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Synthesis of multi-loop automatic control systems by the nonlinear programming method

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Abstract. The article deals with the problem of calculation of the multi-loop control systems optimal tuning parameters by numerical methods and nonlinear programming methods. For this purpose, in the paper the Optimization Toolbox of Matlab is used.

1. Introduction

Multi-loop control systems are widely used for high dynamic control accuracy. However, their calculation is not easy. The traditional variant of the synthesis of multi-loop systems uses a consistent calculation circuit diagram, beginning from the internal loop. This method has a serious disadvantage consisting in arising of additional errors. When consecutive calculation is used, the source of two errors appears. There are calculation errors and error distribution desired indicators of quality and accuracy of the loops. This decomposition can be done only approximately, and as a result an addition error occurs.

It is clear that to eliminate this drawback, we must replace the two-step scheme of calculation, which include decomposition of the desired properties of the contours and calculation of regulators, in a single-stage scheme, at which the stage of decomposition is excluded. This variant was proposed in [4]. However, solution of nonlinear equations is a large difficulty. It is especially difficult to obtain a solution in an ill-posed problem [5]. Mostly, these difficulties are computational in nature, since the procedure solving a system of nonlinear equations is laborious. In the paper, the method of solving the problem of the synthesis of multi-loop systems by nonlinear programming is considered. Here, the possibility of using software environment Matlab to solve the problem of synthesis is investigated.

2. Statement of the problem

The practice of using the approach outlined in [5] has shown that it has limitations. For example, it is the number of coefficients to be determined. Therefore, the desire to apply the nonlinear programming method enquires research of its features, positive attributes and limitations. For simplicity, let us turn to the dual-loops ACS, a diagram of which is shown in Figure 1. The following notation is introduced: $W_{OU}(p)$ – the transfer function of control object, $W_{reg1}(p)$, $W_{reg2}(p)$ – transfer functions of regulators, K_1, K_2 – feedback gains.

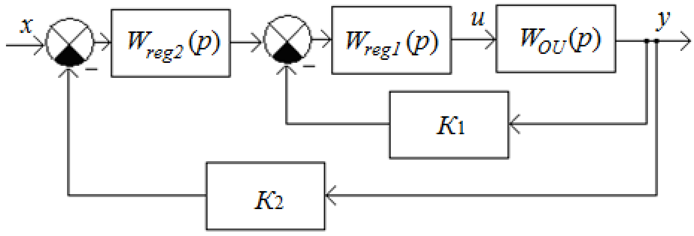

Figure 1. An operator-block diagram of dual-loop ACS

To solve the problem of synthesis, it is necessary to solve the equation of the synthesis, in the general feed having form

$$W_{жел}(p) \approx F[W_{reg1}(p), W_{reg2}(p), K_1, K_2, W_{OU}(p),] \quad (1)$$

On the solutions of equations (1), we have two difficulties. Firstly, the equation has a nonlinear form, so its solution is difficult. And, secondly, it is necessary to decompose (1) in a system of equations.

The obvious way to solve is the frequency method. In solving the problem by this way, we need to replace original function $F(p)$ to image function $F(j\omega)$. Wherein variable $p=\delta+j\omega$ is replaced by imaginary variable $j\omega$, $\delta=0$. We shall perform numerical calculations in the subsequent transition to a digital model of the set task, where $\omega_i, i=1,2,...,\eta$ [6, 7, 8]. However, this approach has a significant disadvantage. In solving this problem by this method, the amount of computation is doubled. It is due to the transformation of the real and imaginary parts. Another way is to replace variable $p=\delta+j\omega$ by real variable $\delta \in [C,\infty], C \geq 0$, $\omega=0$ [9]. As a result, image function $F(\delta)$ is real with real variable. Further, it is necessary to move to numerical form $F(\delta_i), i=1,2,...$ by discretisation. A positive feature of this way is that the method uses well-developed numerical methods and algorithms, as well as digital hardware and software for their implementation. Let us onsider the features of the approach to the calculation of multi-loops systems.

The technology of calculation of the real interpolation method is consecutive interpretation of synthesis equation $W^{cl}_{des}(p) \cong W^{cl}_{sint}(p)$ in real form $W^{cl}_{des}(\delta) \cong W^{cl}_{sint}(\delta)$ and in numerical form $W^{cl}_{des}(\delta_i) \cong W^{cl}_{sint}(\delta_i)$, $i=1,2,...$, with its subsequent decision. The first step of the generated sequence is performed formally [9].

The transition from discrete to continuous functions is more difficult. To reduce the difficulties, [9] has recommendations that allow formalizing a transition to the numerical model. In this case, we obtain:

$$W^{cl}_{des}(\delta_i) \cong \frac{W_{reg2}(\delta_i)\cdot\dfrac{W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}{1+K_1\cdot W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}}{1+K_2\cdot W_{reg2}(\delta_i)\cdot\dfrac{W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}{1+K_1\cdot W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}}, \quad i=\overline{1,\eta}. \quad (2)$$

Now the problem is reduced to solving a system of η nonlinear equations. Typically, two proportional integral differential controllers (PID-controllers) are used. Then there are six coefficients of these regulators and two coefficients of feedback. Thus, it is required to calculate the eight unknowns of equation (2). It can be assumed that the difficulties in solving the problem may be significant, perhaps insurmountable. It is they that are the target of further consideration.

