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A method for calculating the productivity of cable communications networks

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Abstract. A probabilistic-mathematical instrument was used to develop a method for calculating the productivity of a cable line. The effect of deviation of factors from data of recording devices was determined when identifying random stream characteristics. The developed method was used to perform predictive calculation of the productivity of the modern cable communication line.

1. Introduction

One of the tendencies visible in the most recent and modern developments of communication networks is a change in terms of a transmission structure. The traffic has become multimedia-based in large networks. They are characterized as networks with a non-uniform information stream where the emphasis has been shifted towards the use of various network applications. The increased cost of creating and building the system itself necessitates an improvement of the actual quality of the design solutions, particularly the accuracy of the channel throughput determining the attenuation factor, and other productivity characteristics [1–3]. One of the possible approaches to the estimation of these essential design indicators is probabilistic modelling, with the systems being represented as a set of resources used one by one.

The validity of probabilistic modelling of results using the waiting-line theory and other methods is highly dependent on the conformity of the models utilised to the real systems. Network equipment designers and developers need to gather data about the behaviour of networks of various scales, architecture and configuration, as well as about their qualitative characteristics. Therefore, such modelling aids are required to account for all the specific features of the network operation, but also to allow defining initial information and lastly, to obtain reliable network characteristics [4].

2. The principle of communication line productivity modelling

In real cable communication systems the known and time-invariant parameters are those that characterize service facilities. The intensity of incoming/outgoing streams typically varies over time, and these variations are of random nature. However, it is clear that the cable system operation is directly dependent on the intensity of incoming streams or calls: the higher the intensity of incoming streams, the heavier the service mode, which requires, for instance, connection of additional service facilities. In view of this, an important task is to estimate the parameters of event streams at an arbitrary point in time based on observation of this stream.

Based on their randomness, the streams of events with time-dependent intensity can be divided into two types. The first one comprises the streams with intensity $\lambda(t)$, which is a continuous random process. The second type includes only streams having intensity $\lambda(t)$ as a piecewise-constant process with a finite number of states $(\lambda_1, \lambda_2, \ldots, \lambda_n)$. The transition of process $\lambda(t)$ from one state to another occurs at random points in time, with the stream of events behaving as stationary intensity flow λ_i during those time intervals when process $\lambda(t)$ is in state λ_i. Such streams are the most typical for real networks. These piecewise-stationary streams of events can be divided into three categories:

1) *synchronous* twice stochastic streams of events – streams with the intensity for which state transition occurs only at random points in time that are the moments of events occurrence;
2) *asynchronous* twice stochastic streams of events – streams with the intensity for which state transition does not depend on the moments when the events occur;
3) *half-synchronous* twice stochastic streams of events – streams where for one set of states it is 1 that is true, and for the rest of the states it is 2 that is true.

Taking into account all the above-stated requirements for the construction of the model, this paper suggests researching the problem of estimating the parameters of asynchronous streams of events with intensity $\lambda(t)$ being a piecewise-constant Markovian process that takes two values $\{\lambda, 0\}$ of the alternating stream of events, with the events being partially observable. The partial observability is due to the formation of a so-called dead time scheme, implying that an event occurrence in the alternating stream is accompanied by a fixed period of time (a dead time period) during which no other events can be observed. One of the major distorting factors when determining the characteristics of random streams is the dead time of recording devices. Dead time in this respect is the period when a recorded event is being processed, while any other event entered into the system input during this period is lost.

In view of this, the need to estimate the parameters (characteristics) of the real stream of events received at the system input arises. The amount and the character of dead time of certain devices depend on numerous factors. At a first approximation it can be assumed that this period lasts for certain (fixed) time T. All recording devices with a sufficient degree of approximation can be separated into two groups. The first group comprises devices with non-extending dead time, which is independent of the receipt of other events in the range of its action. Non-extending dead time is sometimes called first genus dead time, and corresponding recording devices are called counters, or type I recorders. Devices with extending dead time (type II recorders) belong to the second group. In this case dead time emerges after any event obtained at the system input regardless of it being recorded or not. In terms of data transmission lines, dead time can be represented by disturbances, failures, i.e. any effects on the network operation. Since these effects are also of random nature, recorders of the II type with extending dead time will be used for this study.

Let us consider a stationary stream of events with intensity λ that is observed only at certain time intervals and has two states – 0 and 1. If the control process is in state 1, the stream can be observed. But if it is in state 0, the stream cannot be observed. The lengths of stationary sections within the control process are distributed according to exponential law [5]

$$F_1(t) = 1 - \exp(-\alpha t) \text{ or } F_1(t) = 1 - \exp(-\beta t),$$

where α and β are intensities of the control process transition from state 1 to state 0 and vice versa, respectively.

