Synthesis of a PID-controller of a trim robust control system of an autonomous underwater vehicle

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Abstract. Autonomous underwater vehicles are often used for performing scientific, emergency or other types of missions under harsh conditions and environments, which can have non-stable, variable parameters. So, the problem of developing autonomous underwater vehicle motion control systems, capable of operating properly in random environments, is highly relevant. The paper is dedicated to the synthesis of a PID-controller of a trim robust control system, capable of keeping an underwater vehicle stable during a translation at different angles of attack. In order to synthesize the PID-controller, two problems were solved: a new method of synthesizing a robust controller was developed and a mathematical model of an underwater vehicle motion process was derived. The newly developed mathematical model structure is simpler than others due to acceptance of some of the system parameters as interval ones. The synthesis method is based on a system poles allocation approach and allows providing the necessary transient process quality in a considered system.

1. Introduction
Modern underwater operations and activities dedicated to scientific, industrial or emergency issues cannot be always performed by a human, especially in harsh environments. This makes a problem of autonomous underwater vehicles (AUVs) development relevant. In order to operate effectively, an AUV must be equipped with a control system, manipulating several parameters: course, roll, trim, translation velocity and depth. In this paper a problem of a trim stabilization control system development is explored.

2. Formulation of a problem
First of all, in order to set the problem properly, a structure of a considered type of AUVs must be stated. AUVs of the considered type are usually cigar-shaped and are being driven by several thrusters: four main thrusters, one horizontal steering thruster and two vertical steering thrusters. The course is manipulated by a horizontal steering thruster, the depth and trim are manipulated by vertical steering thrusters, translation velocity is manipulated by main thrusters. The scheme of thrusters allocation is given below in Figure 1.

The construction of the propulsion and steering system allows considering the trim stabilization problem separately from manipulating the course and roll, so, in this paper it is considered that the course and roll are properly controlled by other subsystems of the AUV.

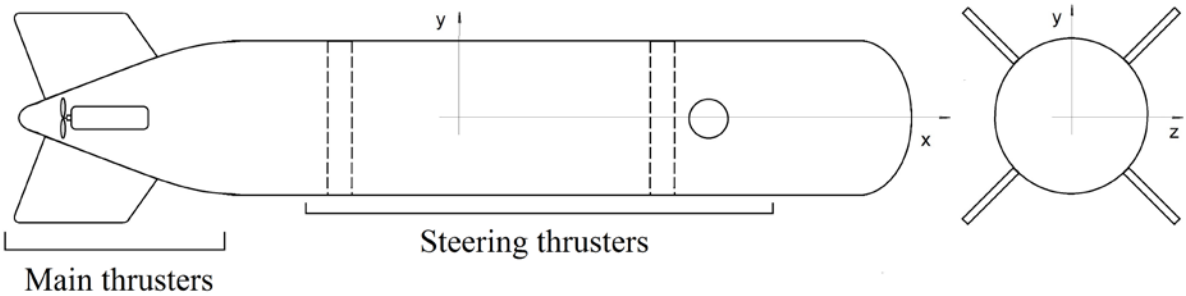

Figure 1. Thrusters allocation

Manipulating the trim of the AUV is considered to be performed by applying the moment of a couple of forces, generated by vertical steering thrusters, to a body of the AUV as it is shown in the Figure 2.