3. Statement and solution of the problem of synthesis of multi-loop control systems by the real interpolation method and nonlinear programming

The problem of nonlinear programming in the framework of program FMINCON of section

Optimization Toolbox Matlab posed as a problem of finding the minimum of the nonlinear problem

$$\min_x f(x)$$

with limitations

$$c(x) < 0; \;\; ceq(x) = 0;$$

provided that

$$Ax \le b; \;\; Aeg \cdot x = beg; \;\; lb < x < ub; \;\; lb \le x \le ub,$$

where x- vector of unknowns; b - vector of inequality constraints, beq - vector constraints of equality, lb, ub - vectors of restrictions placed above and below, A - matrix of constraints-inequalities; Aeq - matrix of constraint equations, $c(x)$ and $ceq(x)$ - the function of non-linear constraints.

We define numerical characteristics of desired and synthesized systems in the form of n samples p_i in the nodes of interpolation δ_i. So the controller synthesis problem can be formulated as a nonlinear programming problem: it is required to minimize functional $f(x)=e^{T}He$ under conditions

$$b_2\delta_1^2 + b_1\delta_1 + b_0 - p_1\delta_1^2 a_2 - p_1\delta_1 a_1 + e_1 = p_1,$$
$$\ldots$$
$$b_2\delta_n^2 + b_1\delta_n + b_0 - p_n\delta_n^2 a_2 - p_n\delta_n a_1 + e_n = p_n,$$
$$b_2 \ge 0,\; b_1 \ge 0,\; b_0 \ge 0,\; a_2 \ge 0,\; a_1 \ge 0.$$

Vector e is of the residual of the approximate solution of a system of linear equations in the nodes of interpolation. A general vector of unknown parameters x includes controller parameters b_2, b_1, b_0, a_2, a_1 and the components of the vector of residuals e. The restrictions are imposed only on controller parameters b_2, b_1, b_0, a_2, a_1.

4. The example of solving the example of the turning machine control system

We apply the proposed approach to a common dual-management structure of the electric drive [10]. For this system, solutions are known, therefore, in the future it will be possible to conduct just a comparative analysis of the results.

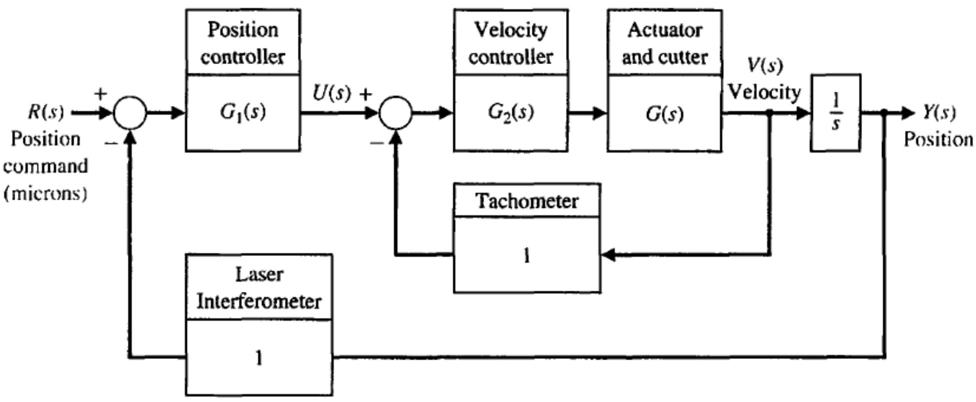

Figure 2. The turning machine control system

The transfer function of control object has form

$$G(p) = \frac{4500}{p+60}.$$

Work [10] has certain requirements for the system. Based on these requirements, the controller structure have the following forms:

$$G_1(p) = K_1 \frac{p + b_{1,0}}{p + a_{1,0}},\ G_2(p) = K_2 \cdot \frac{p^2 + b_{2,1}p + b_{2,0}}{p(p + a_{2,0})}.$$

Also, the transfer function of the desired system have the form of $W_{des}^{cl}(p)$.

Let us select the variant when the problem was computationally simplified to the limit, but retained its main feature: the system of equations is nonlinear. For this, we assume that the unknown parameters are coefficients K_1 and K_2.

The target function is given as $f(x) = e_1^2 + e_2^2$, where e is the vector of the residual of synthesis equations in a numerical form.

Let us impose restrictions in the form of linear inequalities on the values of the unknown parameters $K_1 > 0$, $K_2 > 0$. As a result of solving the quadratic programming problem by program FMINCON, the values of the unknown parameters are obtained: K_1=1000, K_2=2. The found coefficients are close enough to decision [10].

For further investigation, the number of unknown coefficients has been increased to five. It was found that the increase in dimension of the problem entailed a significant deterioration in the terms of solving the problem. In particular, the computational difficulties increase due to ill-posed problems.

5. Conclusion

In the present paper, we investigated the possibility of using the real interpolation method in combination with a non-linear programming to solve the controller synthesis problem.

These results allow us to recommend the real interpolation method in combination with a non-linear programming in the Matlab to solve the problem of synthesis of multi-loop system regulators.

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