.ru

Experimental and computation fission neutron age data up to the indium resonance in mixtures Zr-H_2O, Al-H_2O, Fe-H_2O often used in reactor physics have been analyzed. The approximations obtained on the basis of the least square method are introduced, their error is estimated. The necessity of experimental neutron age definition in pure metals and search of alternative methods of computation are shown.

Introduction

Age is the important neutron migratory characteristic in physics of thermal reactors. It determines slowing-down neutron leakage from core region and it is necessary for solving critical task. Age analytic definition and its experimental verification in various material combinations do not still give a good fit owing to inaccuracy of nuclear-physical constants by single elements. Definition of fission neutron age in various mixtures requires, first of all, awareness of mixture component age. The accuracy of experimental data is achieved, as a rule, by application of expensive equipment, more advanced techniques and devices that determines, in its turn, their deficiency. In this work the experimental values of fission neutron age up to the indium resonance in metal-water mixtures obtained by Paschall and colleagues [1–5] at the devices of the International nuclear center for testing barriers, the results of numerical computation by Monte Carlo method [6, 7] and method of spatial-angle moments of neutron distribution function [8] and approximated dependences of experimental and designed values of age [9–11] are discussed.

Age in single components of metal-water mixture

Light water as one of the components of thermal reactor core region was carefully studied on fission neutron age to indium resonance [12]. Spread in the results of different researches is explained first of all, in our opinion, by the quality of modeling the fission neutron source, perfection of indium detector construction, the influence of the effect of neutron geometric absorption, quality of water treatment. The experimental equipment defines the procedure of determining age and higher-order moments, quantity and value of corrections. The consistence of the test with the results of theoretical calculation by various computer codes is the criterion of the experiment perfection.

The plane source of fission neurons of the bounded diameter and plane detectors almost equivalent to unbounded plate were used in the work [1] at measurement of fission neutron age at slowing-down in water to indium resonance 1,46 eV. In comparison with measurements where axial detectors with large plane sources were used these results did not require rated correction on extrapolation to infinite geometry source and corrections were insignificant and well consistent. The experimental neutron age in water with density 1 g/cm^3 amounted to

$$\tau_{1,46}^{\text{экс}}=26,48\pm0,32\ \text{cm}^2,$$

that is well agreed with the results of slowing-down calculation by Monte Carlo method by the program TYCHE [1]

$$\tau_{1,46}^{\text{рас}}=25,99\pm0,18\ \text{cm}^2.$$

The code took into account inelastic scattering and anisotropy of elastic scattering at oxygen. Such good coincidence determined our choice and the results of the experiment [1] were taken as the reference for water. Later the results close to this magnitude were obtained by other researches.

The experimental data on fission neutron age in zirconium, aluminum and iron were not found. Therefore, the results of numerical calculation by the program KDZ FEI in homogeneous infinite medium with plane isotropic source in multigroup approximation [8]: for Zr – $\tau_{1,01}$=1912 cm^2 and $\tau_{1,00}$=2381 cm^2; for Al – $\tau_{1,01}$=6294 cm^2 and $\tau_{1,00}$=8327 cm^2, and for Fe – $\tau_{1,01}$=548 cm^2 and $\tau_{1,00}$=3170 cm^2 were accepted for comparison.

The age rated values by 18-group constant system were obtained for energy 1,01 eV, and by the 26-group – for energy 1,00 eV. The difference in age value in the range of 1,00...1,46 eV was neglected as it amounts to 2,5...5,0 % by estimations [12].

Age approximation in the mixture Zr-H_2O

The experiments on age determination in the mixture Zr-H_2O [2] were carried out in aluminum container of the length 234 cm, width 132 cm and depth 183 cm. 6804 kg of zircalloy-2 including more than 98 % of zirconium with the density of 6,55 g/cm^3 were used for plates of various size. Such sizes reduced to minimum the geometric leakage influence. The results of the experiment given in Fig. 1 accord satisfactorily with slowing down calculation by Monte Carlo method with an accuracy of $\delta\leq3$ % [6].