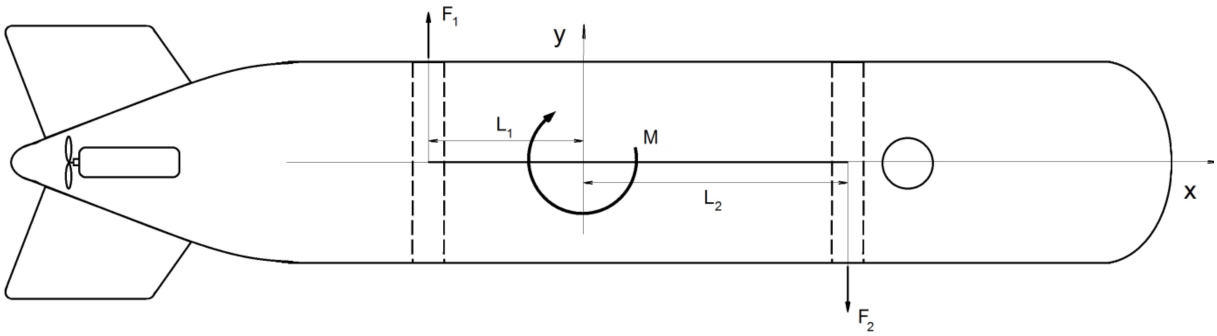

Figure 2. Manipulating principle

A structure of a control loop is shown in Figure 3.

The operating principle of the system shown in figure 3 can be described as follows: a setpoint of the trim is set by a programmable positioner or another control system 1. The error between the actual trim value and the setpoint is calculated by comparator 2 and then arrives to controller 3. The control signal from the output of controller 3 arrives to the propulsion and steering system (PSS) 4. The PSS generates a moment of vertical steering thrusters traction forces, which is applied to the body of AUV 5. After changing the trim value, it is measured by trim sensor 6 and comes back to the comparator. So, the considered system works as a classic closed-loop control system.

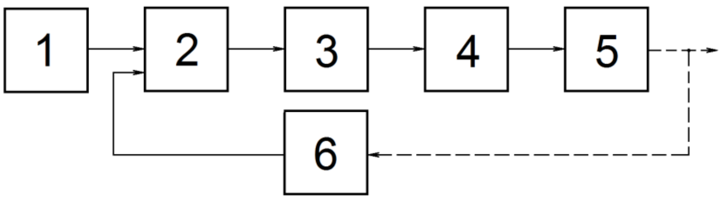

Figure 3. Flowchart of the considered control loop

Finally, the aim of the considered research can be formulated as follows: considering some parameters of the system as interval, it is necessary to synthesize a linear PID-controller for a robust control system shown in figure 3, dedicated to stabilize an AUVs trim during its translation at different angles of attack.

In order to achieve the main aim of the research, following objectives must be accomplished:

- deriving a transfer function of the process of an AUV motion in a longitudinal vertical plane;
- deriving transfer functions of the PSS and a trim sensor;

- developing a controller synthesis method;
- synthesizing a controller, providing a necessary transient process quality.

3. Modelling a process of an AUV motion into a longitudinal vertical plane

The motion of an AUV, having a symmetric body, is fully determined by the following system of differential equations [1]:

$$\begin{cases} (m+\lambda_{11})\cdot\frac{d\upsilon}{dt}-(m\cdot y_g-\lambda_{16})\cdot\frac{d^2\psi}{dt^2}+(m+\lambda_{11})\cdot\frac{d\psi}{dt}+(m+\lambda_{22})\cdot\upsilon\cdot\alpha\cdot\frac{d\psi}{dt}-\lambda_{12}\cdot\alpha\cdot\frac{d\upsilon}{dt}- \\ -\lambda_{12}\cdot\upsilon\cdot\frac{d\alpha}{dt}-\lambda_{12}\cdot\upsilon\cdot\omega_z=T-\frac{1}{2}\cdot c_x\cdot\rho\cdot V^{2/3}\cdot\upsilon^2-p\cdot\sin(\psi); \\ -(m+\lambda_{22})\cdot\alpha\cdot\frac{d\upsilon}{dt}-(m+\lambda_{22})\cdot\upsilon\cdot\frac{d\alpha}{dt}+(m\cdot x_g+\lambda_{26})\cdot\frac{d^2\psi}{dt^2}+\lambda_{21}\cdot\frac{d\upsilon}{dt}+ \\ +(m+\lambda_{11}-\frac{1}{2}\cdot c_y^{\omega_z}\cdot\rho\cdot V^{2/3}\cdot L)\cdot\upsilon\cdot\frac{d\psi}{dt}=\frac{1}{2}\cdot c_y\cdot\rho\cdot V^{2/3}\cdot\upsilon^2-p\cdot\cos(\psi); \\ (J_z+\lambda_{66})\cdot\frac{d^2\psi}{dt^2}-(m\cdot x_g+\lambda_{26})\cdot\alpha\cdot\frac{d\upsilon}{dt}-(m\cdot x_g+\lambda_{26})\cdot\upsilon\cdot\frac{d\alpha}{dt}-(m\cdot y_g-\lambda_{16})\cdot\frac{d\upsilon}{dt}+ \\ +\left[m\cdot x_g+\lambda_{26}-\frac{1}{2}\cdot m_z^{\omega_z}\cdot\rho\cdot V\cdot L-(m\cdot y_g-\lambda_{16})\cdot\alpha\right]\cdot\upsilon\cdot\frac{d\psi}{dt}+\lambda_{21}\cdot\upsilon^2=\frac{1}{2}\cdot m_z\cdot\rho\cdot V^{2/3}\cdot\upsilon^2- \\ -G\cdot\left[x_g\cdot\cos(\psi)+y_g\cdot\sin(\psi)\right] \end{cases}$$

