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# ON CALCULATION OF PARAMETERS AND EFFICIENCY OF ENERGY TRANSFORMATION WITH RAILGUN

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The calculation formulas of pulse duration and amplitude of growing current at specified values of maximum velocity and weight of accelerated body by railgun have been obtained. To provide minimum values of railgun current and source strength it is necessary to obtain current impulse close to rectangular shape from the source and to have the most railgun inductance. The dependencies defining power conversion factor of railgun including the energy losses in rails possible in electric arch and remaining energy in the railgun magnetic field are presented for current rectangular impulse. It is shown that conversion factor increases with increasing mass of accelerate body and its maximum velocity as well as with decreasing the railgun length at optimal values of accelerated body's cubic density and its initial velocity.

Railgun is an electromechanical device converting electromagnet energy of current impulse into mechanical energy of accelerated body. At present railguns are considered to be promising electromagnet accelerators of bodies with the weight of 0,001...1 kg to velocities amounting 10 km/s, to be applied in space engineering, scientific investigations. Railgun consists of two parallel rails (buses), between which the accelerated body moves, Fig. 1. When flowing current along the rails and body due to electrodynamic force the body accelerates and can achieve the velocity significantly higher than 1,8 km/sec. The velocity 1,8 km/sec is maximum for accelerators using gas-dynamic pressure of chemical combustion materials, for example, powder. However, to achieve such velocities it is necessary to supply railgun from a very power impulse source of electromagnet energy capable of generating current pulses with the amplitude to 1 MA and more, durability to 5 msec and energy to 1 MJ and more [1]. Rotating generator with discontinuous inductance can be used as such source [2].

Calculation of parameters and revealing the factors causing the increase in efficiency  $\eta$  of electromagnetic energy transformation into kinetic energy of accelerated body by railgun for the purpose of decreasing source power is an urgent problem.

To calculate the parameters and efficiency  $\eta$  of railgun let us consider inductance and resistance of railgun depend on the distance covered by the accelerated body in railgun approximately and linearly:

$$Lp(t) \approx L_0 \cdot x(t); \quad Rp(t) \approx R_0 \cdot x(t);$$

where  $L_0$ ,  $R_0$  is the railgun inductance and resistance per unit of length.





Then the force accelerating the body we define as [3]

$$F = \frac{d}{dx} \left[ \frac{Lp(t) \cdot i(t)^2}{2} \right] \approx \frac{i(t)^2}{2} \cdot L_0$$

If one neglects friction on rails and insulation and assumes that accelerated body of m mass moves in vacuum, then on the basis of the second Newton's law one can write down the equation

$$F = m \cdot \frac{dv(t)}{dt} = \frac{i(t)^2}{2} \cdot L_0$$

from whence velocity is found out

$$v(t) = V_0 + \frac{L_0}{2m} \cdot \int_0^t i(t)^2 dt$$
 (1)

and the covered distance is determined

$$x(t) = \int_{0}^{t} v(t) dt.$$
 (2)

To state the influence of form and amplitude of current impulse on achieving predetermined maximum velocity at minimum source power we assume that current impulse i(t) with duration  $\tau$  and amplitude  $I_m$  in the time interval  $0 \le t \le \tau$  is written down as  $(n \ge 0)$ 

$$i(t) = I_m \left(\frac{t}{\tau}\right)^n$$

then with the formulas (1, 2) we find out  $(V_0$  is the initial velocity)

$$v(t) = V_0 + \frac{I_m^2 L_0 t}{2m(2n+1)} \left(\frac{t}{\tau}\right)^{2n};$$
  
$$x(t) = V_0 t + \frac{I_m^2 L_0 t^2}{2m(2n+1)(2n+2)} \left(\frac{t}{\tau}\right)^{2n}$$