Age computation by the method of spatial-angle moments of neutron distribution function [8] with different number of groups gives approximation «down» and «up» with an error not more than 7,5 and 6 %, respectively. The computation data [8] approximate satisfactorily by rational functions:

$$\tau_{1,01}=\frac{26,48-8,838\cdot V_{\text{Zr}}/V_0}{1-0,9908\cdot V_{\text{Zr}}/V_0}, \tag{1}$$

$$\tau_{1,0}=\frac{26,48-5,002\cdot V_{\text{Zr}}/V_0}{1-0,991\cdot V_{\text{Zr}}/V_0} \tag{2}$$

with an accuracy not more than 1,7 and 2,5 %, without points with values V_{Zr}/V_0, equal 0,258 and 0,280, where the error amounted about 4,0 and 4,5 %. Quite possible that it is conditioned by inaccuracy in publications.

In Fig. 2 one can see that the data [2] conform well ($\delta \leq 1$ %) with the equation

$$\tau_{1,01} = \frac{26,48 - 6,635 \cdot V_{Zr}/V_0}{1 - 0,9907 \cdot V_{Zr}/V_0} \qquad (3)$$

and differ considerably from the rated values [13].

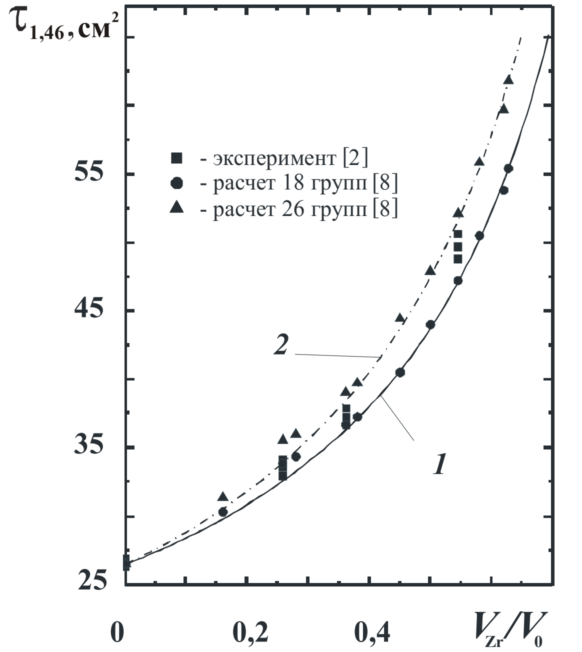

Fig. 1. *Dependence of fission neutron age on metal part in the mixture Zr-H_2O: 1) by eq. (1); 2) by eq. (2)*

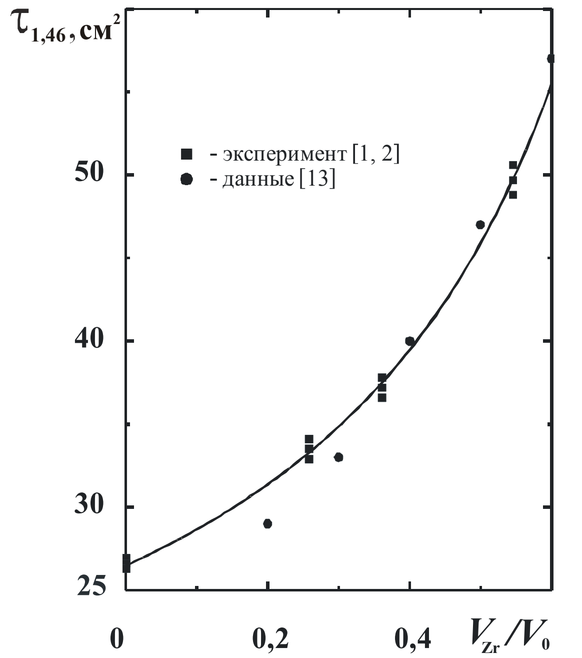

Fig. 2. *The experimental data approximation by fission neutron age by [1, 2] for the mixture Zr-H_2O, eq. (3)*