The parameters of this system of differential equations are designated as follows: m – AUVs weight; λ_{ij} – coefficients of added masses of the AUV; α – angle of attack; x_g, y_g – coordinates of the mass center; ρ – water density; V – AUV displacement; c_x, c_y – coefficients of hydrodynamic forces; m_z – coefficient of a hydrodynamic moment; $c_y^{\omega_z}$, $m_z^{\omega_z}$ – derivative of hydrodynamic forces and moments with respect to angle of attack α; T – traction force of main thrusters; J_z – AUV moment of inertia on Oz axis; p – negative buoyancy; ψ – AUV trim; υ – AUV translation velocity.

The only equation which links rotational characteristics of the AUV motion process with each other is the third one. In this equation the impact of the PSS is not taken into consideration. In order to consider this impact, the moment of a couple of vertical steering thrusters traction forces M was added into the right part of the equation.

Also, this equation describes a multivariable control object. In order to simplify the mathematical model of an object, a translation velocity, an angle of attack and their derivatives were considered as interval parameters. Values of cosines and sinus of the AUV trim are also considered as intervals with minimum and maximum values equal to -1 and 1 accordingly.

Considering all assumptions, the differential equation of a control object can be written as follows:

$$(J_z+\lambda_{66})\cdot\frac{d^2\psi}{dt^2}+\left[m\cdot x_g+\lambda_{26}-\frac{1}{2}\cdot m_z^{\omega_z}\cdot\rho\cdot V\cdot L-(m\cdot y_g-\lambda_{16})\cdot[\alpha]\right]\cdot[\upsilon]\cdot\frac{d\psi}{dt}=M+\left(\frac{1}{2}\cdot m_z\cdot\rho\cdot V^{2/3}-\lambda_{21}\right)\cdot[\upsilon]^2-$$
$$-G\cdot\left[x_g+y_g\right]\cdot[-1;1]+(m\cdot x_g+\lambda_{26})\cdot([\alpha]\cdot[d\upsilon]+[\upsilon]\cdot[d\alpha])+(m\cdot y_g-\lambda_{16})\cdot[d\upsilon]$$

In this equation some variables are put into square brackets in order to show, that they have interval values. Here [α] – angle of attack, [υ] – AUV translation velocity, [d α] and [d υ] are interval values of their derivatives accordingly. The transfer function of the control object can be written as follows:

$$W_{AUV}(s)=\left\{(J_z+\lambda_{66})\cdot s^2+\left[m\cdot x_g+\lambda_{26}-\frac{1}{2}m_z^{\omega_z}\cdot\rho\cdot V\cdot L-(m\cdot y_g-\lambda_{16})\cdot[\alpha]\right]\cdot[\upsilon]\cdot s\right\}^{-1}$$

4. Modelling a trim control loop

Besides the control object, which transfer function was determined previously, a trim control loop includes an actuator – a couple of steering thrusters - and a feedback device – a trim sensor. In order to complete the mathematical model of it, it is necessary to derive mathematical models of thrusters and sensors firstly.

