Radiation from relativistic electrons in "light" and in conventional undulators. Classical and quantum approaches

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Abstract. A photon spectrum of undulator radiation (UR) is calculated in the semi-classical approach. The UR intensity spectrum is determined by an electron trajectory in the undulator neglecting by energy losses for radiation. Using the Planck's law, the UR photon spectrum can be calculated from the classical intensity spectrum both for linear and nonlinear regimes. The radiation of an electron in a field of strong electromagnetic wave (radiation in the "light" undulator) is considered in the quantum electromagnetic frame. Comparison of results obtained by both approaches has been shown that UR spectra in the whole cone coincide with high accuracy for the case $x \ll 1$. Characteristics of the collimated UR beam were simulated with taking into account the discrete process of photon emission along an electron trajectory in both kinds of undulators.

1. Introduction

One of the options of the future International Linear Collider (ILC) includes the polarized positron source based on a helical undulator with length ~100 m placed downstream the main linear accelerator. The energy of electron beam is about 150-250 GeV, the photon yield is $\sim 10^2$ photon/electron with energy ≥10 MeV, and polarization ~0.5 [1]. As a rule, characteristics of UR are calculated using the classical electrodynamics [2] and such an approach was used to calculate UR characteristics for the ILC positron sources [3, 4].

In our work we have compared characteristics of UR calculated using the classical electrodynamics and the quantum electrodynamics. We used the analogy between radiation of electrons in a "light" undulator and in the conventional undulator. In the former case, radiation is treated as the Compton back scattering process [5].

2. Photon spectrum from conventional magnetic undulator.

For the sake of simplicity, we consider UR from a helical undulator with a period λ_u, an undulator parameter K (nonlinearity parameter), and number of period N_u. In such an undulator, the electron trajectory is the helix. In the system where an electron is at rest (in average) (*R*-system) the trajectory of an electron is a circular with a radius [6]:

$$R \approx \frac{K\lambda_u}{\pi\gamma_0}, \text{ if } K \sim 1, \quad (1)$$

here γ_0 is the Lorentz-factor.

Using the well known formulas for synchrotron radiation, one can easily obtain formulas for radiation intensity after Lorentz-transformation from *R*-system to laboratory system [7]

$$\frac{dW}{d\Omega} = \frac{8\alpha\hbar\omega_0 N_u \gamma_0^4}{(1+K^2+\gamma^2\theta^2)^3} K^2 \sum_{n=1}^{\infty} n^2 \left[J_n'^2(nZ) + \left(\frac{\gamma_0\theta}{K} - \frac{1}{Z} \right)^2 J_n^2(nZ) \right], \quad (2)$$

where $\omega_0 = \frac{2\pi c}{\lambda_u}\left(1 - \frac{1+K^2}{2\gamma_0^2}\right)$ is the frequency of the fundamental harmonic, θ is outgoing photon angle, *n* is the number of the harmonic, $Z = \frac{2K\gamma_0\theta}{1+K^2+\gamma_0^2\theta}$.

The formula (2) is obtained in the small outgoing photon angle approximation for the long undulator ($N_u >> 1$). There is the well known relation connecting the frequency of *n*-th harmonic and outgoing angle [6]:

$$\omega^{(n)} = n \frac{2\gamma_0^2}{1+(\gamma_0\theta)^2+K^2} \frac{2\pi c}{\lambda_u}. \quad (3)$$

Using this relation, it is conveniently to transform the expression (2) into the spectral distribution ($dW/d\Omega \rightarrow dW/d\omega$) and after that to use a dimensionless spectral variable instead a frequency:

$$S^{(n)} = \frac{\omega^{(n)}}{2\gamma_0^2 2\pi/\lambda_u} = \frac{n}{1+(\gamma_0\theta)^2+K^2}, \; 0 \le S^{(n)} \le S_{\max}^{(n)}, \; S_{\max}^{(n)} = \frac{n}{1+K^2}. \quad (4)$$

The photon UR spectrum for *n*-th harmonic can be obtained from the UR intensity spectrum dividing by the emitted photon energy $\hbar\omega^{(n)}$:

$$\frac{dN^{(n)}}{dS^{(n)}} = 4\pi\alpha K^2 N_u \left\{ \frac{\left[n - S^{(n)} 2(1+K^2)\right]^2}{4S^{(n)} K^2 \left[n - S^{(n)}(1+K^2)\right]} J_n^2(nZ) + J_n'^2(nZ) \right\}. \quad (5)$$

