Thermodynamic and relativistic uncertainty relations

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Abstract. Thermodynamic uncertainty relation (UR) was verified experimentally. The experiments have shown the validity of the quantum analogue of the zeroth law of stochastic thermodynamics in the form of the saturated Schrödinger UR. We have also proposed a new type of UR for the relativistic mechanics. These relations allow us to consider macroscopic phenomena within the limits of the ratio of the uncertainty relations for different physical quantities.

1. Introduction

Two fundamental theories formed at the beginning of the 20^{th} century: the quantum mechanics and the theory of relativity. First advances of quantum theory are related to the works of Max Planck. The well-known Planck's formula [1] describes the radiation intensity of black body $R(\nu, T)$ as a function of temperature and radiation frequency ν

$$R(\nu, T) = \frac{2\pi h\nu^3}{c^2}\frac{1}{exp(h\nu/k_BT) - 1}; \tag{1}$$

In this formula Planck introduces two constants which will be later called as Planck's constant h and Boltzmann constant k_B. The third constant c, the speed of light, is also present in equation (1). These three constants have important fundamental meaning in modern physics. Planck's constant gives the minimal action, Boltzmann constant gives the smallest entropy and the speed of light is the maximal speed of field or matter propagation in vacuum. The meaning of Planck's constant is the most highlighted in the Heisenberg uncertainty principle which is generalized later in the works of Robertson and Schrödinger [2]. The modern form of Heisenberg uncertainty principle is

$$\Delta q \Delta p \geq \frac{\hbar}{2}, \tag{2}$$

where $\hbar = \frac{h}{2\pi}$ and Δq, Δp are the standard deviations of position and momentum.

2. Thermodynamic uncertainty relation

Statistical mechanics (i.e. microscopic theory) was developed at the same time as quantum mechanics due to several efforts, including those of Gibbs. However, careful analysis of the 9th book of Gibbs [3] by Sukhanov [4] showed that Gibbs developed the foundations of macroscopic theory. Gibbs considered thermodynamical characteristics of the system as an ensemble of dynamical variables of all objects from this system. Sukhanov managed to derive a thermodynamical analog of uncertainty relations from Gibbs equations:

$$\Delta \left(\frac{1}{T(\epsilon)} \right) \Delta\epsilon \geq k_B, \tag{3}$$

where $T(\epsilon)$ is the temperature of the system, ϵ is the macroscopic parameter characterizing the energy of the system. However, we need to note that ϵ in inequality 3 is not the energy of the system but only the parameter which characterize the energy. This important remark which allows to correctly interpret thermodynamic uncertainty helped us to verify it experimentally. In this work we found an experimental confirmation of thermodynamic uncertainty principle. In out work we used an installation consisting of two transistors 11 mm. Both transistors shared the same semiconductor crystal therefore formed a united semiconductor-device. One transistor served as a thermometer, the other was used to measure the parameters characterizing the energy of the system E. The value of ϵ was found using equation:

$$\epsilon = E \cdot \eta(N), \tag{4}$$

where $\eta(N)$ is a function depending on the number of particles participating in the process which defines the energy of the system . In our case:

$$\eta(N) = \frac{1}{\sqrt{N}}; \tag{5}$$

This approach allows to solve several inconsistencies arising during theoretical analysis. Experimental investigations were performed at different physical conditions: at equilibrium conditions and at conditions far from equilibrium. The total number of experiments was around 10 000. It should be noted that at thermodynamic equilibrium we were limited by the precision of measured parameters. Although in the most cases we prove the accuracy of thermodynamic uncertainty relation.

The analogy between thermodynamic and quantum-mechanic uncertainty relations is not so deep and has its boundaries [5, 6, 7]. For more information one can consult the book of Bazarov [8].

3. Relativistic uncertainty relations

In addition we introduce new relativistic uncertainty principle:

$$\Delta q \Delta \left(\frac{1}{t} \right) \leq c, \tag{6}$$

where q is coordinate and $1/t$ is inverse time. This uncertainty principle comes as a result of theoretical analysis of the nature of such type of uncertainties. In our work we do not consider this uncertainty extensively and only give its formula without theoretical analysis and experimental verification. Careful analysis of this uncertainty will be given in the future work.

4. Conclusion

In this work we confirm experimentally the thermodynamic uncertainty relation. We expect that experimentally confirmed thermodynamic uncertainty using one macroscopic object, semiconductor device, will hold true for other macroscopic objects as well. We assume that our result will open broad possibilities for application of thermodynamic uncertainty relation I various engineering and scientific problems. We expect that our relativistic uncertainty principle which relates standard deviations of coordinates and of inverse time will be verified or disproved in future works.

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