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Rolling Friction in Loose Media and its Role in Mechanics Problems

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Abstract: Rolling friction between particles is to be set in problems of granular material mechanics alongside with sliding friction. A classical problem of material passive lateral pressure on the retaining wall is submitted as a case in point. 3D method of discrete elements was employed for numerical analysis. Material is a universe of spherical particles with specified size distribution. Viscose-elastic properties of the material and surface friction are included, when choosing contact forces. Particles' resistance to rolling relative to other particles and to the boundary is set into the model. Kinetic patterns of medium deformations are given. It has been proved that rolling friction can significantly affect magnitude and nature of passive lateral pressure on the retaining wall.

1. Introduction

It is known, that the mechanics of granular material is based on Coulomb's law:

$$|\tau_n| = -tg\varphi \cdot \sigma_n + k\,, \tag{1}$$

где σ_n, τ_n – respectively are normal and tangential stresses at limit state area, k – adherence, tgφ – internal friction coefficient (in fact, sliding friction only). While solving continuum mechanics problems, proceeding to invariants is to be done in equation (1), and on this basis various options of closed mathematical models are built [1-4]. Since the original equation (1) involves sliding friction only, corresponding models and numerous solutions are also based on sliding friction. It is known that while deformation of soils and granular materials (including some rocks), freedom degrees, associated with particles rotation are significant. Therefore, along with sliding friction, rolling friction should be involved. Within the framework of continuum mechanics, it significantly complicates models because of additional internal variables [5-9].

The purpose of this paper is to demonstrate the importance of rolling friction by means of the numerical method of discrete elements (DEM). As an example, we choose a retaining wall classical problem, which is used in studies of natural phenomena (landslides), and in design of engineering structures (road and rail embankments, quarries sides). Consider the following problem: let a rigid vertical wall to hold heavy loose ground. Evaluate the required pressure P, acting on the wall by the material if its displacement towards the ground (passive pressure case).

In practice, the limit equilibrium theory is widely used for calculations of slope and retaining walls stability; according to this theory, wall's displacement forms sliding surfaces in the material, wherein displacement towards the material forms a sliding wedge. This is supported by numerous laboratory

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experiments and field observations. Depending on a model, the sliding wedge is limited either by a plane or by a curved surface. Coulomb, Rankine, Sokolovsky's models are conventional for calculations of retaining walls [1, 10 -12]. For example, according to Rankine, in absence of friction between ground and the retaining wall, horizontal pressure P is expressed by:

$$P = \frac{1}{2}\rho g H^2 tg^2\left(\frac{\pi}{4} \pm \frac{\varphi}{2}\right), \tag{2}$$

where ρ is material density ; φ - internal friction angle; g - gravity acceleration; H - height of the filling; "+" corresponds to passive pressure, and "-" to active pressure.

A limitation of these approaches is that they do not consider a material stress history, preceding a limit state. On the other hand, need for limit state hypotheses is eliminated in continuum statements, based on the theories of elasticity and plasticity. However, there is a need for formulation of closed systems of equations, which have not been set up yet. Numerous experiments on various stress modes show formation of localized currents in discrete medium [13-15].

We suggest employing, as a research tool, of the method of discrete elements, which allows determining evolution of stress-strain state of loose medium both at a pre-limiting stress stage and at the stage of localized displacements formation. The advantage of this method is that it does not require either limiting state hypotheses or continuum equations.

At present, method of discrete elements is widely used to solve problems of geo-mechanics [16–19]. Its essence is that a real medium is replaced with a pack of discrete particles, among which particular interaction laws are postulated. Particles have free multi-dimensional parameters which are to be chosen. Under this method, engineering problems of large deformations and rotations are solved without further difficulties. In addition, localization displacements and physically nonlinear effects can be described without major complications, while data on the constitutive equation are not required. Thus, this method is a fundamental alternative to the classic methods based on traditional concepts of the continuum mechanics.