The number of photons emitted at the *n*-th harmonic can be calculated after integration of the spectral distribution (5):

$$N^{(n)} = \int_0^{S_{\max}^{(n)}} \frac{dN^{(n)}}{dS^{(n)}} dS^{(n)}. \quad (6)$$

The total number of emitted photons can be found by summing up the contributions from the *n*-th harmonic:

$$N_{tot} = \sum_{n=1}^{\infty} N^{(n)} \approx \sum_{n=1}^{n_{\max}} N^{(n)}. \quad (7)$$

For the typical case in UR ($K \sim 1$), the sum (7) is converges quickly and criterium for the upper harmonic number $n_{\max}$ can be chosen from the following condition [8]:

$$\frac{N^{(n_{\max+1})}}{N_{tot}} \leq 0.005. \quad (8)$$

The dependence of the emitted photons number on the harmonic number per an electron and per an undulator period for the case $K = 1$ is shown in figure 1.

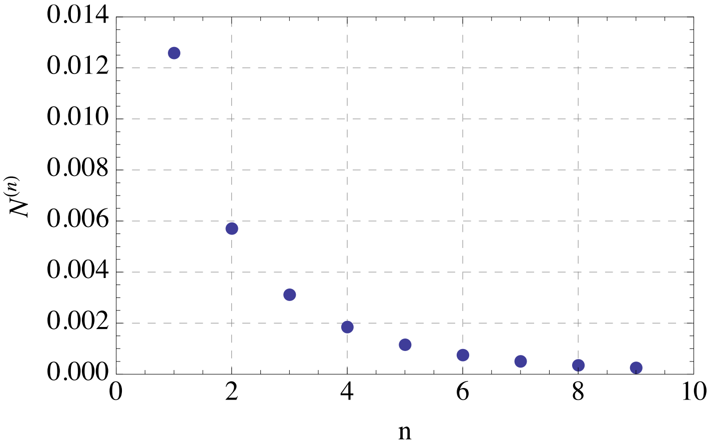

Figure 1. Dependence of the emitted photons number on the harmonic number.

The criterion (8) is fulfilled for $n_{\max} = 9$.
The spectral distribution for different harmonic numbers are presented in figure 2 and the resulting spectrum in figure 3. Both distributions were calculated per an undulator period.

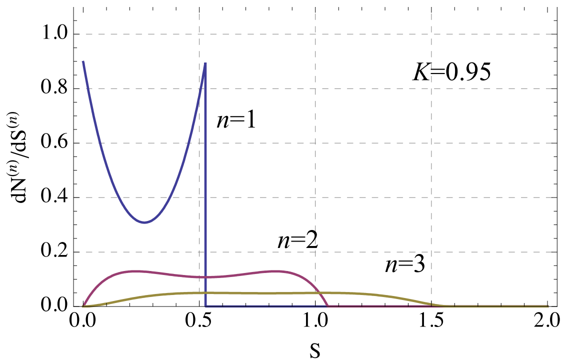

Figure 2. Spectral distribution for the first three harmonics.

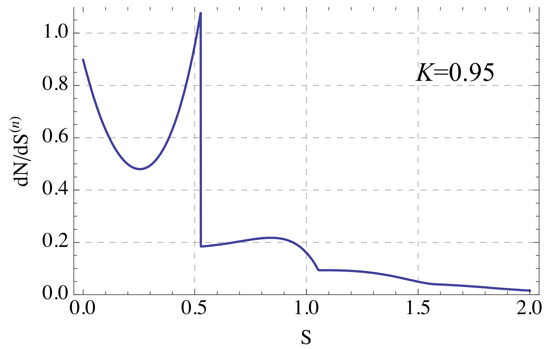

Figure 3. Resulting spectrum of the summarized harmonics.

The spectra presented above depend on the parameter K only and have an universal character allowing to transform them into real photon spectra for any values of γ_0 and K.

The circular polarization of UR depends on the photon outgoing angle or, using relation (3), can be written as the spectral dependence:

$$\xi_2(S^{(n)}) = \sum_{n=1}^{n_{\max}} 2\pi\alpha K N_u \frac{\left[2(1+K^2)S^{(n)} - n\right]}{\sqrt{S^{(n)}\left[n - S^{(n)}(1+K^2)\right]}} J_n(nZ) J_n^{'}(nZ) \Bigg/ \sum_{n=1}^{n_{\max}} \frac{dN^{(n)}}{dS^{(n)}} . \quad (9)$$

Such a dependence is shown in figure 4, which also can be used for calculations of energy dependence for fixed parameters γ_0 and λ_u.

