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CONJOINT EFFECT OF MOBILE NORMAL AND TORQUE LOADS ON TUNNEL WITH CIRCULAR SUPPORT

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The problem solution on influence of mobile normal and torque loads on an infinitely long cylindrical shell in elastic inertial half-space has been obtained. The load functions are supposed to be developed in Fourier series by angular coordinate and Fourier integral by axial coordinate. The movement of shell is described by classical equation of thin shell theory, but elastic half-space is done by elasticity theory dynamic equations in Lame potentials that are solved by means of Fourier integral transformation method. The problem is a model for rock mass deflected mode calculation under inequality of dynamic loads transferred to each of rails in a cylindrical tunnel or under treatment facility's rotary motion in an underground pipework.

1. The problem on influence of mobile axisymmetric torque load on a cylindrical shell in elastic environment has been considered in the work [1]. Motion of periodic load along a cylindrical cavity in elastic half-space has been studied in [2].

Similarly to [2] let us consider infinite circular cavity of R radius located in elastic, homogeneous and isotropic half-space with Lame parameters λ, μ and density ρ. In contrast to [2] the cavity is strengthened by a thin elastic shell (in view of small thickness of the shell we suppose that environment contacts with the shell along its median surface), on the internal surface of which aperiodic normal and tangential (torque) loads translate with the constant speed c. Introduce the Cartesian coordinate system, the axis Z of which coincides with plane axis, parallel to load free plane boundary of the half-space, but the axis X is perpendicular to this boundary: $x \le h$, where h is the distance from the cavity axis to the half-space boundary.

Owing to the fact that we consider a stable process one can go to the Cartesian mobile (X, Y, $\eta=Z-ct$) or cylindrical (r, θ, $\eta=Z-ct$) coordinate systems.

In mobile coordinate system let us use the classical equations of the fine shell theory to describe the shell motion (1), but to describe environmental motion we use the dynamic equations of the elasticity theory (2):

$$\left[1-\frac{(1-\nu_0)\rho_0 c^2}{2\mu_0}\right]\frac{\partial^2 u_{0\eta}}{\partial\eta^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0\eta}}{\partial\theta^2}+$$
$$+\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial\eta\partial\theta}+\frac{\nu_0}{R}\frac{\partial u_{0r}}{\partial\eta}=-\frac{1-\nu_0}{2\mu_0 h_0}q_\eta,$$
$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\eta}}{\partial\eta\partial\theta}+\frac{(1-\nu_0)}{2}\left(1-\frac{\rho_0 c^2}{\mu_0}\right)\frac{\partial^2 u_{0\theta}}{\partial\eta^2}+$$
$$+\frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial u_{0r}}{\partial\theta}=\frac{1-\nu_0}{2\mu_0 h_0}(P_\theta-q_\theta),$$
$$\frac{\nu_0}{R}\frac{\partial u_{0\eta}}{\partial\eta}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2 u_{0r}+$$
$$+\frac{(1-\nu_0)\rho_0 c^2}{2\mu_0}\frac{\partial^2 u_{0r}}{\partial\eta^2}+\frac{u_{0r}}{R^2}=-\frac{1-\nu_0}{2\mu_0 h_0}(P_r-q_r), \qquad (1)$$

where $u_{0\eta}$, $u_{0\theta}$, u_{0r} are the points displacements of shell median surface; q_η, q_θ, q_r are the constituents of environment reaction to the shell (at $r=R$ $q_\eta=\sigma_{r\eta}$, $q_\theta=\sigma_{r\theta}$, $q_r=\sigma_{rr}$, where σ_{rj} are the stress tensor components in the environment, $j=\eta, \theta, r$); ν_0, μ_0, ρ_0 are the Poisson coefficient, the modulus of elasticity in shear and the shell material density correspondingly, h_0 is its thickness; $P_\theta(\theta,\eta)$, $P_r(\theta,\eta)$ are the intensities of torque and normal loads correspondingly;

$$\left(\frac{1}{M_p^2}-\frac{1}{M_s^2}\right)\text{grad div}\,\boldsymbol{u}+\frac{1}{M_s^2}\nabla^2\boldsymbol{u}=\frac{\partial^2\boldsymbol{u}}{\partial\eta^2}, \qquad (2)$$

where $\boldsymbol{u}$ is the vector of elastic environment displacement; $M_p=c/c_p$, $M_s=c/c_s$ are the Mach numbers; c_p, c_s are the speeds of expansion wave propagation – compression and shear in the environment.