Besides, we determine the voltage at railgun input for the most unfavourable conditions of acceleration in terms of voltage  $u_d(t)$  at arc in narrow slot channel [4], formed after current stop at the accelerated body and burning between rails behind the accelerated body

$$u(t) = \frac{d[L_0 \cdot x(t) \cdot i(t)]}{dt} + R_0 \cdot x(t) \cdot i(t) + u_d(t) = I_m \left(\frac{t}{\tau}\right)^n \left[\frac{nL_0 x(t)}{t} + L_0 v(t) + R_0 x(t)\right] + u_d(t).$$

From whence at  $t=\tau$  we obtain the maximum velocity

$$V_m = V_0 + \frac{I_m^2 L_0 \tau}{2m(2n+1)},$$
(3)

railgun length

$$l = V_0 \tau + \frac{I_m^2 L_0 \tau^2}{2m(2n+1)(2n+2)}$$
(4)

and maximum value of voltage

$$U_{m} = I_{m} \left( \frac{nL_{0}l}{\tau} + L_{0}V_{m} + R_{0}l \right) + u_{d} (\tau).$$
 (5)

If the values of m,  $V_0$ ,  $V_m$ , l and  $L_0$  are given, from (3–5) we find out the necessary parameters of current impulse and maximum power  $P_m$  of source

$$\tau = \frac{2(n+1)l}{(2n+1)V_0 + V_m};$$

$$I_m = \sqrt{\frac{m(2n+1)[V_m^2 - V_0^2 + 2nV_0(V_m - V_0)]}{(n+1)L_0l}};$$

$$P_m = U_m I_m = I_m^2 (\frac{nL_0l}{\tau} + L_0V_m + R_0l) + u_d(\tau) \cdot I_m.$$
(6)

Kinetic energy obtained by the body in railgun forms

$$W_{m} = \frac{m(V_{m}^{2} - V_{0}^{2})}{2}$$

at maximum value of accelerating force

$$F_m = \frac{I_m^2 L_0}{2}$$

and maximum pressure of magnetic field at accelerated body and rails

$$\sigma_m \approx \frac{F_m}{b^2}.$$

From the relationships (5) and (6) it follows that in order to provide the given values of m,  $V_0$ ,  $V_m$ , l at minimal values  $I_m$ ,  $\tau$  and  $P_m$  one should have the maximum inductance of railgun  $L_0$  and current impulse of squared shape, i. e. when n=0 and in the time interval  $0 \le t \le \tau$  the current i(t) is direct and equal to  $I_m$ . To generate such a current impulse from the equation

$$\frac{d\psi}{dt} + Ri(t) + u(t) = 0$$

one can determine the necessary regularity of transformation in linkage of operating winding in rotating generator

$$\psi(t) = \psi_0 - I_m \left( R + \frac{U_d}{I_m} + L_0 V_0 \right) t - I_m \left( \frac{I_m^2 L_0^2}{4m} + \frac{R_0 V_0}{2} \right) t^2 - I_m \left( \frac{I_m^2 L_0 R_0}{12m} \right) t^3$$

where *R* is the operating winding resistance of generator;  $\psi_0$  is the initial value of linkage;  $U_d$  is the direct voltage at possible arc.

Railgun possesses the approximate maximum inductance with relation of geometric sized shown in Fig. 1. At any size of *b* the railgun with rails made of nonferromagnetic material, at constant current density has  $L_0=0,5881$  mkHn/m [5]. Increase in volume density  $\rho$  of accelerated body material results in decrease of size *b*, that leads to growth of current density in rails, increase of  $R_0$  and growth of heat energy losses in rails, however energy losses decreases in possible arc.

Determine the efficiency of railgun for current impulse of squared shape. For this purpose calculate the possible electromagnetic energy losses in railgun. Let us assume that rails are made of separate isolated highstrength stripes to provide uniform current density (Fig. 1), for example, of beryllium bronze with specific conductivity  $\gamma \approx 10^7$  Sm/m [6], then

$$R_0 = \frac{2}{\gamma \cdot (2b) \cdot b} \approx \frac{0,1}{b^2}, \text{ mkOhm/m}$$
$$b \approx \sqrt[3]{\frac{m}{2,3\rho}}.$$