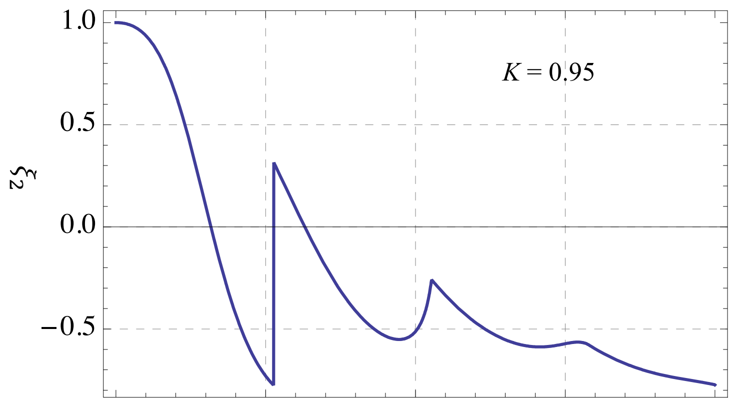

Figure 4. Degree of circular polarization.

The circular polarization of collimated UR is calculated using the same expression (9) but all harmonics are calculated in the limit:

$$S^{(n)}_{\min coll} \le S^{(n)} \le S^{(n)}_{\max}\,,\ S^{(n)}_{\min coll} = \frac{n}{1+(\gamma_0\theta_c)^2 + K^2}\,, \quad (10)$$

where θ_c is the angular collimator aperture.

As was shown in [9, 10], the distribution of the number of emitted UR photons is described by the Poisson law with the mean value N_{tot}. As a rule $N_{tot} > 1$ and it means that a photon may be emitted from an arbitrary point along a trajectory of an electron.

The simulation of such a process in a "light" undulator (the quantum Compton backscattering process) was performed in [9, 10] with taking into account a stochastic process of radiation along a trajectory. The quantum recoil effect for the case of a "light" undulator leads to so-called "red shift" in resulting spectra, but for the case $x = \frac{4\gamma_0\hbar\omega_0}{mc^2} << 1$ ($\hbar\omega_0$ is the energy of a laser photon) such an effect is negligible but the quantum discreetness of radiation process continues to play the important role. The comparison of the calculated "semi classical" UR spectrum using formulas (5, 6) with simulated spectra based on the quantum approach is shown in figure 4a for radiation in whole cone and for collimated UR spectrum in figure 5b.

Comparison were performed for an undulator with following parameters: $E_0 = 150$ GeV (γ_0 = 293500); λ_u = 11.5 mm; K = 0.95, (see [11]) and for “light” undulator with the wavelength twice less due to difference between both kinds of undulators [5].

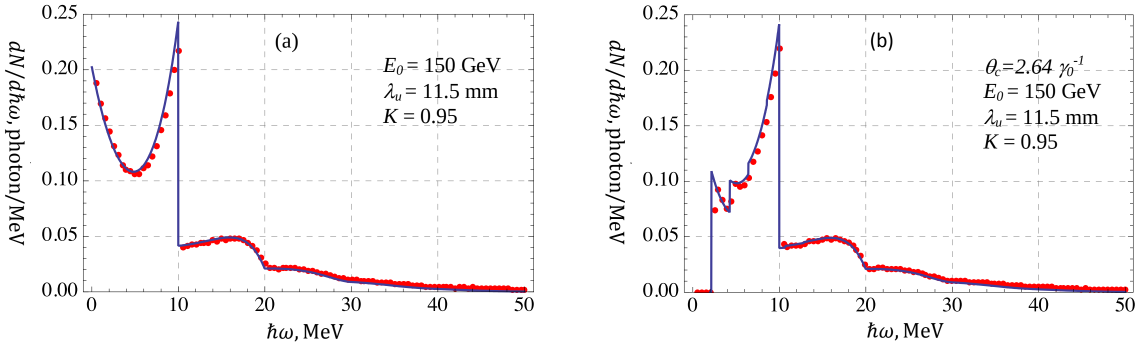

Figure 5. Comparison of the semiclassical UR spectrum (solid line) with simulated spectra based on the quantum approach in the whole cone (a) and collimated (b).

