

Spherical gearing with intermediate ball elements: parameter ranges with a high contact ratio

M V Gorbenko¹ and T I Gorbenko^{1,2}

¹ Tomsk Polytechnic University, 30, Lenina ave., Tomsk, 634050, Russia

² Tomsk State University, 36, Lenina ave., Tomsk, 634050, Russia

E-mail: gmvski@tpu.ru

Abstract. The paper presents analytical research of the geometry and kinematical parameters of spherical gearing with ball intermediate elements. The main attention is paid to the influence of the offset coefficient on the tooth geometry generation, the contact ratio and the motion transmission angle. Intermediate ball element racetracks on the gear are trochoidal curves on a spherical surface. Two areas for the offset coefficient values providing a high value of the contact ratio – basic trochoid (without offset) and prolate trochoid with abutting racetracks of adjacent ball elements — were revealed. Analysis of the investigated parameters showed that for power transmission, it is preferable to use spherical gearing without an offset, and for kinematic transmission, it is possible to use profiles with a large offset. The present study allows making a rational choice of geometrical parameters depending on the transmission predestination.

1. Introduction

In the modern mechanical engineering, a significant number of gearing types are applied. The development of technology today makes new demands on the reliability, accuracy, stiffness and power efficiency to the transmission devices. This applies both to traditional kinds of transmissions and to newly developed ones. Developing of transmissions with intermediate rolling elements is a challenging trend in the synthesis of gearing because they have high service properties, technological effectiveness as well as low noise, high accuracy parameters and great loading capacity [1-7].

This paper presents an analytical investigation of the offset coefficient influence of spherical gearing with intermediate ball elements [1] on the tooth geometry generation, the contact ratio and the motion transmission angle.

Spherical gearing can be used in transmissions as an alternative to their bevel-gear counterparts, and, thus, calculation of rational gear parameters are important.

2. Spherical gearing geometry

The considered transmission (figure 1) consists of pinion-holder 1 with hemispherical sockets for intermediate bodies – balls (3), gear 2, designed as a sphere, on the surface of which there are tracks for balls of a semicylindrical cross-section, the axial line of the track is a trochoidal curve on a spherical surface [1, 8]. Intermediate elements are standard balls produced by the ball-bearing industry. In order to enable free rotation of ball intermediate bodies (BIB), both radii of nests in the pinion-holder and the radius of channel surfaces are slightly larger than the radius of BIB. To prevent



the loss of BIB from nests when leaving the working area, the geometrical closing by the spherical surface of the gear is used.

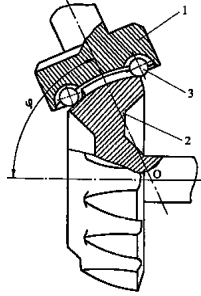


Figure 1. Spherical gearing with intermediate ball elements.

1 – pinion-holder, 2 – gear, 3 – balls.

Let us denote the $\delta = \xi^{(0)}$ kinematical cone angle, $\xi = \delta(x+1)$ – the design cone angle, $x = (\xi - \delta)/\delta$ – the offset factor, φ – the axes intersection angle, φ_1 – the pinion rotation, φ_2 – the gear rotation, U – the gear ratio and R_b is the rated radius of ball centers concerning the gear sphere center, Z – the number of intermediate ball elements and K is the ball center radius in the pinion-holder in terms of the ball radius.

In the spherical coordinate system, placed on the centre of the gear sphere (R, θ, φ_2), the trajectory of the ball is described by the following system of equations [8]:

$$\begin{cases} \sin \theta = \cos \xi \cos \varphi - \cos \varphi_1 \sin \varphi \sin \xi, \\ \varphi_2 = \varphi_1 / U, \\ R = R_b. \end{cases}$$

