MATEC Web of Conferences **72**, 01120 (2016) DOI: 10.1051/matecconf/20167201120
HMTTSC-2016

TWO-DIMENSIONAL PROBLEM OF TURBULENT NATURAL CONVECTION IN A SEMI-OPEN CAVITY WITH RADIANT HEATING OF INTERNAL BOUNDARIES

Alexander Nee[1,*], and *Liliya* Valieva[1]

[1]National Research Tomsk Polytechnic University, Tomsk, Russia

Abstract. Mathematical modelling of turbulent natural convection in a semi-open cavity with a heat-conducting walls of finite thickness with radiant heating of internal boundaries was performed. Two-dimensional problem of the conjugate heat transfer was solved by means of the finite difference method. Scale influence of open boundaries and radiant heating of the gas – wall interfaces on the formation of differential and integral heat transfer characteristics was established. An increase in the dimensionless time (τ) led to displacement of extremum temperatures in the typical cross section (Y = 0.5) to an open vertical boundary. The average Nusselt number monotonically increased at the gas - a wall interfaces in a range of $400 < \tau < 1200$.

1 Introduction

It is known [1] that the convective heating of air masses is appropriate only for heating of small premises. In the case of large industrial buildings, where only a small part of the useful area (local working areas) is used prospectively to apply radiant heating systems based on gas infrared emitters (GIE) [2,3]. However, the use of radiant heating systems is currently not widespread due to the lack of a basic theory of the processes occurring when operating the GIE. For this reason, it is advisable to investigate the basic laws of heat transfer under conditions of radiant energy supply to the heating facilities. To solve these complex problems of heat transfer is prospectively to use the methods of mathematical modelling based on the solution of mass, momentum and energy transfer equations in the conjugate formulation.

It should be noted that the main specificity of the heat transfer process in the large-scale areas is turbulent flow regime. At the same time, the working zones of industrial facilities are locally distributed along the useful building area. In these cases, thermal regime modelling of locally based working area is of interest.

The purpose of this study is mathematical modelling of turbulent natural convection in a semi-open cavity under conditions of conjugate heat exchange and radiant heating of the internal boundaries.

* Corresponding author: nee_alexander@mail.ru

2 Problem formulation and solution method

Two-dimensional problem of turbulent natural convection was considered. Solution domain was presented as a cavity of rectangular cross section with an open vertical boundary. It was assumed that the radiant flux was uniform distributed along the bottom horizontal and vertical gas – wall interfaces. Turbulent flow was calculated by means of the large eddy simulation method [4]. In order to calculate the subgrid scale viscosity, I used a classical Smagorinsky model [5]. It was assumed that the thermal characteristics of the air and the walls were temperature-independent. Conditions of temperature and heat flux equalities were set at the gas - wall interfaces. Thermal insulation conditions were accepted at the external boundaries. I considered the model of a viscous gas in the Boussinesq approximation. “Mild” condition was set for the open boundary.

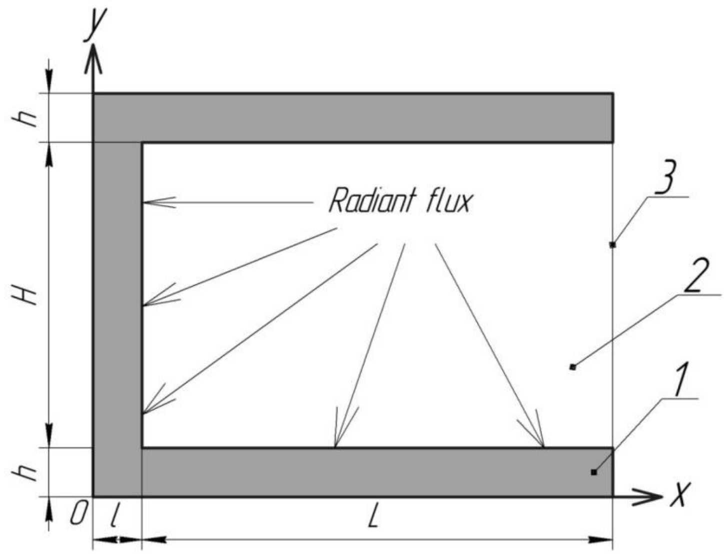

Fig. 1. Solution domain: 1 – thermally conductive wall; 2 - air; 3 - open boundary.