We would like to emphasize that spectra from conventional and "light" undulators were obtained in absolute units (photon per 1 MeV) and comparison showed the good agreement between them.

A tight collimation of UR generated in a long undulator which can be used for the ILC positron source design may lead to some distortion of spectra and, essentially, of resulting polarization due to different distances from an emission point and the collimator.

We simulated the effect of a "long" undulator for the following undulator parameters [12]:

$L_u = 231$ m
$\lambda_u = 11.5$ mm
$N_u = 2 \times 10^4$
$K = 0.92$
$E_e = 250$ GeV
$R_c = 0.7$ mm
Distance from the center of
the undulator to collimator = 412 m

and calculated the number of emitted photons per electron per period for each harmonic (table 1).

Table 1. Number of emitted photons per electron per period for each harmonic.

n	1	2	3	4	5	6	7	8
$N^{(n)}$	0.01167	0.00494	0.00251	0.00138	0.00079	0.00047	0.00028	0.00017

It means that the number of photons emitted from the all undulator length

$$N_0 = N_{tot} \frac{L_u}{\lambda_u} \approx 446.$$

For this case, instead of the Poisson distribution it is possible to use the Gaussian distribution

$$F_g(k, \bar{k}, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(k-\bar{k})^2}{2\sigma^2}}, \tag{11}$$

where $\bar{k} = N_0$, $\sigma = \sqrt{N_0}$.

Because the distribution F_g is very "narrow" for the sake of simplicity, we have changed simulation of the random emission points with taking into account a different number of emitted photons $\bar{k}$ by the uniform distribution along trajectory with fixed number of emitted photons N_0.

The simulation was performed using the following algorithm:

- calculation of $n_{\max}$, N_{tot} and spectral distributions for $n \le n_{\max}$;
- simulation of the harmonic number n;
- simulation of the spectral variable $S^{(n)}$ and corresponding angle θ;
- the condition of passing collimator $\theta \le \theta_c$ is applied for each point of photon creation;
- for "useful" events, creation of histograms according to the formulas (6) and (9);
- averaging by points of photons' creation along the trajectory where length L_u was divided uniformly by 200 points. Further increasing of the trajectory dividing doesn't lead to improvement of simulation results;
- total number of simulated photons was two million.

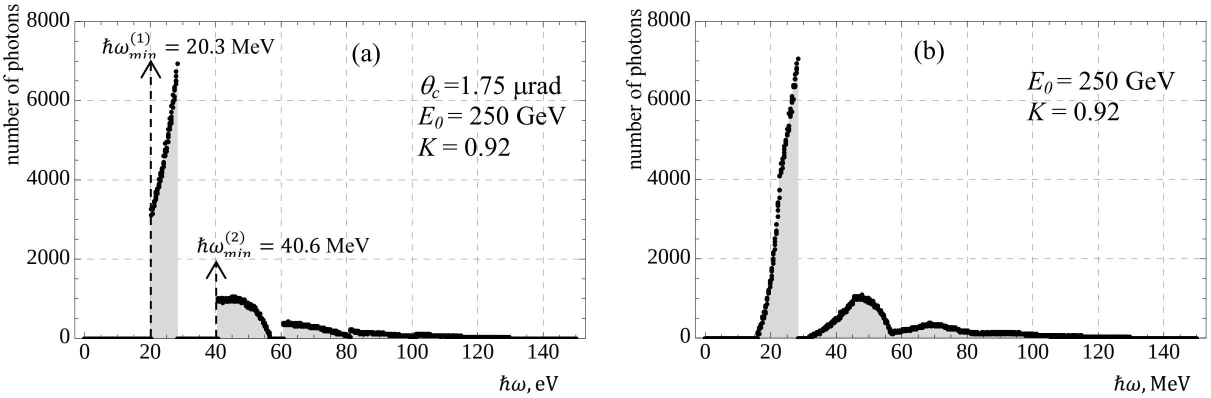

Figure 6. Spectrum of collimated UR emitted from the center of undulator (a) and along the length of undulator (b).

The simulation results are shown in figure 6. The effect of a "long" undulator leads to the "smoothing" of the spectral line. In comparison with the "ideal" case (figure 6a), such a smoothing leads to an appearance of photons with energies less than $\hbar\omega_{min}$. For the considered case, a part of such photons is about 8% for the first harmonic and about 15% for the second one.

The distortion of the circular polarization degree due to this reason will be considered elsewhere.

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