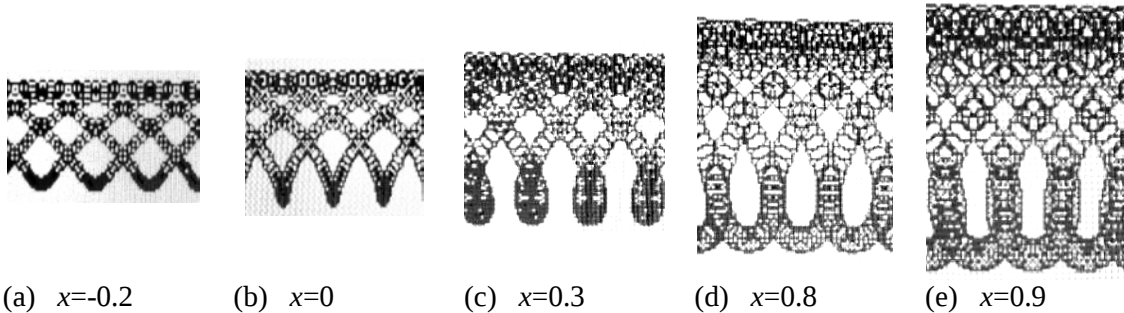
Some calculations are convenient to do in the Cartesian coordinate system, connected with the gear center:

$$\begin{cases} X_{3(s)} = KR_s(\sin \varphi_1 \cos \varphi_2 - \cos \varphi_1 \cos \varphi \sin \varphi_2 - \operatorname{ctg} \xi \sin \varphi \sin \varphi_2), \\ Y_{3(s)} = KR_s(\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi \cos \varphi_2 + \operatorname{ctg} \xi \sin \varphi \cos \varphi_2), \\ Z_{3(s)} = KR_s(\operatorname{ctg} \xi \cos \varphi - \cos \varphi_1 \sin \varphi). \end{cases}$$

The velocity transmission angle at a tangent to the sphere surface plane, perpendicular to the radius-vector of the ball centre, is

$$\mu = \operatorname{arctg} \left(\frac{-\sin \varphi \sin \varphi_1}{\sin \varphi \cos \varphi_1 \cos \xi - \sin \xi (U - \cos \varphi)} \right).$$

The tooth form significantly changes depending on the offset coefficient retaining all other parameters. Figure 2 (a–e) represents the tooth form evolution and ball element trajectories for five different offset coefficient values.



(a) $x=-0.2$

(b) $x=0$

(c) $x=0.3$

(d) $x=0.8$

(e) $x=0.9$

Figure 2. The tooth profile evolution vs the offset factor for gearing of $U=4$, $\varphi=90^\circ$, $Z=6$, $K=8$ (picture scales are the same)

The angle of contact beginning with the tear-shaped teeth may be found from the equation for self-intersecting of the contact line on the gear:

$$\begin{cases} [\cos \lambda \sin \mu \cos \varphi_{1H2} + \sin \varphi_{1H2} (\cos \lambda \cos \mu \cos \xi + \sin \lambda \sin \xi + K)] \cos \varphi_{2H2} \\ - [\cos \varphi_{12} (\cos \lambda \cos \mu \cos \xi + \sin \lambda \sin \xi + K) - \sin \varphi_{1H2} \cos \lambda \sin \mu] \cos \varphi \sin \varphi_{2H2} \\ - (\sin \lambda \cos \xi - \cos \lambda \cos \mu \sin \xi + K \operatorname{ctg} \xi) \sin \varphi \sin \varphi_{2H2} = 0, \\ \varphi_{2H2} = \frac{\varphi_{1H2}}{U}. \end{cases}$$

Angle λ depends on the nest radius and the depth of the pinion-holder, the radius and the depth of the ball track and on the clearance between the top land of the teeth and the inner spherical surface of the pinion-holder. For represented gearing without a separator for balls, this angle is about 15–25°, while for schemes with separator [4], it may reach 30–40°, that results in the decreased efficiency. The edge contact pinion-holder nest-ball– semicylindrical track corresponds to $\lambda \approx 5$ –9°.