Age approximation in the mixture Al-H_2O

A minor spread of experimental [3, 4] and rated data is typical for Al-H_2O mixtures (Fig. 3). The results of computation by the 18-group program [8] approximate by the following dependence with c $\delta \leq 1,5$ %

$$\tau_{1,01} = \frac{26,48 + 7,9346 \cdot V_{Al}/V_0}{1 - 0,995 \cdot V_{Al}/V_0}. \qquad (4)$$

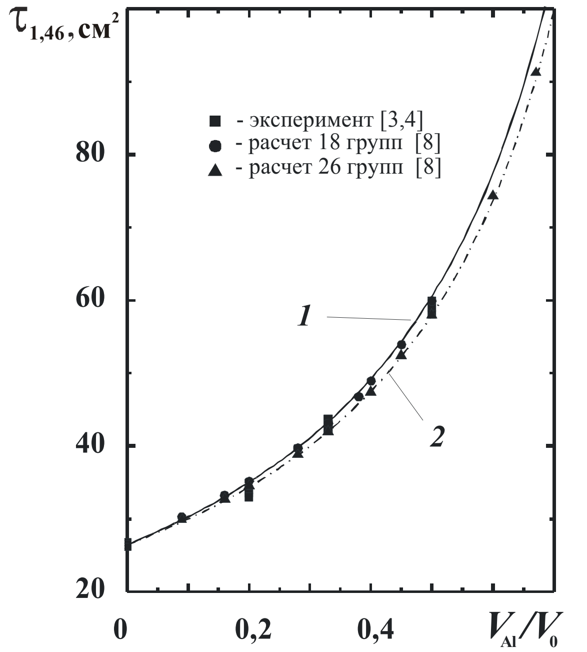

Fig. 3. *Dependence of fission neutron age on metal part in the mixture Al-H_2O: 1) by eq. (4), 2) by eq. (5)*

Approximation of this dependence is observed «down» in comparison with the case with Zr.

The results of calculation by the 26-group program are generalized with an accuracy of ~1 % [3]

$$\tau_{1,00} = \frac{26,48 + 5,2604 \cdot V_{Al}/V_0}{1 - 0,9962 \cdot V_{Al}/V_0}. \qquad (5)$$

This approximation conforms better to the experiment (Fig. 3), therefore it may be recommended for computations.

Age approximation in the mixture Fe-H_2O

The importance of estimating age in iron-water mixture is connected, first of all, with iron-water tampers the integral structural element of the most widespread reactors with uranium-water lattice of the type WWER and WK.

The range of metal part variation in experimental works is rather low and depends on power of thermal neuron converter (Fig. 4). Steel St3, containing 0,14...0,22 % C, 0,05 % Si, 0,3...0,65 % Mn, 0,3 % Cr, 0,05 % S, 0,04 % P, 0,01 % N was used as a test material in the work [14]. Soft steel with density of 7,86 g/cm^3, including 0,2 % C and 0,4 % Mn was applied in the work [5]. Steel content in these works may probably influence the difference in the results, besides the technique.

In order to generalize the experimental data [5, 14] in the studied range of changing metal part in the mixture the exponential function the coefficients of which were determined by the least square method was accepted. The obtained approximations are given in Fig. 4 and generalize the experimental data of the work [14] in the range of iron part variation in the mixture V_{Fe}/V_0 – [0;0,43]:

$$\tau_{1,46} = 23,5 + 5,81 \cdot \exp\left(\frac{V_{Fe}/V_0}{0,279}\right) \quad (6)$$

with relative error δ<8 %, and work [5] in the range [0;0,634]

$$\tau_{1,46} = 23,94 + 2,28 \cdot \exp\left(\frac{V_{Fe}/V_0}{0,2766}\right) \quad (7)$$

with the error δ<3 %.