Thus, the control process induces an alternating stream of events. The alternating stream of events, in its turn, is also partially unobserved. When an event occurs in this stream, there comes a certain time of duration T (dead time) within which other events cannot be observed. Therefore only the alternating stream event that comes after the end of the last period of non-observability will be observed. The occurring situation is shown in Figure 1, where the hatching is the indication of dead time, 0 and 1 are the control process states; t – current time, and $\{t_1, t_2, t_3, t_4, \ldots\}$ – moments of event occurrence in the stream under observation.

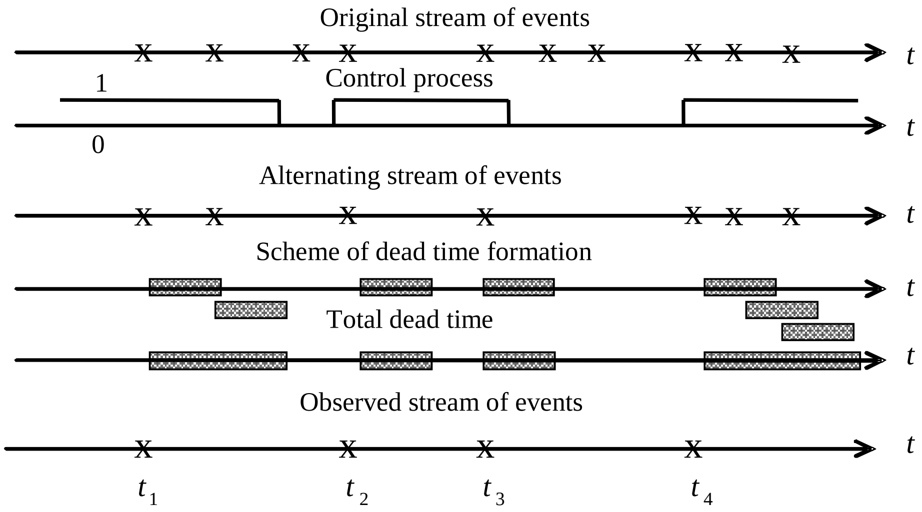

Figure 1. A scheme of the alternating stream.

The observations $\{t_1, t_2, \ldots, t_n\}$ at the final time interval are required to construct the estimation of original stream intensity λ, the estimation of intensities α, β for the control process state transition (i.e. to construct estimates of parameters for the alternating stream of events), as well as the estimation of dead time duration T. Assuming that $\{\tau_1, \tau_2, \ldots, \tau_n\}$ is the time interval between neighbouring events in the stream under observation, four statistical estimates - sampling moments - can be constructed:

$$C_k = \frac{1}{n} \cdot \sum_{i=1}^{n} \tau_i^{\,k}\ ,\ k = 1,2,3,4\,.$$

Thereafter the equations for determining parameters estimations $\{\lambda, \alpha, \beta, T\}$ will be formulated as follows:

$$M\left(\tau^k\right) = C_k\,,\ k = 1,2,3,4\,.$$

By solving the probabilistic model, six equations are obtained for determining transmission parameters estimations [6]:

$$M(\tau) = -\frac{\alpha}{\alpha+\beta} \cdot \frac{\left[(z_2-\lambda)e^{-z_1 T} + (\lambda - z_z)e^{-z_2 T}\right] \cdot \left[2(\alpha+\beta)^2 + \lambda(\alpha+\beta) + \lambda\beta\right]}{2(z_2 - z_1)\cdot(\alpha+\beta)^2 e^{(\alpha+\beta)T} + \lambda\cdot(\beta - z_1)\cdot(z_2+\alpha+\beta)e^{-z_1 T} + \lambda(z_2-\beta)\cdot(z_1+\alpha+\beta)e^{-z_2 T}} -$$

$$-\frac{1}{\alpha+\beta} + \frac{1}{\lambda\beta} \cdot \frac{(z_2 - z_1)\cdot(\alpha+\beta) + \lambda\cdot(\beta - z_1)e^{-z_1 T} + \lambda\cdot(z_2 - \beta)e^{-z_2 T}}{(z_2 - \lambda)\cdot e^{-z_1 T} + (\lambda - z_z)\cdot e^{-z_2 T}}, \qquad (1)$$