Energy heat losses in rails are determined as

$$W_T = \int_0^\tau i(t)^2 R_0 x(t) dt = \frac{I_m^2 R_0 V_0 \tau^2}{2} + \frac{I_m^4 R_0 L_0 \tau^3}{12m},$$

then taking into account residual energy in railgun magnetic field

$$W_L = \frac{I_m^2 L_0 l}{2}$$

and energy losses in possible arc of *b* length at intensity  $E_d \approx 30 \text{ kV/m} [7]$ 

$$W_d = U_d I_m \tau \approx E_d b I_m \tau$$

the efficiency of energy conversion by railgun is

$$\eta = \frac{W_m}{W_L + W_T + W_d + W_m}.$$

For example, at m=0,1 kg;  $V_0=0,1$  km/sec;  $V_m=3$  km/sec;  $W_m=0,45$  MJ; l=1,94 m in terms of the above formulas we calculate  $\tau=1,25$  msec;  $I_m=0,89$  MA,  $F_m=0,232$  MN. If accelerated body is made of aluminium alloy with volume density  $\rho=3$  g/sm<sup>3</sup>, then average rail temperature for adiabatic heating process due to  $W_T$ energy increases by 24 °C and using the formulas we obtain b=2,4 sm;  $\sigma_m\approx403$  MPa;  $R_0\approx171$  mkOhm/m;  $U_m=2,59$  kV;  $W_T=0,11$  MJ;  $W_d=0,8$  MJ;  $P_m=2305$  MW;  $\eta\approx0,247$ , but if accelerated body is made of material with volume density  $\rho=20$  g/sm<sup>3</sup>, average rail temperature increases by 491 °C and b=1,3 sm;  $\sigma_m\approx1373$  MPa;  $R_0\approx606$  mkOhm/m;  $U_m=3$  kV;  $W_T=0,4$  MJ;  $W_d=0,43$  MJ;  $P_m=2670$  MW;  $\eta\approx0,261$ .

At the accepted assumptions according to calculations, railgun efficiency  $\eta$  grows with increase in weight of accelerated body *m* (Fig. 2) and its maximum velocity  $V_m$  (Fig. 3), as well as with decrease in railgun length *l* (Fig. 4) at optimal values of volume density of accelerated body  $\rho$  and initial body velocity  $V_0$ , when the efficiency is maximum (Fig. 5, 6).



Fig. 2. Dependence of transformation efficiency on weight of accelerated body (in kg) at V<sub>0</sub>=0,1 km/sec; V<sub>m</sub>=3 km/sec; I=1,94 m; ρ=3 g/sm<sup>3</sup>

Thus, to provide minimal values of railgun current and power supply it is necessary to generate current impulse from the source close to squared shape and to have the maximum railgun inductance. Efficiency of railgun energy transformation increases with growth of accelerated body weight and its maximum velocity as well as with decrease of railgun length at optimal values of accelerated body volume density and its initial velocity. Undoubtedly, consideration of friction [8] and air resistance decrease the efficiency of railgun, however the formulas and dependence graphs obtained allow us to calculate and optimize railgun and power supply parameters.



Fig. 3. Dependence of transformation efficiency on maximum velocity of accelerated body (in m/sec) at V<sub>0</sub>=0,1 km/sec; m=0,1 kg; l=1,94 m; ρ=3 g/sm<sup>3</sup>



Fig. 4. Dependence of transformation efficiency on railgun length (in meters) at V<sub>0</sub>=0,1 km/sec; V<sub>m</sub>=3 km/sec; m=0,1 kg; ρ=3 g/sm<sup>3</sup>



**Fig. 5.** Dependence of transformation efficiency on volume density of accelerated body (in  $g/sm^2$ ) for its different weight m (in kq) at  $V_0=0,1$  km/sec;  $V_m=3$  km/sec; I=1,94 m



**Fig. 6.** Dependence of transformation efficiency on relative initial body velocity at  $V_m=3$  km/sec; l=1,94 m: 1)  $m=10^{-3}$  kg and  $\rho=1g/sm^3$ , 2)  $m=10^{-2}$  kg and  $\rho=3$  g/sm<sup>3</sup>, 3)  $m=10^{-1}$  kg and  $\rho=10$  g/sm<sup>3</sup>, 4) m=1 kg and  $\rho=20$  g/sm<sup>3</sup>

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## REGULARISING ALGORITHM OF PARAMETER IDENTIFICATION OF ELECTRIC CHARGE EQUIVALENT CIRCUIT. PART II.

