FORMATION OF SECURITY PORTFOLIO TA THE LEVEL OF RISK AVERSION AND DEGREE OF TRUST TO MANAGEMENT COMPANY

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ФОРМИРОВАНИЕ ПОРТФЕЛЯ ЦЕННЫХ БУМАГ С УЧЕТОМ УРОВНЯ НЕПРИЯТИЯ РИСКА И СТЕПЕНИ ДОВЕРИЯ К УПРАВЛЯЮЩЕЙ КОМПАНИИ

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Annotation. *Описан процесс управления портфелем ценных бумаг, созданного по классической схеме Марковица, с учетом уровня неприятия риска и степени доверия инвестора к управляющему. Проводится проверка эффективности управления при помощи коэффициентов качества, основанных на взаимосвязи степени доверия и размера комиссионных сборов.*

One of the factors of the development of the world economy is the high degree of participation of the society in the investment processes on the stock market at home and abroad. In turn, the entry of citizens to the trading floors is carried out through the professional participants of the stock market, namely through brokers or managers of securities. The main responsibility of managers is to preserve and increase the clients capital, which is also achieved with the help of portfolio investment.

The modern theory of portfolio investment was founded in the articles of Harry Markowitz, where he pays great attention to the optimal choice of the assets, basing on the desired ratio of return/risk [1-2].

Suppose, that the vector of share assets in the portfolio is $x = (x_1, \dots, x_d)$. In this case $\sum_{i=1}^{d} x_i = 1$.

Net return at time is t: $r(t) = \frac{P(t+1) - P(t)}{P(t)}$, where $P(t)$ – is a price at time t .

According to the theory of Markowitz, the expected value is the rate of return, and the risk measure is calculated by the standard deviation [3].

- return of the portfolio≡ expected return μ_x

$$\mu_x = E[r_x(t)] = \sum_{i=1}^{n} E[r_i(t)]x_i = \sum_{i=1}^{n} \mu_i x_i,$$

- portfolio risk ≡ volatility σ_x

$$\sigma_{ij} = \mathrm{cov}(r_i(t), r_j(t)) = \rho_{ij}\sigma_i, \sigma_j,$$

where $\rho_{ij} = \mathrm{cov}(r_i(t), r_j(t))$ - the correlation coefficient of random variables .

$$\sigma_x^2 = \mathrm{var}(r_x(t)) = \mathrm{var}(\sum_{i=1}^{n} r_i x_i) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{cov}(r_i(t), r_j(t)) x_i x_j . \quad [4\text{-}5]$$

The problem of constructing the optimal portfolio can be considered with two different aspects:

• minimization of the risk at which the income, that is greater than or equal to the expected level of profitability, is guaranteed:

$$\min_x \sigma_x^2 \equiv \min \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} x_i x_j ,$$

$$\mu_x \geq r ; \sum_{i=1}^{d} \mu_i x_i \geq r ; \sum_{i=1}^{d} x_i = 1.$$

• maximization of return, providing a risk that is less than or equal to the risk of investments:

$$\max_x \mu_x \equiv \max_x \sum_{i=1}^{d} \mu_i x_i ,$$

$$\sigma_x^2 \leq \bar{\sigma}^2 ; \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} x_i x_j \leq \bar{\sigma}^2 ; \sum_{i=1}^{d} x_i = 1 .$$

Maximization of return adjusted for risk:

$$\max_x \mu_x - a\sigma_x^2 \equiv \max \sum_{i=1}^{d} \mu_x x_i - a(\sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} x_i x_j) ,$$

where a – coefficient of risk aversion [6-8].

On the basis of the model given above, manager will build a portfolio of securities. But before they embarked on this, the investor must choose it, one of the main factors is the choice manager level of trust and the fees charged.

Suppose there are two manager j=A,B. The expected utility of investor i delegating to manager j an amount $x_{i,j}$ of risky investment is equal to

$$U_{ij}(x_{i,j}, f_j) = R_f + x_{i,j}(R - f_j) - \frac{a_{i,j}}{2} x_{i,j}^2 \sigma ,$$

f_j

where R_f - the risk-free rate of return of the portfolio; $a_{i,j}$ - coefficient of risk aversion; (R-)- the investor's excess return net of the management fee; σ - volatility.

This function affects the choice of the investor manager A or B. It allows to assess the quality of the portfolio management. And also shows the relationship between the size fees and return of management.

The investor chooses A over B provided that $U(x_{i,A}, f_A) \geq U(x_{i,B}, f_B)$, which is equivalent to

$$\frac{a_{i,B}}{a_{i,A}} \geq \frac{(R - f_B)^2}{(R - f_A)^2},$$

where $a_{i,B}$ - coefficient of risk aversion of the investor's *i* to such a manage B; $a_{i,A}$ - coefficient of risk aversion of the investor's *i* to such a manage A; (R-f_j)- the investor's excess return net of the management fee, where j=A,B.

The investor chooses manager A over manager B provided that the investor's relative trust for A is sufficient to compensate for the relative excess return (net of fee) expected under B. Because of constant absolute risk aversion, higher variance σ of investment reduces overall risk-taking but not the choice between A and B. That choice is pinned down only by the differential anxiety and excess return obtained by the investor with the two managers.

In addition, as it was mentioned earlier, trust plays an important role for the decisions taken by investors to select manager. Typically, the investor goes to "trusted" manager that is to the person advised by the friends or he heard about someone in advertising. Trust is also related to the fees charged and the return, the greater investor confidence, the greater future return.

This statement can be written as follows:

$$f_j^* = \left(\frac{\theta}{1+\theta}\right) * \frac{R}{2},$$

where θ - level of trust, subjective magnitude which is measured in the range from 0 to 1.

The fee charged by each manager is a constant fraction of the expected excess return R. Intuitively, the manager extracts part of the expected surplus R that he enables the investor to access. The equilibrium fee does not depend on a: The ability of a manager to extract rents from his trusting clients does not depend on their level of anxiety, but on the increase in their anxiety when they switch managers. Parameter θ captures exactly this point. In fact, the fraction of excess return extracted by the manager increases in θ. When θ = 0, all investors trust the two managers equally, so competition between identical managers drives equilibrium fees to zero. In contrast, when θ > 0, fees are positive. Now investors bear an anxiety cost of leaving their more trusted manager, which allows him to charge a positive fee. However, investors take more risk with their more trusted manager than with the less trusted one (or on their own). At the maximal dispersion of trust (θ = 1), the two managers have huge market power and extract 1/4 of the excess return from their investors. The model predicts that fees should be higher in sectors in which dispersion of trust is higher, perhaps owing to the absence of a market index or of established measures of risk [9-10].

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