$$M\left(\tau^2\right) = -\frac{\lambda}{\alpha+\beta} \cdot \frac{2\cdot(z_1+z_2)}{z_1^{\,2} z_2^{\,2}} \cdot \left[\beta\cdot g_\xi(0) + \beta\cdot(\alpha+\beta)\cdot g'_\xi(0) + \alpha\cdot g_\xi\cdot(\alpha+\beta)\right] +$$

$$+\frac{\lambda}{\alpha+\beta} \cdot \frac{2\cdot\left(z_1^{\,2} + z_1 z_2 + z_2^{\,2}\right)}{z_1^{\,3} z_2^{\,3}} \cdot \beta\cdot(\alpha+\beta)\cdot g_\xi(0) +$$

$$+\frac{\lambda}{\alpha+\beta} \cdot \frac{1}{z_1 z_2} \cdot \left[2\beta\cdot g'_\xi(0) + \beta\cdot(\alpha+\beta)\cdot g''_\xi(0) + 2\alpha\cdot g'_\xi\cdot(\alpha+\beta)\right], \qquad (2)$$

$$M\left(\tau^3\right)=\frac{6\lambda}{\alpha+\beta}\cdot\frac{(z_1+z_2)\cdot\left(z_1^{\ 2}+z_2^{\ 2}\right)}{z_1^{\ 4}z_2^{\ 4}}\cdot\beta\cdot(\alpha+\beta)\cdot g_\xi(0)+$$
$$+\frac{6\lambda}{\alpha+\beta}\cdot\frac{(z_1+z_2)}{z_1^{\ 2}z_2^{\ 2}}\cdot\left[2\beta\cdot g'_\xi(0)+\beta\cdot(\alpha+\beta)\cdot g''_\xi(0)+2\alpha\cdot g'_\xi\cdot(\alpha+\beta)\right]-$$
$$-\frac{\lambda}{\alpha+\beta}\cdot\frac{1}{z_1z_2}\cdot\left[3\beta\cdot g''_\xi(0)+\beta\cdot(\alpha+\beta)\cdot g'''_\xi(0)+3\alpha\cdot g''_\xi\cdot(\alpha+\beta)\right],\qquad(3)$$

$$M\left(\tau^4\right)=\frac{12\lambda}{\alpha+\beta}\cdot\frac{z_1^4+(z_1+z_2)^2\cdot\left(z_1^2+z_2^2\right)+z_2^4}{z_1^5z_2^5}\cdot\beta\cdot(\alpha+\beta)\cdot g_\xi(0)-$$
$$-\frac{24\lambda}{\alpha+\beta}\cdot\frac{(z_1+z_2)\cdot\left(z_1^{\ 2}+z_2^{\ 2}\right)}{z_1^{\ 4}z_2^{\ 4}}\cdot\left[\beta\cdot g_\xi(0)+\beta\cdot(\alpha+\beta)\cdot g'_\xi(0)+\alpha\cdot g_\xi\cdot(\alpha+\beta)\right]+$$
$$+\frac{12\lambda}{\alpha+\beta}\cdot\frac{z_1^{\ 2}+z_1z_2+z_2^{\ 2}}{z_1^{\ 3}z_2^{\ 3}}\cdot\left[2\beta\cdot g'_\xi(0)+\beta\cdot(\alpha+\beta)\cdot g''_\xi(0)+2\alpha\cdot g'_\xi\cdot(\alpha+\beta)\right]-$$
$$-\frac{4\lambda}{\alpha+\beta}\cdot\frac{z_1+z_2}{z_1^{\ 2}z_2^{\ 2}}\cdot\left[3\beta\cdot g''_\xi(0)+\beta\cdot(\alpha+\beta)\cdot g'''_\xi(0)+3\alpha\cdot g''_\xi\cdot(\alpha+\beta)\right]+$$
$$+\frac{\lambda}{\alpha+\beta}\cdot\frac{1}{z_1z_2}\cdot\left[4\beta\cdot g'''_\xi(0)+\beta\cdot(\alpha+\beta)\cdot g^{IV}_\xi(0)+4\alpha\cdot g'''_\xi\cdot(\alpha+\beta)\right]\qquad(4)$$

$$z_1=\frac{\lambda+\alpha+\beta-\sqrt{(\lambda+\alpha+\beta)^2-4\lambda\beta}}{2},\ z_2=\frac{\lambda+\alpha+\beta+\sqrt{(\lambda+\alpha+\beta)^2-4\lambda\beta}}{2}.\qquad(5)$$

The system of the equations is determined by formulas (1–5), which make it possible to estimate the parameters of system transmission at any point in time. This system contains six equations concerning six unknowns $\{\lambda,\alpha,\beta,T,z_1,z_2\}$. The obtained system has only a numerical solution.

