STATISTICAL ASSESSING INVESTOR PREFERENCES FOR PORTFOLIO CONSTRUCTED AND MANAGEMED

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Introduction

One of the factors of the development of the world economy is the high degree of participation of the society in the investment processes on the stock market at home and abroad. In turn, the entry of citizens to the trading floors is carried out through the professional participants of the securities market, namely through brokers or managers of securities. The main responsibility of managers is to preserve and increase the clients capital, which is also achieved with the help of portfolio investment.

The modern theory of portfolio investment was founded in the articles of Harry Markowitz, where he pays great attention to the optimal choice of the assets, basing on the desired ratio of return / risk. [1]

Suppose, that the vector of assets in the portfolio is $x = (x_1, \dots, x_d)$. In this case $\sum_{i=1}^{d} x_i = 1$.

Net return at time t

$r(t) = \frac{P(t+1) - P(t)}{P(t)}$, where $P(t)$ – price at time t.

According to the theory of Markowitz, the expected value is the rate of return, and the risk measure is calculated by the standard deviation. [2]

- return of the portfolio≡ expected return μ_x

$$\mu_x = E[r_x(t)] = \sum_{i=1}^{n} E[r_i(t)]x_i = \sum_{i=1}^{n} \mu_i x_i,$$

- portfolio risk ≡ volatility σ_x

$$\sigma_{ij} = \mathrm{cov}(r_i(t), r_j(t)) = \rho_{ij}\sigma_i, \sigma_j,$$

where $\rho_{ij} = \mathrm{cov}(r_i(t), r_j(t))$ - the correlation coefficient of random variables .

$$\sigma_x^2 = \mathrm{var}(r_x(t)) = \mathrm{var}(\sum_{i=1}^{n} r_i x_i) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{cov}(r_i(t), r_j(t)) x_i x_j.$$

[3-4]

The problem of finding the optimal portfolio can be considered with two different aspects:

• minimization of the risk at which the income, that is greater than or equal to the expected level of profitability, is guaranteed:

$$\min_x \sigma_x^2 \equiv \min \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} x_i x_j,$$

$$\mu_x \ge r;\ \sum_{i=1}^{d} \mu_i x_i \ge r;\ \sum_{i=1}^{d} x_i = 1.$$

• maximization of return, providing a risk that is less than or equal to the risk of investments:

$$\max_x \mu_x \equiv \max_x \sum_{i=1}^{d} \mu_i x_i,$$

$$\sigma_x^2 \le \bar{\sigma}^2;\ \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} x_i x_j \le \bar{\sigma}^2;\ \sum_{i=1}^{d} x_i = 1.$$

Maximization of return adjusted for risk:

$$\max_x \mu_x - \tau\sigma_x^2 \equiv \max \sum_{i=1}^{d} \mu_x x_i - \tau(\sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} x_i x_j)$$

where τ – coefficient of risk aversion.[5]

After constructing the portfolio, evaluation of efficiency of management is done. It occurs due to analysis of the various coefficients, the main of which are alpha, beta, and the Sharpe ratio. [6-7]

Beta ratio - a measure of market risk, which reflects the variability of return of a security relative to the index return.

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)},$$

where β_i - beta ratio; R_m – return of an asset; R_i – index return (taken as standard).[8]

Coefficient alpha - the difference between the real return of the security for the period and return, it had to show taking into account changes of the market and the beta coefficient of this security:

$$\alpha_{i,t} = R_{i,t} - \beta_{i,t} R_{m,t},$$

where $\alpha_{i,t}$ – estimate of the coefficient alpha for the asset at time t; $R_{i,t}$ – return of the asset at time t; $\beta_{i,t}$ – beta ratio for the asset at time t; $R_{m,t}$ – return of standard index at time t.

Sharpe ratio reflects a portfolio returns excess over the return on risk-free asset, taking into account the overall portfolio risk, measure of which performs the standard deviation.

$$Sh = \frac{E(r_p - r_I)}{\sigma_r},$$

where *Sh* – sharpe ratio; r_p – return of portfolio; r_I – yield of standard index; σ_r - standard deviation of the return tools.

We need to assess portfolio taking into account the whole set of coefficients. [9-10]

The formation of securities portfolio with the help of the Markowitz model.

Based on the preferences of the investor and the analysis of the changes of dynamics of return for the global stock markets and commodity markets and raw materials sector on the interval 2004-2014 yrs., those financial instruments were selected that will be involved in the construction of portfolios. Namely, it is German stock index (DAX) and the Russian stock index RTS index, the yield of which on the selected time interval was 127% and 111%, respectively.

For comparison, we will construct three different portfolio composition: separately from the securities of index DAX (Siemens, Daimler, RWE, E.On, Bayer, Allianz, Volkswagen, BMW, Commerzbank, Adidas.), RTS (Gazprom, Sberbank, Magnet, Norilsk nickel, Novate, Rosneft, System, Transneft, Megafon, Alrosa) and a portfolio of securities on the basis of both indexes. Portfolios will be based on data from the stock price 7.07.2013-7.07.2014 yrs., which are available on the website of the company Finam, (http://www.finam.ru/),online (http://xetra.com/), etc.