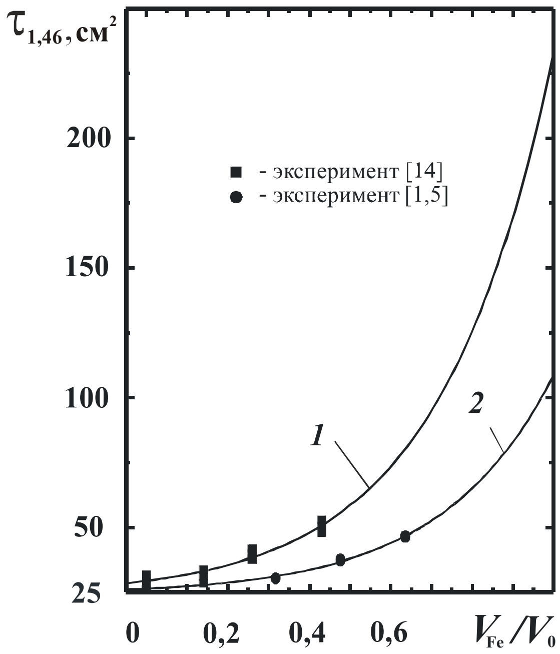

Fig. 4. *Test data approximation by age in Fe-H_2O mixtures: 1) by eq. (6), 2) by eq. (7)*

The results of the experiment [5] are also confirmed with slowing-down calculations by the probabilistic Monte Carlo method by the program TYCHE-III [7].

Age values in the mixture Fe-H_2O in the range V_{Fe}/V_0>0,634 were determined only by numerical multi-group programs by the method of spatial-angle moments of neutron distribution functions [8]. The peculiarity of the program with 26-group system of constants in slowing-down problem is a more detailed account of inelastic scattering, resonance self-shielding in average group trapping cross-section, full and inelastic scattering.

The results of calculations using 18 and 26 groups of neutrons age introduced in Fig. 5. The results of calculation with various numbers of groups up to V_{Fe}/V_0=0,8 differ insignificantly and exceed slightly the experimental data [5]. It allowed approximating them by one dependence

$$\tau_{1,00} = \frac{26,95 - 14,758 \cdot V_{Fe}/V_0}{1 - 0,9956 \cdot V_{Fe}/V_0}, \quad (8)$$

which describes the results of calculation subject to 18 groups with an accuracy of δ<1 % in the interval [0;0,67], and subject to 26 groups – with δ<3 % in the range [0;0,8].

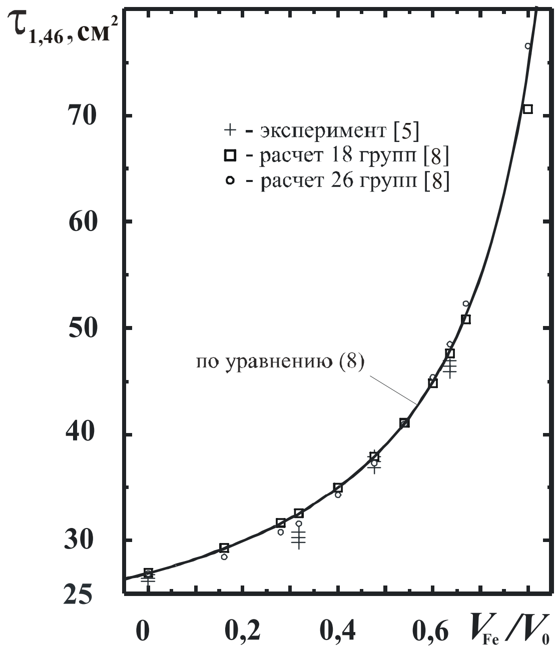

Fig. 5. *Matching calculation results by the method of multi groups with the experiments [5]*