2. Research method

DEM represents discrete material as a universe of N spherical particles (discrete elements). Each i-th element has a radius r_i and a set of physical properties: density, elastic and viscous moduli, friction, adherence, etc. $(i = 1, \ldots, N)$. Single discrete element motion is both translation and rotation and is described by the following equations:

$$m_i \ddot{\boldsymbol{x}}_i = \sum_j \boldsymbol{f}_{ij} + m_i \boldsymbol{g}, \tag{3}$$

$$I_i \ddot{\boldsymbol{\theta}}_i = \sum_j (\boldsymbol{r}_{ic} \times \boldsymbol{f}_{ij} + \boldsymbol{M}_{r,ij}). \tag{4}$$

Here the dots denote differentiation of time t; $\boldsymbol{x}_i$ - radius-vector of i-th particle's gravity center; $\boldsymbol{\theta}_i$ - its rotation relative to coordinate axes; m_i - mass; I_i - moment of inertia; $\boldsymbol{g}$ - gravity acceleration; $\boldsymbol{r}_{ic}$ - vector directed from the i-th particle center to the point of contact; contact force $\boldsymbol{f}_i$ affects to i-particle with another j-particle (or border), (it depends on their overlapping as well as on elastic and viscous moduli), its normal component is calculated according to Hertz law [20]; $\boldsymbol{M}_{r,ij}$ - resisting moment of particles (while particles interaction or along the wall).

In (3) and (4) all elements and borders, being in contact with a current i-particle, are summed up. Since forms of discrete elements are supposed to be constant during the contact, deformation degree is described by magnitude of contacting particles overlapping. A special feature is a spherical shape of elements, which does not change during the entire numerical experiment. This imposes certain limitations on the scope of this method, since, in an actual situation, rolling resistance of particles, when they are in reciprocal contact or contact with a border, is conditioned by their shapes and deformations. To remove this limitation, you can either use particles of any form or create clusters, which are considered as separete elements. These methods require larger computational resources.

In this paper we use the method presented in [21-24]; we propose the model of rolling resistance as an adjunct to the contact interaction model; rolling resistance hinders relative particles' mutual motion or their motion along the border.

From the right side of the equation (4) the contact moment is represented as two summands. The first summand is the moment of direct contact or relative mutual sliding of particles. The second summand is the moment, hindering relative rotation of discrete elements:

$$\boldsymbol{M}_{r,ij} = -\frac{\boldsymbol{\omega}_i - \boldsymbol{\omega}_j}{\left|\boldsymbol{\omega}_i - \boldsymbol{\omega}_j\right|} \mu_{ij} r_{ij} \left|\boldsymbol{f}_{ij} \cdot \boldsymbol{n}_{ij}\right|, \tag{5}$$

where $\boldsymbol{\omega}_i$ and $\boldsymbol{\omega}_j$ - angular velocities of contacting particles; $\boldsymbol{n}_{ij}$ - unit vector lying on the straight line joining their centers; r_{ij} - reduced radius: $1/r_{ij} = 1/r_i + 1/r_j$.
Dimensionless parameter μ_{ij} is rolling resistance coefficient ($\mu_{ij} \cdot r_{ij}$ - rolling friction coefficient of length dimension). If to draw an analogy with dry friction angle between two bodies, just depict that this ratio can be represented as $\mu_{ij} = \mathrm{tg}\,\psi_{ij}$.
Here ψ_{ij} is the maximum angle of plane inclination at which particles will be in equilibrium due to rolling resistance (if no slipping).

To examine the effect of a set parameter on kinematics of particle motion, let us perform the following numerical experiment. Suppose that radius sector of 0.01m rests on a horizontal plane in gravity field. Contact friction angle between the particle and the boundary is equal to 30.0°. Particle mass center's velocity v_0 = 1.0 m/s and zero angular velocity are set at zero time parallel to the plane. If there are no rolling ($\psi_{ij} = 0$) and viscous component of contact force, particle mass center's velocity will be equal to the initial ($v(t) \equiv v_0$) at any time t; and the traveled distance will be a linear function of time: $s(t) = v_0 \cdot t$.