Generally, only lower teeth for negative and 0-corrected gears (figure 2a and 2b) are taken into consideration. Analytical research of geometrical features for this gear and derived equation systems allow calculating beginnings (φ_{1H}) and endings (φ_{1K}) of ball engagement with all teeth lines. Borderlines for active meshing are shown in figure 3 and the corresponding contact ratio diagram is in figure 4. Equipotential line ($\mu=20^\circ$) must be considered as borderline for φ_K due to extremely low effectiveness in transmitting motion when $\mu < 20^\circ$, even though there is no self-braking up to $\mu \approx 7$ –9°.

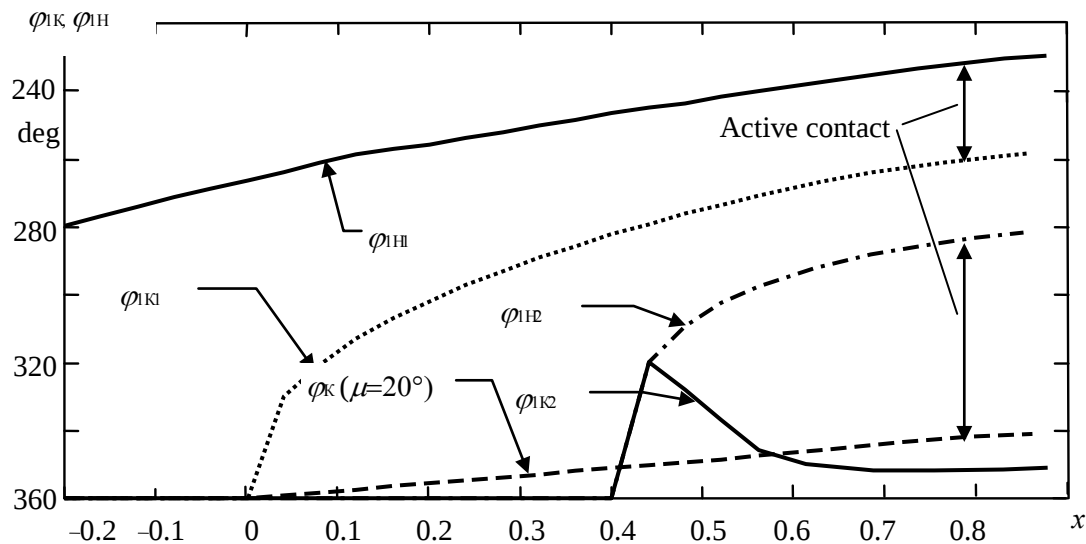


Figure 3. Borders of active meshing in terms of pinion-holder rotation vs the offset factor for transmission: $U=2$, $\varphi=90^\circ$, $Z=6$, $K=5$.

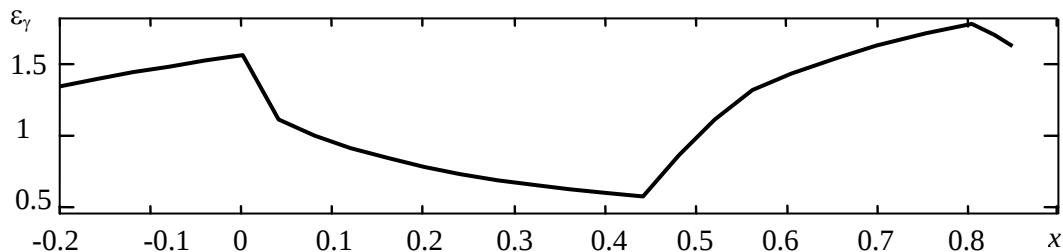


Figure 4. The contact ratio factor vs the factor for transmission: $U=2$, $\varphi=90^\circ$, $Z=6$, $K=5$.

The maximum in the contact ratio (figure 4) corresponds to abutting racetracks of adjacent ball elements (figure 2d). The contact ratio drop within the interval of $0 < x < 0.4$ is explained by the first

rank tooth undercutting, until a tear-shaped tooth increases and comes into operation. The last drop (figure 4, $x > 0.8$) corresponds to undercutting of the tear-shaped tooth (figure 2e). Lines ϕ_{112} and ϕ_{112} in figure 3 mean the active contact of the tear-shaped tooth.