The turbulent natural convection and conjugate heat transfer process under study is described by unsteady two-dimensional Navier - Stokes and energy equations. These equations in vorticity - stream function - temperature dimensionless variables are as follows [4]:

$$\frac{\partial \Omega}{\partial \tau}+U\frac{\partial \Omega}{\partial X}+V\frac{\partial \Omega}{\partial Y}=\frac{\partial^2}{\partial X^2}\left[\left(\sqrt{\frac{\mathrm{Pr}}{Ra}}+\nu_t\right)\Omega\right]+\frac{\partial^2}{\partial Y^2}\left[\left(\sqrt{\frac{\mathrm{Pr}}{Ra}}+\nu_t\right)\Omega\right]+\frac{\partial \Theta_1}{\partial X}, \quad (1)$$

$$\frac{\partial^2 \Psi}{\partial X^2}+\frac{\partial^2 \Psi}{\partial Y^2}=-\Omega, \quad (2)$$

$$\frac{\partial \Theta_1}{\partial \tau}+U\frac{\partial \Theta_1}{\partial X}+V\frac{\partial \Theta_1}{\partial Y}=\frac{\partial^2}{\partial X^2}\left[\left(\frac{1}{\sqrt{\mathrm{Ra}\cdot\mathrm{Pr}}}+\frac{\nu_t}{\mathrm{Pr}_t}\right)\Theta_1\right]+\frac{\partial^2}{\partial Y^2}\left[\left(\frac{1}{\sqrt{\mathrm{Ra}\cdot\mathrm{Pr}}}+\frac{\nu_t}{\mathrm{Pr}_t}\right)\Theta_1\right], \quad (3)$$

$$\frac{\partial \Theta_2}{\partial \tau}=Fo\left(\frac{\partial^2 \Theta_2}{\partial X^2}+\frac{\partial^2 \Theta_2}{\partial Y^2}\right). \quad (4)$$

Subgrid scale eddy viscosity was calculated as in [5]:

$$\nu_t = \left(C_S \cdot \Delta\right)^2 \cdot \sqrt{2 \cdot S_{ij} \cdot S_{ij}},\ S_{ij} = \frac{1}{2}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right).$$

The initial conditions for the equations (1) - (4) are as follows:

$$\Psi(X,Y,0) = 0;\ \Omega(X,Y,0) = 0;$$
$$\Theta_1(X,Y,0) = \Theta_2(X,Y,0) = 0.$$

Boundary conditions for equations (1) - (4) are as follows:
at the external boundary of the solution domain:

$$\frac{\partial \Theta_2(X,Y,\tau)}{\partial n} = 0.$$

at the gas – wall interfaces parallel to the X – axis:

$$\Psi = 0,\ \frac{\partial \Psi}{\partial Y} = 0,\ \begin{cases} \Theta_2 = \Theta_1, \\ \dfrac{\partial \Theta_2}{\partial Y} = \dfrac{\lambda_1}{\lambda_2} \cdot \dfrac{\partial \Theta_1}{\partial Y} + Ki_1. \end{cases}$$

at the gas – wall interfaces parallel to the Y – axis:

$$\Psi = 0,\ \frac{\partial \Psi}{\partial X} = 0,\ \begin{cases} \Theta_1 = \Theta_2, \\ \dfrac{\partial \Theta_1}{\partial X} = \dfrac{\lambda_2}{\lambda_1} \cdot \dfrac{\partial \Theta_2}{\partial X} + Ki_2. \end{cases}$$

at the open boundary:

$$\frac{\partial \Theta}{\partial X} = 0,\ \frac{\partial \Omega}{\partial X} = 0,\ \frac{\partial \Psi}{\partial X} = 0,$$

where Fo – Fourier number; Ki – Kirpichev number; $\Pr$ – Prandtl number; $\Pr_t$ – turbulent Prandtl number; Ra – Rayleigh number; *U, V*– dimensionless velocities corresponding *u,v; X,Y* – dimensionless coordinates corresponding *x,y;* τ – dimensionless time; Θ – dimensionless temperature; Ψ - dimensionless analogue of stream function; Ω – dimensionless analogue of vorticity; C_S – empirical constant (Smagorinsky constant); Δ – filter width. Indices: 1,2 – elements of the system.