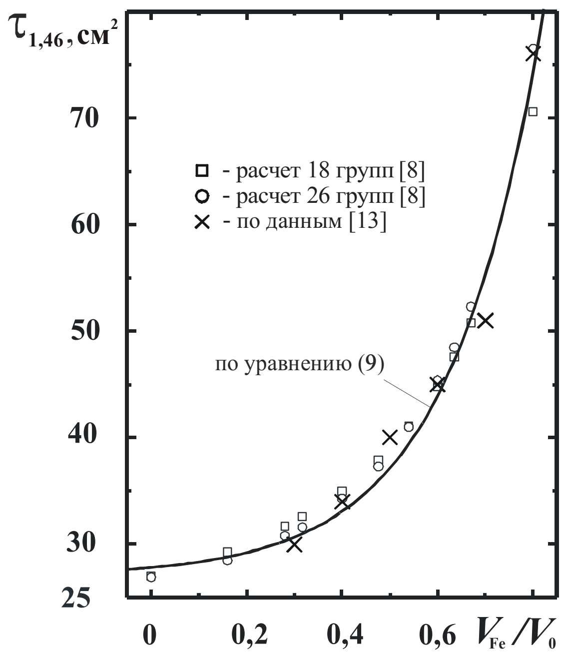

Fig. 6. *Age matching of rated data in the mixture Fe-H_2O with the data [13]*

Age magnitudes in iron-water mixture are proposed in the work [13] by the results of calculation [8,16] (Fig. 6). They are approximated with the initial value [1] by the dependence

$$\tau_{1,46}^{\text{pac}} = 27,075 + 0,765 \cdot \exp\left(\frac{V_{\text{Fe}}/V_0}{0,194}\right), \qquad (9)$$

which has considerable error δ<8 % in the interval [0;0,8].

Discussion of the results

The fundamental difference of the tests in iron-water mixtures [5, 14] besides the dimensions of the device (about 3 m^3 in the experiment [5] to 0,5 m^3 in [14]) is a noticeable divergence in age results in pure water (Fig. 4). In the work [14] the age value to the energy of indium resonance was determined at the same experimental unit and was equal to

$$\tau_{1,46}=30,2\pm2,7 \text{ cm}^2,$$

in water while in tests [1] in water with the same density of 1 g/cm^3 it turned out to be less by 14 %. This value being the result of more perfect technique and procedure of the experiment was confirmed in the same laboratory by Alter theoretical computation by Monte Carlo method and then by other researches.

Let us notice that successful definition of fission neutron age in water [1] was later the base for testing new experimental data by oxygen absorption sections and differential sections of elastic scattering at oxygen in MeV-energy region [15] taking into account its significance in reactor physics on fast neurons with oxide fuel. This task was solved as well by Monte Carlo method by the modified program MAK with point assignment of sections and probabilities, i.e. without traditional approximations in solving the problem on neuron slowing-down in the infinite homogeneous medium and considering more strictly the oxygen anisotropy of elastic scattering.

Awareness of fission neutron age in the studied metals for which there are only rated values is not of less importance. The pattern may be presented, for example, comparing the fission neutron age rated values in iron volume obtained by different authors with the results of interpolation by the proposed approximations, Table.

Table. *The results of fission neutron age computation to energy E**

Source	E*, eV	τ(E*), sm^2
[17]	0,2	160
[14]	1,46	743
[8], 18 group	1,01	548
[8], 26 group	1,00	3170
By eq. (6)	1,46	233
By eq. (7)	1,46	109
By eq. (9)	1,0...1,46	160

The authors of the work [14] computed the age by the technique of A.D. Galanin [17], which he later [13] accepted as an untenable. The computation gave satisfactorily coincidence with the results of the experiment that may be considered as accidental due to incorrect definition of mean cosine of scattering angle.