Calculations of the velocity transmission angle μ through the active part of intermediate body trajectory vs the pinion-holder rotation for the number of offset coefficients are represented in figure 5.

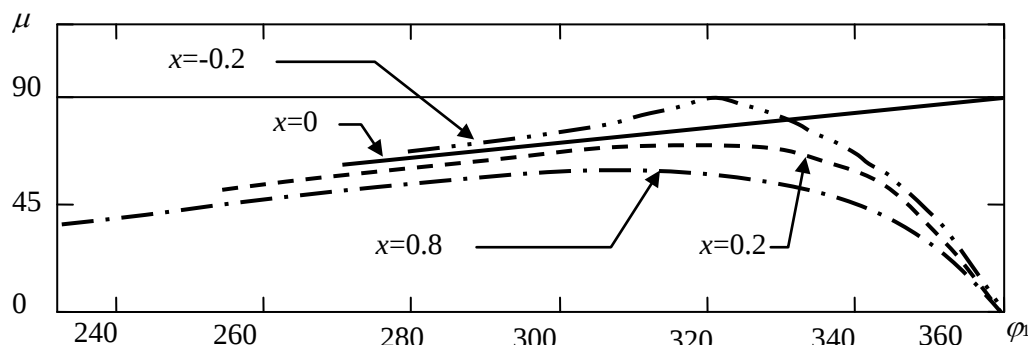


Figure 5. Transmission angle μ vs rotation of the pinion-holder. ($U=2$, $\phi=90^\circ$, $Z=6$, $K=5$).

If $\mu=90^\circ$ the radius-vector of contact point drawn from the ball centre and the vector of the ball centre absolute velocity are in one plane. The angle between them is λ . It may be referred as pressure angle. This is the reason why separatorless schemes are preferable. Study of angle μ shows sufficient values in the low negative and 0 offset factor. An ideal value to transmit motion is 90° .

3. Conclusion

After conducted studies of the geometer-kinematic features of spherical gearing with intermediate ball elements, it is possible to conclude that low positive correction ($x=0.2-0.4$) is ineffective. The use of tear-shaped teeth (a high value of the correction factor: $x=0.6-0.8$) may be useful in kinematic drives, where there is no heavy loads, due to an attractive value of the contact ratio. These gears can smoothly transmit motion avoiding fluctuations in the velocity ratio and impact blows in case of teeth changeover. This kind of transmission possesses a whole number of positive qualities relevant to transmissions with free intermediate rolling elements. The approach developed by the authors provide engineers with a tool to make a justified choice of spherical gearing parameters. The contact conditions are better for separatorless gearing, but in this case, the closing surface is required to prevent loss of BIB.

References

- [1] A E Beljev, An I- Kan, V V Gurin and L D Garanin, USSR. Patent No. 261,072
- [2] A N Morozov, V N Kundel and N V Vasilchenko, USSR. Patent No. 1,173,099
- [3] A E Beljev, An I- Kan, V I Risan and A N Morozov, USSR. Patent No. 658345
- [4] An I-Kan, E Efremenkov and A Cheremnov, Russia. Patent No. 2,457,377
- [5] An I-Kan, A S Il'in and A V Lazurkevich 2016 *IOP Conf. Ser.: Mater. Sci. Eng.* Vol 124 (England: IOP Publishing) 012003
- [6] I Kenji, U.S. Patent No. 4,829,851A
- [7] K Akiyoshi, K Sato, M Nakakoji, EP Patent No. 2,759,740 A1
- [8] Gorbenko M V 2000 *Investigation of errors affect the kinematics and dynamics of a spherical ball-transfer intermediate bodies and the choice of rational parameters* (Tomsk: Tomsk Polytechnic University Press) p 123