If $\psi_{ij} > 0$ solution will differ. Fig. 1 shows results of numerical experiments for different values of ψ_{ij}.

Higher magnitudes for ψ_{ij} were chosen to demonstrate the prominent effect of rolling friction. Fig. 1 shows that the distance, traveled by the particle, is finite and diminishes with increasing of rolling resistance angle ψ_{ij}.

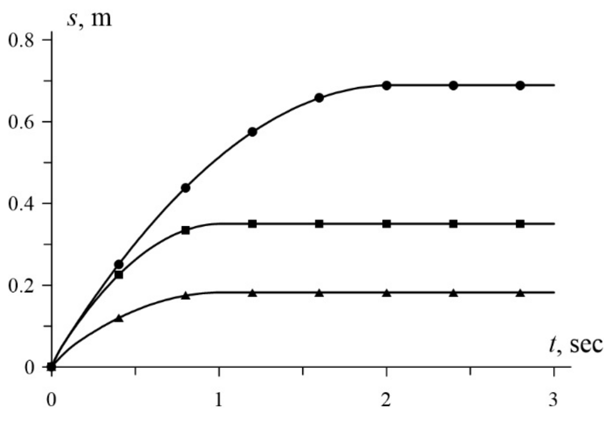

Fig. 1 Distance-time relationship (s is passed by a spherical particle with 0.01m radius) if rolling resistance angles ψ_{ij} are: 5.0° (●); 15.0° (■); 25.0° (▲)

Thus, if there is data on laws of motion and particles contact interaction (equation (3) - (5)) as well as initial and boundary conditions, you can set up and solve equations and determine environment's stress-strain state evolution.

3. Numerical DEM simulation

Based on the model considered above, a software has been developed for numerical analysis of various modes of motion of discrete material consisting of separate spherical particles [25, 26].

Let us consider the following three-dimensional problem. Suppose that Ω volume is set in *Oxyz* space (Ω is a parallelepiped container for loose material, constrained with planes (walls) oriented along coordinate axes); the upper boundary is non-strained (Fig. 2). Length l is equal to 0.2 m, width = 0.01 m and filling height h = 0.1 m.

Discrete material is a universe of discrete elements, its density ρ_i = 2500 kg/m^3, elastic moduli E_i =3 GPa, Poisson's ratio v_i = 0.25; diameters are chosen in uniform distribution over the range 0.0015m to 0.0025m.

Restorability ratio of particles' interaction and that between particles and a retaining wall e_r = 0.75. Gravity vector **g** = (0, 0, -9.81) is directed downwards *Oz*. Front and rear walls are rigid and smooth, i.e. the condition is close to that of planar deformation along *Oy*.