Conclusion

The experimental and rated data in determining fission neutron age to indium resonance in Zr-H_2O, Al-H_2O, Fe-H_2O mixtures were analyzed. On the basis of the least square method their approximations were obtained, determination error was estimated.

The obtained expressions may be used for computing reactors in diffusion-age approximation. The considerable spread of rated data on various codes and approximations (by the example of pure iron) indicates the necessity of experimental adjustment of nuclear physical constants, theoretical base improvement, search for the alternative methods of calculation.

REFERENCES

1. Paschall R.K. The Age of Fission Neutrons to Indium Resonance Energy in Water // Nucl. Sci. and Eng. – 1964. – V. 20. – P. 436–444.
2. Paschall R.K. The Age of Fission Neutrons to Indium Resonance Energy in Zirconium-Water Mixtures – I. Experiment // Nucl. Sci. and Eng. – 1965. – V. 23. – P. 256–263.
3. Paschall R.K. The Age of Fission Neutrons to Indium Resonance Energy in Aluminum-Water Mixtures // Transactions of ANS. – 1965. – V. 8. – № 2. – P. 467–468.
4. Paschall R.K. The Age of Fission Neutrons to Indium Resonance Energy in Aluminum-Water Mixtures // Nucl. Sci. and Eng. – 1966. – V. 26. – P. 73–79.
5. Paschall R.K. The Age of Fission Neutrons to Indium Resonance Energy in Iron-Water Mixtures – I. Experiment // J. Nucl. Energy. Part A/B. – 1966. – V. 20. – P. 25–35.
6. Alter H. The Age of Fission Neutrons to Indium Resonance Energy in Zirconium-Water Mixtures – II. Theory // Nucl. Sci. and Eng. – 1965. – V. 23. – P. 264–271.
7. Alter H. The Age of Fission Neutrons to Indium Resonance Energy in Iron-Water Mixtures – II. Theory // J. Nucl. Energy. Part A/B. – 1966. – V. 20. – P. 37–54.
8. Marchenko L.V., Sergeev Yu.A. Calculation of square slowing-down length for various media in 18- and 26-group P1-approximations and their comparison with the experimental data // Bulletin of nuclear data center. – Moscow: Atomizdat, 1969. – Issue 6. – P. 319–390.
9. Zyk I.S., Kuzmin A.V. Approximations for the age of fission neutrons in mixtures zirconium, aluminum and water // Modern Techniques and Technologie: Transactions XII Intern. Scientific and Practical Conf. – Tomsk, 2006. – P. 154–156.
10. Alekseev A.V., Kuzmin A.V. Approximations of experimental and rated data on fission neutron age in iron-water mixture // Modern technique and technology: Proceedings of the XII International theoretical and practical conference. – Tomsk, 2006. – V. 2. – P. 336–338.
11. Alekseev A.V., Zyk I.S., Kuzmin A.V. On definition of fission neutron age in metal water mixtures // Transfer of technologies, innovations, modern problems of nuclear branch: Proceedings of International theoretical and practical conference. – Snezhinsk, 2006. – P. 215–216.
12. Bekurts K., Virts K. Neutron physics. – Moscow: Atomizdat, 1968. – 456 p.
13. Galanin A.D. Introduction to the theory of nuclear reactors on thermal neurons. – Moscow: Energoatomizdat, 1990. – 536 p.

14. Geraseva L.A., Vavilov V.V. Neutron slowing down in iron-water mixtures // Atomnaya Energiya. – 1960. – V. 8. – Issue 6. – P. 556–557.
15. Naumov V.A., Rozin S.G. Solving the problems of the reactor physics by Monte Carlo method. – Minsk: Nauka i Tekhnika, 1978. – 208 p.
16. Garusov E.A., Petrov Yu.V. Moments of slowing down function and its few-group models for water-metal mixtures // Atomnaya Energiya. – 1974. – V. 36. – Issue 2. – P. 143–144.
17. Galanin A.D. Theory of nuclear reactors on thermal neutrons. – Moscow: Atomizdat, 1959. – 383 p.