In modern AUVs intellectual thrusters are usually used. An intellectual thruster is equipped with an automatic control system, which manipulates the rotation frequency of a thruster's propeller. So, in fact, the control signal of these thrusters determines a traction force of the thruster but not the rotation frequency of its propeller. The inertness of the thruster should be explored experimentally. The mathematical model of the steering thruster can be written as follows:

$$W_T(s) = K_T \cdot (T_T \cdot s + 1)^{-1}$$

After deriving a single thruster's model, a model of PSS must be derived according to the manipulating principle shown in figure 2. According to this principle, the traction force of each steering thruster should be multiplied by its arm in order to calculate the moment of each thruster's traction force and then, these moments should be summed. So, the mathematical model of the part of PSS, responsible for manipulating the trim, can be written as follows:

$$W_{PSS}(s) = K_T \cdot (L_1 + L_2) \cdot (T_t \cdot s + 1)^{-1}$$

The trim can be described by its transfer coefficient K_{TS} only, because it inertness is negligible. To form a control signal a PID-controller will be used. Its transfer function can be written as follows:

$$W_{PID}(s) = K_P + K_I \cdot s^{-1} + K_D \cdot s$$

After deriving mathematical models of all elements of the system shown in figure 3, a model of the whole control loop can be derived. The transfer function of the considered system can be written as follows:

$$W(s) = \frac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{a_4 \cdot s^4 + a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s + a_0}$$

$$a_0 = 2 \cdot K_I \cdot K_{TS} \cdot K_T \cdot (L_1 + L_2); \quad a_1 = 2 \cdot K_P \cdot K_{TS} \cdot K_T \cdot (L_1 + L_2);$$

$$a_2 = 2 \cdot K_D \cdot K_{TS} \cdot K_T \cdot (L_1 + L_2) + [v] \cdot 2 \cdot (\lambda_{26} + m \cdot x_g) + [\alpha] \cdot [\upsilon] \cdot 2 \cdot (\lambda_{16} - m \cdot y_g) - L \cdot V \cdot \rho \cdot \upsilon \cdot m_Z^{\omega_z};$$

$$a_3 = 2 \cdot (J_Z + \lambda_{66}) + [v] \cdot 2 \cdot T_T \cdot (\lambda_{26} + m \cdot x_g) + [\alpha] \cdot [\upsilon] \cdot 2 \cdot T_T \cdot (\lambda_{16} - m \cdot y_g) - L \cdot T_T \cdot V \cdot \rho \cdot \upsilon \cdot m_Z^{\omega_z};$$

$$a_4 = 2 \cdot T_T \cdot (J_Z + \lambda_{66});$$

$$b_0 = 2 \cdot K_I \cdot K_T \cdot (L_1 + L_2); \quad b_1 = 2 \cdot K_P \cdot K_T \cdot (L_1 + L_2); \quad b_2 = 2 \cdot K_D \cdot K_T \cdot (L_1 + L_2);$$

In order to complete the model all parameters of the system must be substituted with their values. For this purpose, the values of interval parameters will be determined. Let us assume that translation velocity υ vary between 0.3 and 3 meters per second; angle of attack α – between -1.5708 and 1.5708 radians. Finally, considering all parameters values, the transfer function of the considered system can be written as follows:

$$W(s, K_P, K_I, K_D) = \frac{K_D \cdot s^2 + K_P \cdot s + K_I}{0.4800 \cdot s^4 + [0.3240; 2.1100] \cdot s^3 + [35.4651 + K_D; 65.0154 + K_D] \cdot s^2 + K_P \cdot s + K_I}$$

5. Synthesizing a robust PID-controller

To synthesize a PID-controller capable of providing the necessary control quality, a newly developed method of synthesis will be used. It is based on a system's poles allocation by applying a robust D-partition method [2].

Before synthesizing a controller, some requirement for the transient process must be determined. It is necessary to provide a 2-3 second long transient process in the considered system. Allocating system's poles allows manipulating the system transient process duration by changing its stability degree. The necessary transient process duration can be provided by achieving a robust stability degree equal to 2 by allocating one system pole at an interval of the real axis of a complex plane between -2.1 and -2; all other poles must be allocated on the left side of the line, which is parallel to the imaginary axis and goes through point (-15;0) of the complex plane.