Formulated boundary value problem was solved by means of the finite difference method analogically [6, 7]. Implicit two-layer scheme [8] was applied for equations (1) – (4) approximation. One-dimensional finite-difference analogues of differential equations were solved by the sweep method [9].

In order to validate mathematical model, solution method and algorithm used were tested on a model problem of turbulent natural convection [10, 11] in closed square cavity. Comparison of obtained temperature fields and average Nusselt numbers with [10, 11] showed their satisfactory agreement.

3 Results and discussion

Investigations of conjugate natural convection regimes were conducted for the following dimensionless criteria corresponding to the turbulent flow: $Ra = 10^8$, $\Pr_t = 1$, Pr=0.71, $Ki_1 = 42.86$, $Ki_2 = 21.43$. fig. 2 presents typical fields of temperature and stream function illustrating the influence of the non-stationarity factor and presence of the open boundary on differential heat transfer characteristics formation in the solution domain under consideration.

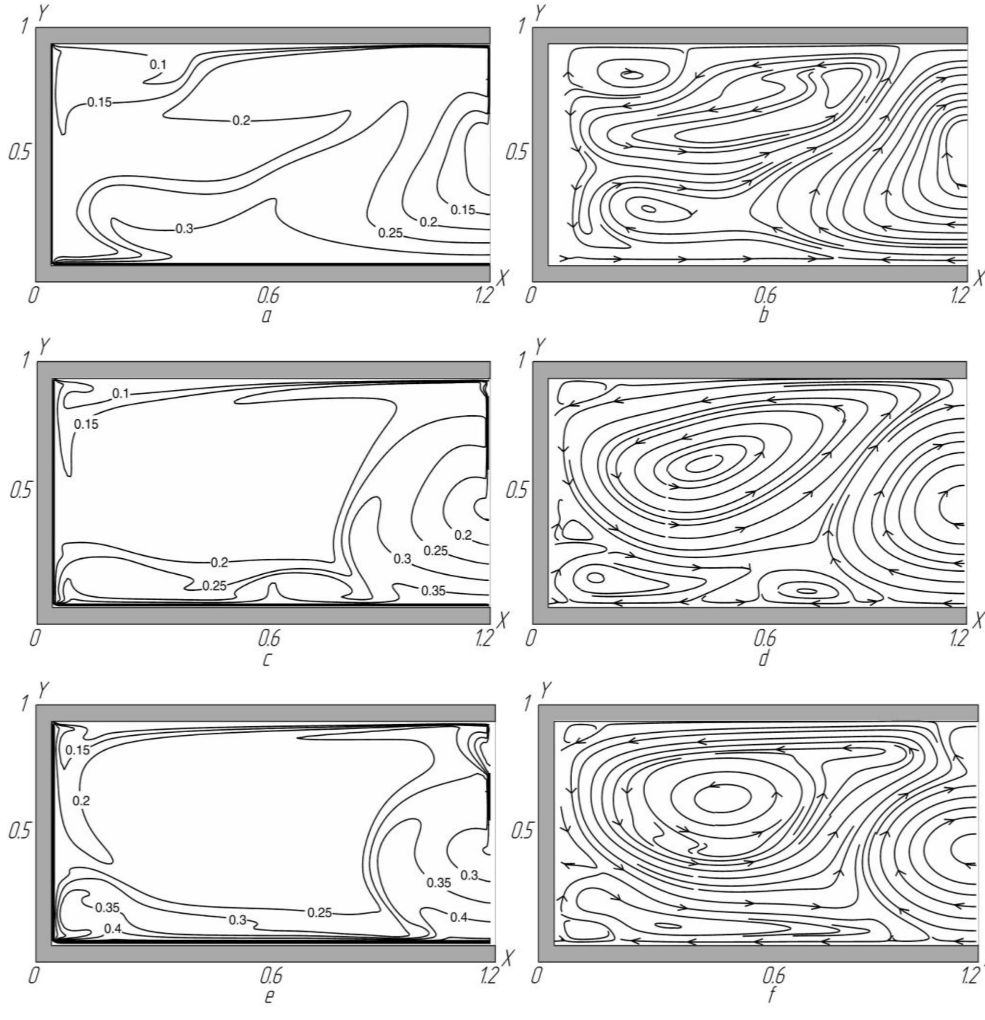

Fig.2. Fields of temperature (a, c, e) and stream function (b, d, f) at $Ra = 10^8$:
a, b) $\tau = 300$; c, d) $\tau = 600$; e, f) $\tau = 1200$.

Based on the analysis of the presented fields of differential heat transfer characteristics (fig. 2), it can be concluded that the thermophysical process under study had significant unsteady nature. When $\tau = 300$, a large-scale convective cell in the center of the air cavity and a small circulation flow were formed in the solution domain, which was obviously due to the radiant heat supply to the air – wall interfaces and the presence of the open boundary. An