Received on 07.12.2006

UDC 536.21

ON THE CHOICE OF TEMPERATURE PROFILE AT SOLVING THE HEAT CONDUCTION EQUATION IN SPHERICAL COORDINATES BY THE METHOD OF THERMAL BALANCE INTEGRAL

R.V. Gorodov, A.V. Kuzmin

Tomsk Polytechnic University
E-mail: gorodov@tpu.ru

Solutions of the heat conduction equation for a sphere and an area limited from within by a spherical cavity have been obtained by means of the integrated method. The influence of the choice of the temperature profile on efficiency of the approached analytical solution is shown. The variant of solution specification in transitive area is offered.

Introduction

Exact solutions of heat conduction problems are rather cumbersome and laborious. Moreover they are almost absent in problems on radial heat flux in spherical coordinates at changing aggregate state [1, 2]. Therefore the diagrams obtained by numerical or approximate methods are usually used for solving practical problems [3]. One of the approximate analytical methods is the method of heat balance integral (HBI), in which its physical clearness, simplicity and rather high accuracy of the results are, first of all attractive, that T. Goodmen shows obviously [4] by numerous examples. The main difficulty which one can face using the method of HBI consists in correct specification of the temperature profile which influences greatly the results accuracy, in T. Goodmen's opinion.

There are several approaches in selecting temperature profiles. A.I. Veinik in the work [5] proposes using temperature profiles in the form of common polynomials for the problems of any geometry that should simplify the solution of the set problem.

Referring to the work of F. Poll and T. Lardner [6] and without solution, T. Goodmen suggests in his article [4] using the temperature profile of the form:

$$T(r,t)=\text{polynomial}/r, \tag{1}$$

is time in the case of spherical symmetry; where $T(r,t)$ is the temperature of the body; r is the reference radius, t.

It is substantiated by the fact that the exact solution of the problems is proportional to the magnitude $1/r$, and use of the profile in the form of a common polynomial at large times gives a considerable error.

For the external problem (area limited from within by a spherical cavity) with the restricted conditions of the second kind of G. Karslow and D. Eger [3] the following solution:

$$T(r,t)=\frac{R^2q}{\lambda r}\left\{\begin{aligned}&\Phi^*\left(\frac{r-R}{2(at)^{1/2}}\right)-\\&-\exp\left(\frac{r-R}{R}+\frac{at}{R^2}\right)\Phi^*\left(\frac{r-R}{2(at)^{1/2}}+\frac{(at)^{1/2}}{R}\right)\end{aligned}\right\}, \tag{2}$$

is given, where $\Phi^*(x)=\frac{2}{\sqrt{\pi}}\cdot\int\limits_x^\infty \exp(-\xi^2)d\xi$ is the error function; q is the heat flux; R is the sphere radius; a, λ are the coefficients of temperature- and heat conduction respectively.

It is seen that solution is proportional to the value $1/r$, and the profile recommended by T. Goodmen is possible to function efficiently here (1). But is it suitable for the inner problem (sphere area) with the same boundary conditions of the second kind in general solution of which only one summand is proportional to $1/r$?

In the book of A.V. Lykov [4] this solution is introduced in the form of series and has the form:

$$T(r,t)=\frac{qR}{\lambda}\left[\begin{aligned}&3\text{Fo}-\frac{1}{10}\left(3-5\frac{r^2}{R^2}\right)-\\&-\sum_{n=1}^{\infty}\frac{2}{\mu_n^2\cos\mu_n}\cdot\frac{R\sin\mu_n\frac{r}{R}}{r\mu_n}\exp(-\mu_n^2\text{Fo})\end{aligned}\right]+T_0, \tag{3}$$

where Fo$=at/R^2$ is the Fourier number; μ_n are the roots of characteristic equation tg$(\mu)=\mu$.