A necessary poles location can be provided by dividing them into two groups: dominant ones, which location on the complex plane is absolutely fixed and determines a system control quality, and poles with undetermined location, which should be allocated as far from dominant poles as it is possible in order to minimize their influence on the system control quality.

In accordance to aforesaid, an interval characteristic polynomial – denominator of the transfer function – can be divided into three parts as shown:

$$D(s) = A(s, s_0) \cdot B(s, \overline{K}, \overline{q}) + R(\overline{K}, \overline{q}).$$

Here roots of polynomial $A(s, s_0)$ are dominant poles of the system; polynomial $B(s, \overline{K}, \overline{q})$ is polynomial $D(s)$ divided by polynomial $A(s, s_0)$, its roots are poles with undetermined location; $R(\overline{K}, \overline{q})$ is a remainder of this division; s_0 is an interval dominant pole; $\overline{K}$ is a vector of PID-controller parameters; $\overline{q}$ is a vector of system interval parameters. In order to allocate system poles properly with the help of method [2] several steps must be taken:

– the location of the interval dominant pole must be determined;

– $R(\overline{K}, \overline{q})$ equality to zero must be provided;

– coefficients of the polynomial $B(s, \overline{K}, \overline{q})$ must be calculated.

The first step has already been taken, the second step is providing the remainder's equality to zero. To calculate the value of the remainder a following expression will be used:

$$R(\overline{K}, \overline{q}) = \sum_{i=0}^{m} a_i \cdot [s_0]^i = D([s_0])$$

After calculating the interval value of the remainder, in order to provide its equality to zero both minimum and maximum values of the remainder must be equated to zero and put into a system of two linear algebraic equations. For the considered system, these equations can be written as follows:

$$-2.1 \cdot K_P + K_I + 4.41 \cdot K_D + 283.81 = 0$$
$$-2.0 \cdot K_P + K_I + 4.0 \cdot K_D + 125.06 = 0$$

By solving this system, dependencies between controller parameters, which provide remainders equality to zero and a necessary location of the dominant pole, can be derived:

$$\begin{aligned} K_I(K_P) &= 1.0244 \cdot K_P + 1423.70 \\ K_D(K_P) &= 0.2439 \cdot K_P - 387.20 \end{aligned} \tag{1}$$

After that, in order to place other poles of the system, coefficients of polynomial B must be calculated with the help of the expression written below:

$$[b]_i = [b]_{i+1} + [a]_{i+2} \cdot [s_0]; i \in n-1...0.$$

Here $[b]_i$ is a coefficient of $B(s, \overline{K}, \overline{q})$ polynomial; $[a]$ is a coefficient of the original characteristic polynomial of the system. After calculating all coefficients, some substitutions must be made: values of K_I, K_D parameters must be substituted with their functions (1); also substitution $s \to -15 + j\omega$ must be made in order to define the maximal real part of the non-dominant poles. Now, a robust D-partition can be plotted in order to find a stability region on a plane of the K_p parameter. An example of D-partition curve is shown in Figure 4. In Figure 4 the start of the curve is marked with a circle, the end – with a bold point; the chosen value of the controller parameter is marked with a square. After choosing the value of one parameter, it is possible to calculate the rest of the parameters by (1).

Finally, parameters of the PID-controller providing the necessary control quality are $K_P = 1758$, $K_I = 3224.60$, $K_D = 41.58$. In order to estimate the control quality of the considered system a step response in all vertexes of a system coefficients polytope is shown in Figure 5, also a poles location diagram is shown in Figure 6. Figure 5 and figure 6 clearly show that the necessary control quality was successfully provided: the duration of the transient process is less than half a second.