increase in the dimensionless time to $\tau = 600$ led to a rise in the temperature of the bottom horizontal interface. Large-scale convective cell in the center of the domain displaced small circulation flow in the left top and bottom corners of the cavity. The isotherms were shifted to the open boundary and followed the shape of the streamlines in the area of 0.9 <X <1.2, 0.1 <Y <0.75. Further increase in the dimensionless time to $\tau = 1200$ did not lead to significant modification of the differential heat transfer characteristics in geometrical and physical conditions under consideration.

Fig. 3 shows the temperature distribution in the gas cavity in the Y = 0.5 section.

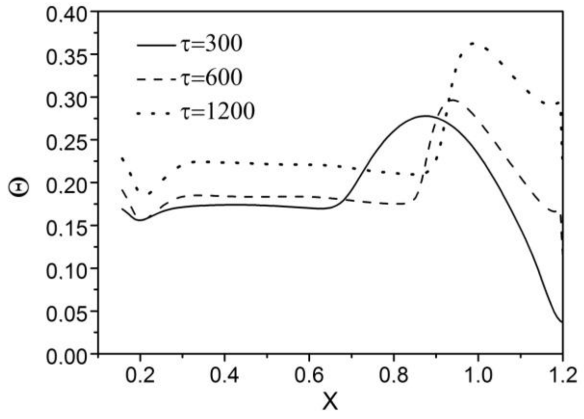

Fig. 3. The temperature distribution in the gas cavity in the Y = 0.5 section at $Ra = 10^8$.

It was clearly seen that an increase in dimensionless time led to a rise in the temperature in the typical cross-section of Y=0.5. Extremum of temperature was shifted to the open boundary, which was obviously due to the air flow pattern in this area (fig. 2 b, d, f).

In order to estimate the heat transfer rate, integral heat exchange characteristics were calculated. Dimensionless heat exchange coefficients at the top (Nu_{av1}), bottom (Nu_{av2}) and vertical (Nu_{av3}) gas – wall interfaces are defined as in [8]:

$$Nu_{av1} = \int_{0,2}^{1,2} \left|\frac{\partial\Theta}{\partial Y}\right|_{Y=0,1} dX; \quad Nu_{av2} = \int_{0,2}^{1,2} \left|\frac{\partial\Theta}{\partial Y}\right|_{Y=0,9} dX; \quad Nu_{av3} = \frac{1}{0,8}\int_{0,2}^{1,2} \left|\frac{\partial\Theta}{\partial X}\right|_{X=0,1} dY.$$

Figure 4 shows the dependence of the average integral Nusselt numbers versus dimensionless time.