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A new regularizing algorithm of function calculation of indicial admittance in discharge gap equivalent circuit using stable differentiation and integral equation algorithms which allow for errors of initial data is suggested. Application of the least perfect square method at error modelling for the function parameters of indicial admittance is an additional way of «smoothing» modeling error of regularizing solution.

# 1. Regularising algorithm for calculation of indicial admittance function

In integral equation (1) of the first part of the given paper [1] the intergrand  $\frac{dU(\tau)}{d\tau}$  is substituted by its estimation  $S_{\lambda}(t)$  of smoothing cubic spline derivative. It is necessary to find the solution of this equation, i. e. indicial admittance g(t). The solution of such equation is known to be ill-posed problem and to calculate stable solution one must use special regularisation methods [2, 3].

In the work [4] regularising algorithm for impulse function identification in stationary dynamic system (kernel of integral equation) in which input and output signals of identified system are known with random error has been suggested. Use of the discrete Fourier transformation (DFT) and algorithm of the fast Fourier transform (FFT) stipulates high calculation efficiency of regulating algorithm. Without repeating the construction of this algorithm, we are giving the basic calculation relations, *adapting them for the problem of function recovery* g(t) and for designations used in this paper.

The calculation algorithm g(t) can be presented by the following steps [4]:

Step 1. Forming periodic (with *N* period) sequences:

$$i_{p}(j) = \begin{cases} \tilde{I}(j \cdot \Delta), & j = 0, ..., N_{I} - 1; \\ 0, & j = N_{I}, N_{I} + 1, ..., N - 1, \end{cases}$$
$$d_{p}(j) = \begin{cases} S_{\lambda}'(j \cdot \Delta) \cdot \Delta, & j = 0, ..., N_{U} - 1; \\ 0, & j = N_{U}, N_{U} + 1, ..., N - 1. \end{cases}$$

Step 2. Calculation of sequence elements

$$D_p(l) = \sum_{j=0}^{N-1} d_p(j) \exp\left(\frac{2\pi i}{N} lj\right), \quad l = 0, ..., N-1, \quad (1)$$

where  $i = \sqrt{-1}$ .

Step 3. Calculation of DFT sequence coefficients  $\{i_p(j)\}$  (direct DFT):

$$I_{p}(l) = \frac{1}{N} \sum_{j=0}^{N-1} i_{p}(j) \exp\left(-\frac{2\pi i}{N} l j\right), \quad l = 0, ..., N-1.$$
(2)

Step 4. Determination of DFT coefficients (denoted as  $\{G_{pa}(l)\}\)$  of regularizing solution (the calculated relations are presented below).

Step 5. Calculation of periodic solution (inverse DFT from the sequence  $\{G_{pq}(l)\}$ ):

$$g_{p\alpha}(j) = \sum_{l=0}^{N-1} G_{p\alpha}(l) \exp(\frac{2\pi i}{N} l j), \quad j = 0, ..., N-1.$$
(3)

Step 6. Formation of  $N_g$ -dimensional vector  $g_{\alpha}$  according to the rule:

$$g_{\alpha_{j}} = g_{p\alpha}(j-1), \quad j = 1, ..., N_{g},$$

where  $N_g = N_I - N_U + 1$ . If the following condition is met  $N \ge N_{II} + N_I - 1$ ,

then projection  $g_{a_j}$  of vector  $g_{\alpha}$  are taken as values of solution regulation  $g_a(t)$  in the nodes  $t_j = j \cdot \Delta$ ,  $j = 0, 1, ..., N_g - 1$ .

Note that in calculations (1-3) the FFT algorithm is used which reduces the number of operations by the or-