The adequacy of the proposed computational method was verified with a specific data transmission system of JSC “RICT” – “Russian-Italian Company of Telephone Installation” that provides various communication services to the city of Mezhdurechensk, Kemerovo region, such as: city telephony, wire communications, radio-access, a multipurpose duplex pay telephone network, Internet, IP-telephony, additional services based on intellectual networks, etc.

3. Calculation of data transmission network productivity

The original stream for the JSC “RICT” data transmission system was an asynchronous stream of events with intensity $\lambda(t)$, a piecewise-constant process with partially observable events. This study examines carefully a part of the system comprising a data source (a non-uniform stream of events), commutator switches 1 and 2 that were used for measuring intensity $\lambda(t)$, and various sites (residential-accommodation spaces, production plants, etc.). The dependences were measured on a working day, during peak time, from 12 to 2 p.m., when the maximum load in the data transmission network can be observed.

The statistical experiment on estimating the unknown parameters of the observed stream of events was based on simulation modelling of the examined stream of events with parameters $\{\lambda,\alpha,\beta,T\}$ at a fixed interval of observations $(0,T)$, assuming that at some point in time an event occurs in the original stream. The stationarity periods within the control process were generated according to the task definition. If the event occurred at the time point when the control process was in state 1, and the time point fell into the non-observability period, then the event was not recorded as an event of the stream under observation. If the event occurred after the dead time was over, then the event was recorded as an event of the stream under observation. The statistical experiment on estimating unknown parameters $\{\lambda, \alpha, \beta, T\}$ for the given task consisted in constructing the estimations with confidence intervals and looks as follows:

1) the alternating stream model with dead time is implemented and it forms sample $\{\tau_1, \tau_2, \dots, \tau_n\}$ of observations with random value τ (a time interval between neighbouring events in the stream under observation);
2) statistical estimates C_1, C_2, C_3, C_4 are accumulated;
3) in order to determine the estimations of alternating stream parameters $\{\lambda, \alpha, \beta\}$, a system of equations is formed using formulas (1–5), with this system of equations being solved by a numerical method;
4) estimations $\{\lambda, \alpha, \beta, T\}$ are determined;
5) average values of estimations $\lambda = \frac{1}{N} \cdot \sum_{i=1}^{N} \lambda_i$ and dispersion $D(\lambda) = \frac{1}{N-1} \cdot \sum_{i=1}^{N} (\lambda_i - \lambda)^2$ are formed, which is followed by the calculation of confidence intervals based upon the obtained statistical data [6];
6) network load factor ρ is calculated.

An imitation experiment was conducted to determine parameters $\{\lambda, \alpha, \beta, T\}$ under this procedure. Table 1 summarizes the intensity values obtained through calculating the alternating stream of events in the scheme with extending dead time for commutator switches 1 and 2.

Table 1 – Intensities of data transmission through commutator switches 1 and 2.

Network channels	Commutator switch 1		Commutator switch 2	
	Transmitted to the server, Mbps	Received from the server, Mbps	Transmitted to the server, Mbps	Received from the server, Mbps
Gi0/01	3.28898915	25.53188615	0.67362369	18.54783908
Gi0/02	0.04298392	33.24441975	0.10073978	12.06851600
Gi0/03	0.80975518	35.48801027	63.98897778	15.39330280
Gi0/04	0.12364604	27.12027801	1.33482974	13.78169399
Gi0/05	0.42751007	26.13877361	0.19634786	13.03044206
Gi0/06	0.64386077	25.89963004	0.75870185	10.06545153
Gi0/07	0.88508078	33.38903852	1.07752315	15.33562257
Gi0/08	4.51858631	33.69003122	0.01963478	11.21360999
Gi0/09	0.13307745	26.30807871	1.42522137	16.58389541
Gi0/10	0.93511678	32.63097632	0	0
Gi0/11	15.06527933	23.80922425	0	0
Gi0/12	31.39562755	33.27545442	31.55281399	22.42059607

The developed model adequacy is verified by forming confidence intervals based upon the obtained statistical data. To this end, let us calculate the average of estimations and dispersion. Let us tabulate the obtained numerical dispersion data for commutator switches 1 and 2 in Table 2.

Therefore, the confidence interval of data numerical calculation obtained from commutator switch 1 is equal to 0, while that obtained from commutator switch 2 is $1.236 \cdot 10^{-15}$.