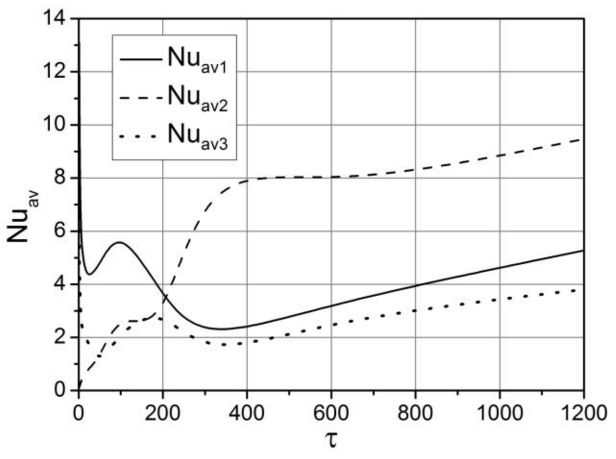

Fig. 4. Dependence Nu_{av} on τ when $Ra = 10^8$.

The obtained dependences (fig. 4) of the integral heat transfer characteristics illustrated the unsteady nature of the conjugate heat transfer process under study. The average Nusselt number at the bottom horizontal (Nu_{av1}) and vertical (Nu_{av3}) gas – wall interfaces had a nonlinear dependence before $\tau = 300$, which was connected with the radiant energy supply to these interfaces. The air layer was formed with an increase in temperature in the sections of X = 0.1 and Y = 0.1 (fig. 2 a, c. e) in a range of $1 < \tau < 40$. As the result, Nu_{av1} and Nu_{av3} reduced. Hereafter (before $\tau = 100$), the average dimensionless heat exchange coefficients at the bottom horizontal and vertical gas - a wall interfaces increased, which was connected with the formation of convective plumes as the result of natural convection. The average integral criteria Nu_{av} monotonically increased in a range of $400 < \tau < 1200$, which was obviously due to a rise in the circulation gas flow rate. At the same time, the thermophysical process under study gradually entered the quasi-stationary mode. The average temperature increased in the solution domain, but the shape of the isotherms did not substantially modify.

4 Conclusion

An approach for the modelling of turbulent natural convection in the semi - open cavity under conditions of the radiant heating of gas – wall interfaces was suggested. According to the numerical simulation results, scale influence of open vertical boundary and radiant heating of the gas – wall interfaces on the formation of differential and integral heat transfer characteristics was established. The formulated boundary value problem allows to decrease the spent of computing resources by reducing the modelling area size when the analyzing the thermal regimes of systems heated by infrared emitters.

Acknowledgments

The reported research was supported by Russian Federation President Grant for state support of the Russian Federation leading scientific schools SS-7538.2016.8.

References

1. K.-N. Rhee, K. W. Kim, Build Environ **91** (2015)
2. Buhmirov, S. A. Krupennikov, Yu. S. Solnyshkova, Her. Ivan. St. Pow. Un. **4** (2010)
3. G.V. Kuznetsov, V.I. Maksimov, T.A. Nagornova, N.I. Kurilenko, G. Ya. Mamontov, J. Engin. Ph. Thermoph. **86**, 3 (2013)
4. Sh. Chen, X.-h. Huang, Ch. Yang, G.-f. Liu, C.-j. Ding, B. Han, Appl. Math. Mech. - Engl. Ed. **34** 11 (2013)
5. H. Sajjadi, R. Kefayatib, Therm. Sci. **19**, 1 (2015)
6. G.V. Kuznetsov, P.A. Strizhak J. Eng. Thermophys-Rus. **17**, 3 (2008)
7. G.V. Kuznetsov, T.A. Nagornova, A.E. Ni, JEPT. **88**, 1 (2015)
8. G.V. Kuznetsov and M.A. Sheremet, Int. J. Heat Mass Transf. **52**, 1-2 (2009)
9. P. J. Roache, *Computational Fluid Dynamics* (Hermosa Publishers, Albuquerque, 1976)
10. H.N. Dixit, V. Babu, Int. J. Heat Mass Transf. **49** (2006)
11. N.C. Markatos, K.A. Pericleous, Int. J. Heat Mass Transf. **27** (1984)