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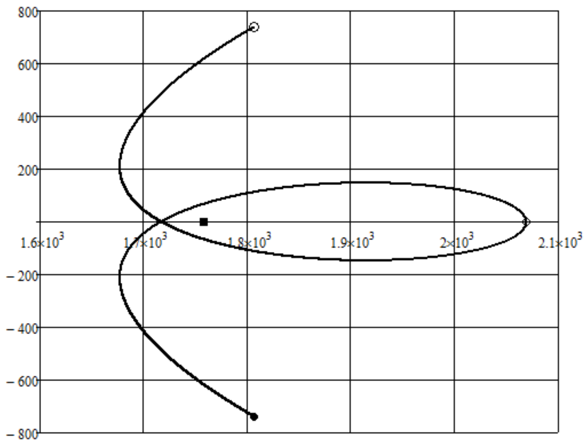

Figure 4. Example of D-partition curve

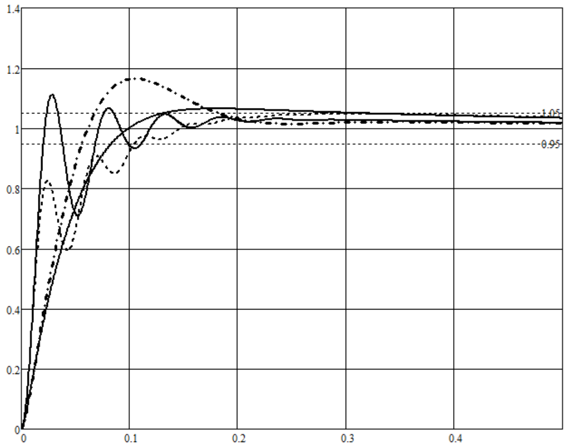

Figure 5. Step responses in all vertexes of coefficients polytope

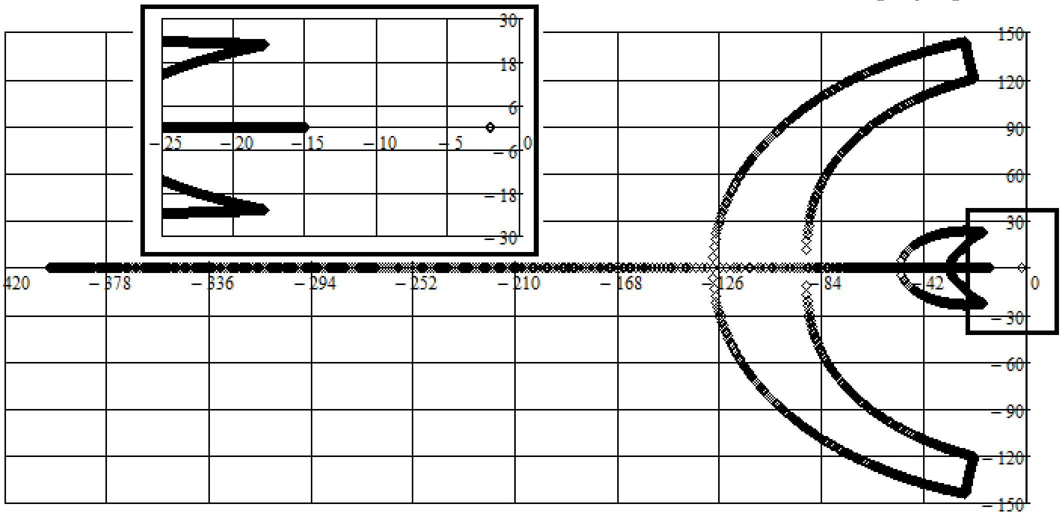

Figure 6. System poles location diagram

6. Conclusion

The main aim of this research has been successfully reached: a robust PID-controller, capable of providing the necessary control quality for the trim control system, has been synthesized. To do this, a new method of synthesis was developed, which allows determining the system robust stability degree and the transient process duration by allocating a single interval dominant pole. This method can only be applied to systems with the simplest interval type of characteristic polynomial coefficients uncertainty. In future papers the method under discussion will be developed to be applicable to systems with an affine or multilinear type of uncertainty. The model of the trim control loop will also be developed later: its interconnection with control loops of the course and roll will be taken into consideration.

References

[1] Pantov E N, Makhin N N and Sheremetov B B 1973 *Saint-Petersburg: Sudostroeniye* 173
[2] Polyak B T and Sherbakov P S 2002 *Moscow: Science* 229