In order to find the optimal value of the securities to be included in portfolio, it is necessary to solve the problem of profitability maximization. The solution to this problem has been realized in the package "Solver" embedded in Microsoft Office Excel 2007. Thus, we obtained the following portfolios:

The formed portfolio of securities index DAX, the maximum yield of which is equal to 25.6% at the level of volatility of

$$\pi_t = 0,15x_1 + 0,35x_2 + 0,18x_3 + 0,2x_4 + 0,12x_5 ,$$

19%:
where the assets of the company: x_1- Siemens; x_2- Daimler; x_3- Bayer; x_4- BMW; x_5- RWE.

The formed portfolio of shares included in the RTS index, the maximum yield of which is equal to 32, 99% at the level of volatility, 19%: $\pi_t = 0,09x_1 + 0,46x_2 + 0,19x_3 + 0,09x_4 + 0,17x_5$ where the shares of the company: x_1- Magnet;x_2- Norilsk Nickel;x_3- System;x_4- the Transneft;x_5- Alrosa.

Finally, the portfolio constructed from German shares and the Russian stock market at the level of volatility of 16%, has the yield about 35%. Eight securities are included into the portfolio, with the following proportions:

$$\pi_t = 0,42x_1 + 0,02x_2 + 0,01x_3 + 0,05x_4 + 0,28x_5 + 0,1x_6 + 0,05x_7 + 0,08x_8$$

where the shares of the company: x_1- Daimler; x_2- Bayer; x_3- RWE; x_4- Magnet; x_5- Norilsk Nickel; x_6- System; x_7- the Transneft; x_8- Alrosa.

Evaluation of the portfolio management efficiency.

After the formation of the portfolios the statistical evaluation of their effectiveness was conducted, with the help of estimates of the coefficients alpha, beta and Sharpe ratio. Data for the calculation were taken from 8.07.2014-8.08.2014, daily data were considered. Indexes of DAX and RTS were selected as reference assets. The coefficients calculations were performed daily after the portfolio formation. Also, we found the values of coefficients estimations for the entire period of the research:

Table 1. Analytical coefficients portfolios

Portfolio	alpha	beta	sharp
1. Portfolio of stocks DAX	0	1	-0,01
2. Portfolio of stocks RTS	0	0,03	2,4
3.Portfolio of stocks RTS and DAX	9,1	0,02	3,1

This table shows that for all portfolios betas are greater than zero, it indicates the presence of a direct relationship between the price and value of the index portfolio. Furthermore, beta of the first portfolio is equal to one, and it means, the portfolio will exactly follow the change in the market. Coefficient alpha for the first and second portfolio is statistically equal to zero with a probability of 95%, it means, that yield of the control is equal to return of indexes of DAX and RTS, respectively. Finally, for the last portfolio (RTS + DAX) alpha is 9.1, ie, portfolio brought a positive return that exceeds the income index. Sharpe ratio is approximately equal to zero for the first portfolio, ie the behavior of the portfolio is comparable with the behavior of the index DAX. The two remaining portfolios are managed well enough, and the higher the ratio the more effectively it is managed.

In addition, all the daily values of the coefficients used in the analysis were tested for normal distribution with the help of chi-square test. At a significance level of 0.05, all the coefficients alpha and beta ratio for a portfolio composed of German assets take the hypothesis of normal distribution. But betas for portfolios composed of stocks DAX + RTS and RTS are not normally distributed and, therefore, are not statistically significant in our calculations.
But in spite of this, taking into account the values of other coefficients, portfolio composed of stocks DAX + RTSI, is most effectively controlled with respect to the ratio of return / risk and is mostly diversified. And it is this portfolio which will participate in further research.

References

1 A.D. Radigina, L.P. Habarova, L.B. Shapiro. Basic Course on Securities Market. - M: Financial Publishing House "Business Express", 1998 - 408 p.
2. Martin Haugh, Garud Iyengar. Financial Engineering and Risk Management. Mean Variance Optimization. URL: https://www.coursera.org/course/fe.
3. Harry Markowits. Portfolio Selection // Journal of Finance. 1952. Vol. 7. № 1. pp. 71-91.
4. William Sharpe. Investments.-M.: INFA-M, 2003.-X11, 1028
5. A. N. Burenin. Portfolio management. - Moscow: Scientific and Technical Society named after academician Vavilov, 2008 - 440 p.
6. O.A. Belsner, O.L. Kritskii. Optimizing portfolio of financial instruments // Mining and Exploration 2013, № 36, p. 35-41.
7. E.V. Masterova. Markowitz model / / Proceedings of the IX International Conference of Students and Young Scientists "Prospects for the development of basic sciences." - Tomsk, 2012. - Vol.1. - S. 598-597.
8. James D. Gwartney and Richard L. Stroup, "What Everyone Should Know About Economics And Prosperty".- Kolonna Publication, 1993.
9. L.J. Gitman., M.D. Junks. Investing basics. - M .: Business, 1999 - 992 p.
10. Bollerslev T., Engle R.F., and Nelson D.B. ARCH Models // In R.F. Engle and D.L. McFad-den (Eds.). – Handbook of Econometrics. – Elsevier Science, Amsterdam.– 1994.