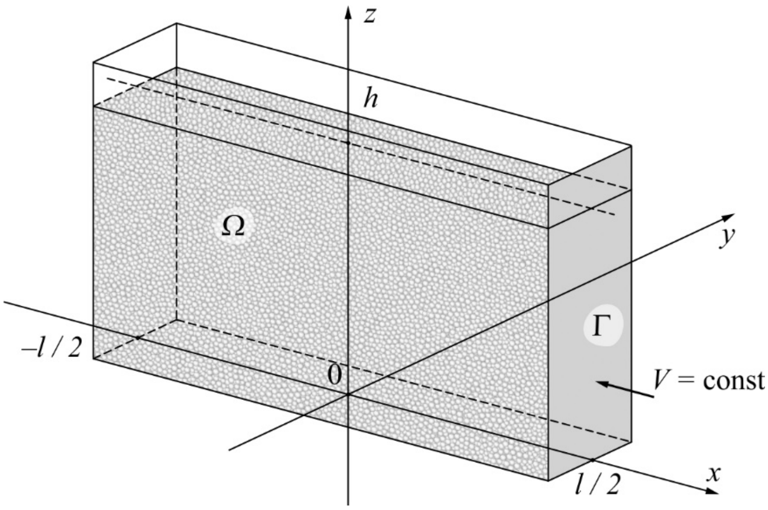

Fig. 2 Scheme of numerical experiment, straining condition

Creating of equilibrium pack of solid spherical particles is the initial step in solving problems with use of DEM and is to be investigated as a separate independent task. In our study, filling of the volume (container) was performed under dynamic algorithm of spherical particles packing. The initial layer of non-segregated discrete elements was created at a certain height from the fixed lower boundary $z = 0$. Further, under gravity, particles deposited in the container interacting together and with boundaries. On reaching equilibrium state, elements were removed if vertical coordinates of their gravity centers did not correspond to $z_i \le h$. We suggested frictionlessness between the particles and between the particles and the walls; that gives maximum density magnitude.

When equilibrium pack was completed, the material constituted a universe of discrete elements with total weight of 0.3kg. Angle φ_{ij} of contact friction between the particles was set (in all subsequent numerical experiments φ_{ij} = 30°). For evaluation of rolling resistance coefficient's effect on localized flow pattern of discrete material and on evolution of lateral pressure P, numerical calculations were performed for the case $\psi_{ij} = 0°$ and $\psi_{ij} = 30°$. Here, greater ψ_{ij} magnitude was chosen to denote a prominent effect of rolling friction. Pressing displaced the wall $\boldsymbol{\Gamma}$ towards the discrete material at a constant low speed $\boldsymbol{V}$; that is the case of passive pressure. The distance of wall displacement was 0.02m; total deformation of the sample ε_x = 0.1.

4. Results and Discussion

Authors examined effect of rolling resistance angle on material deformation. Fig. 3 shows the final state of discrete material in non-rolling conditions and while high rolling resistance. Kinematics deformation is colored for visualization. We see that if $\psi_{ij} = 0°$ (Fig. 3a) free surface configuration is close to a straight line, i.e. without rolling resistance particles, under action of gravity, can roll down to the fixed left edge, forming a smooth slope.

If $\psi_{ij} > 0°$, strain state of the material differs. In this case, the sliding wedge may form a horizontal area of free surface (Fig. 3b).

a *b*

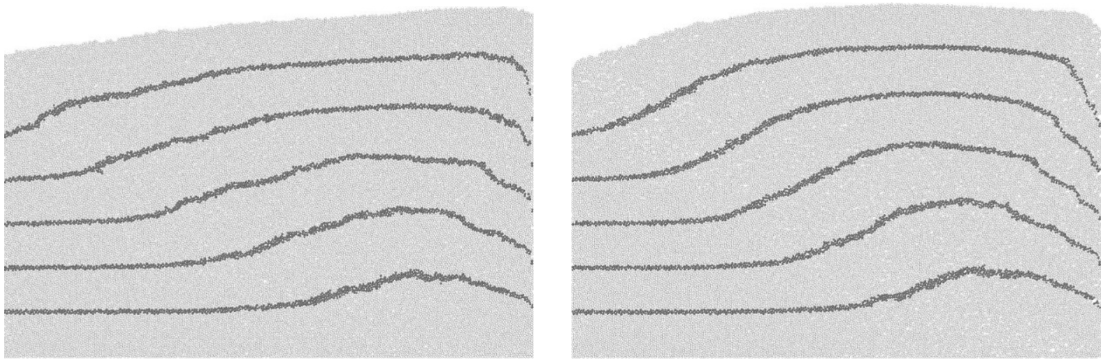

Fig. 3 Deformed state of sample; no rolling resistance (a) high rolling resistance (b)

Sliding wedges are formed by distribution of vertical components of displacement vectors, as shown in Fig. 4. Low magnitudes of vertical displacements are colored dark; great magnitudes are colored light. In the material two or more sliding lines are formed; that differs significantly from the theories based on hypotheses for granular medium ultimate state.