The obtained values of confidence intervals make it possible to confirm the adequacy of the constructed model.

Table 2 – The estimation of dispersion for commutator switches 1 and 2.

Network channels	Commutator switch 1		Commutator switch 2	
	incoming	outgoing	incoming	outgoing
Gi0/01	13.994	8.249	9.725	8.149
Gi0/02	17.24	15.961	10.298	1.670
Gi0/03	16.473	18.205	53.590	4.995
Gi0/04	17.159	9.837	9.064	3.383
Gi0/05	16.856	8.856	10.202	2.632
Gi0/06	16.639	8.616	9.640	0.333
Gi0/07	16.398	16.106	9.321	4.937
Gi0/08	12.765	16.407	10.379	0.815
Gi0/09	17.150	9.025	8.974	6.185
Gi0/10	16.348	15.348	10.399	10.399
Gi0/11	2.218	6.526	10.399	10.399
Gi0/12	14.112	15.992	21.154	12.022
Intensity estimation	17.283		10.398	

The next step was to determine the load factor. Assuming that maximum intensity $\lambda_{\max}$ of the examined line is 100 Mbps, the load factor in our case is the ratio between the intensity, obtained through calculating the alternating stream of events in the scheme with extending dead time, and the maximum load factor:

$$\rho = \frac{\lambda'}{\lambda_{\max}},$$

where λ' is the intensity obtained through calculating the alternating stream of events in the scheme with extending dead time; $\lambda_{\max}$ is maximum intensity of information stream transmission within the network. The obtained data are summarized in Table 3.

Table 3 – The load factor for commutator switches 1 and 2.

Network channels	Commutator switch 1		Commutator switch 2	
	incoming	outgoing	incoming	outgoing
Gi0/01	0.0328898915	0.2553188615	0.0067362369	0.1854783908
Gi0/02	0.0004298392	0.3324441975	0.0010073978	0.1206851600
Gi0/03	0.0080975518	0.3548801027	0.6398897778	0.1539330280
Gi0/04	0.0012364604	0.2712027801	0.0133482974	0.1378169399
Gi0/05	0.0042751007	0.2613877361	0.0019634786	0.1303044206
Gi0/06	0.0064386077	0.2589963004	0.0075870185	0.1006545153
Gi0/07	0.0088508078	0.3338903852	0.0107752315	0.1533562257
Gi0/08	0.0451858631	0.3369003122	0.0001963478	0.1121360999
Gi0/09	0.0013307745	0.2630807871	0.0142522137	0.1658389541
Gi0/10	0.0093511678	0.3263097632	0	0
Gi0/11	0.1506527933	0.2380922425	0	0
Gi0/12	0.3139562755	0.3327545442	0.3155281399	0.2242059607

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Having calculated the line productivity, a predictive calculation of this communication line productivity was performed. To this end, it was necessary to calculate the maximum data transmission rate within the network so that there would be no overload of servicing facilities, no losses of transmitted and received data calls, as well as to ensure that the service and calls processing time would not lead to server overload. In other words, the formulated task was reduced in order to find a maximum acceptable network load factor, followed by the calculation of optimal intensity for this network.

Let us take a maximum acceptable line load factor as $\rho = 0.99 \ldots 0.95$. Knowing the load factor, the maximum possible transmission rate equal to 1 Gbps and taking into account the number of channels, let us calculate the acceptable transmission rate for each channel. By applying the developed probabilistic-mathematical instrument it was found that, considering the maximum acceptable load factor and ignoring dead time, the maximum rate is within the range of 950 – 999 Mbps. However, taking into account dead time and stated requirements, the data transmission intensity within the network can fall between 193 and 200 Mbps.

4. Conclusion

1. In order to calculate the communication network with a non-uniform stream of information, we have constructed a model that was used as a basis for performing the task of estimating an asynchronous stream with partially observable events of various intensities, which is a piecewise-constant process.
2. It has been established that one of the major distorting factors when determining the characteristics of random streams is the dead time of data recording devices. Upon the event occurrence in the stream there comes a certain time of fixed duration, within which other events cannot be observed, i.e. dead time is the period when a recorded event is being processed, while any other event entered into the system input during this period is lost.
3. The intensity obtained through calculation by a probabilistic-mathematical method and the data transmission intensity in the examined real communication network is two times higher than the existing one, i.e. the communication line is unloaded. This conclusion serves as a recommendation to a possible data transmission rate increase and, hence, the improvement of the communication system productivity as a whole.

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