a *b*

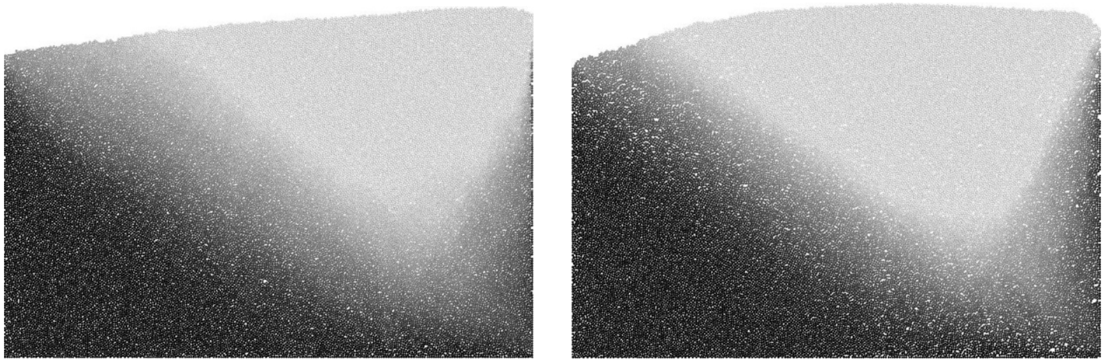

Fig. 4 Vertical component of particles' displacement vector; no rolling resistance (a) high rolling resistance (b)

Let us define material lateral pressure $P(t)$, acting on the retaining wall Γ during deformation. At a fixed time t we suggest:

$$P(t) = \frac{1}{h}\sum_{i=1}^{N_p} F_{x,i}, \tag{6}$$

where $F_{x,\mathrm{i}}$ - a horizontal component of i-th particle's contact force vector acting on Γ (summing is performed only over particles being in contact with a boundary (wall)); N_p - number of inter-contacts between particles and boundaries; h - maximum vertical contact coordinate.

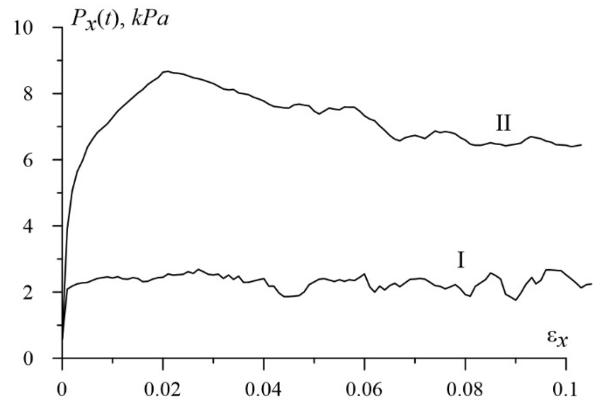

Fig. 5 Change of horizontal pressure *P(t)* on retaining wall during straining; rolling resistance angle ψ_{ij} = 0° (I); ψ_{ij} = 30° ; (II)

Fig. 5 shows variables of $P(t)$ during all the experiment. Diagram I corresponds to rolling resistance angle ψ_{ij} = 0°, diagram II corresponds to rolling resistance angle ψ_{ij} = 30°.

It is evident that if specifying external friction between particles (without rolling resistance), material passes to limiting state immediately (Figure I). If rolling friction, material state can be considered as pre-limiting with deformation magnitude 0.02. If further wall's displacement, granular medium passes to limit state; resulting pressure varies significantly.

5. Conclusions

In problems of loose material mechanics rolling friction should be considered as well as sliding friction. In the 3D-model of discrete material deformation caused by displacement of the retaining wall (passive pressure), effect of rolling resistance coefficient on emergence of localized flow patterns has been confirmed as well as change of horizontal lateral pressure towards the retaining wall while further straining.

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