As the boundary of half-space is load free, then at $x=h$

$$\sigma_{xx}=\sigma_{xy}=\sigma_{x\eta}=0. \qquad (3)$$

In the case of rigid shell coupling with environment

$$u_j\big|_{r=R}=u_{0j}, \quad j=\eta,\theta,r. \qquad (4)$$

Here u_r, u_θ, u_η are vector components $\boldsymbol{u}$.

Having expressed the displacement vector of elastic environment by Lamé potential [3] $\boldsymbol{u}=\text{grad}\varphi_1+$ $+\text{rot}(\varphi_2\boldsymbol{e}_\eta)+\text{rot rot}(\varphi_3\boldsymbol{e}_\eta)$, we transform (2) to the form

$$\nabla^2\varphi_j=M_j^2\frac{\partial^2\varphi_j}{\partial\eta^2},\ j=1,2,3, \qquad (5)$$

where $M_1=M_p$, $M_2=M_3=M_s$.

Having applied to (5) the Fourier transformation by η, we find

$$\nabla_2^2\varphi_j^*-m_j^2\xi^2\varphi_j^*=0, \quad j=1,2,3. \qquad (6)$$

Here $m_j^2=1-M_j^2,\ m_1\equiv m_p,\ m_2=m_3\equiv m_s,$

$$\varphi_j^*(r,\theta,\xi)=\int_{-\infty}^{\infty}\varphi_j(r,\theta,\eta)e^{-i\xi\eta}d\eta.$$

Having presented the components of environment stress-deformation state through Lamé potentials and having applied the inverse Fourier transform by η, one can obtain the expressions for displacement u_i^* and stress σ_{ij}^* transforms in the Cartesian ($i=x,y,\eta$, $j=x,y,\eta$) and cylindrical ($i=r,\theta,\eta$, $j=r,\theta,\eta$) coordinate systems as function from φ_j^*.

Suppose load speed is less than the speed of displacement wave propagation in cavity environment. In this case M_s<1 (m_2=m_3=m_s>0) and the equation solutions (6) can be presented in the form

$$\varphi_j^* = \Phi_j^{(1)} + \Phi_j^{(2)}, \qquad (7)$$

where $$\Phi_j^{(1)} = \sum_{n=-\infty}^{\infty} a_{nj} K_n(k_j r) e^{in\theta},$$

$$\Phi_j^{(2)} = \int_{-\infty}^{\infty} g_j(\xi,\zeta) \exp(iy\zeta + (x-h)\sqrt{\zeta^2 + k_j^2})\, d\zeta.$$

Here $K_n(k_jr)$ are the Macdonald functions, k_j=$m_j\xi$, $g_j(\xi,\eta)$; a_{nj} are unknown functions and coefficients being the subject to determination.

As shown in [4], presentation of potentials in the form of (7) results in the following expressions for potential transforms in the Cartesian coordinate system:

$$\varphi_j^* = \int_{-\infty}^{\infty} \left[\frac{e^{-xf_j}}{2f_j} \sum_{n=-\infty}^{\infty} a_{nj}\Phi_{nj} + g_j(\xi,\zeta) e^{(x-h)f_j} \right] e^{iy\zeta} d\zeta, \quad (8)$$

where $f_j = \sqrt{\zeta^2 + k_j^2}, \Phi_{nj} = \left(\frac{\zeta + f_j}{k_j} \right)^n, \; j = 1, 2, 3.$

Let us use boundary conditions rewritten for the transforms (3), taking into account (8). Separating the coefficients at $e^{iy\zeta}$ and equalling them to zero due to arbitrariness of y, we obtain the system of three equations, from which we express $g_j(\xi,\zeta)$ through the coefficients a_{nj}:

$$g_j(\xi,\zeta) = \sum_{k=1}^{3} \frac{\Delta_{jk}}{\Delta} e^{-hf_k} \sum_{n=-\infty}^{\infty} a_{nk}\Phi_{nk}. \qquad (9)$$

The type of determinant Δ and algebraic adjuncts Δ_{jk} is defined in [4]. In the same work it is shown that Δ is the Reley determinant, which is not reduced to zero if the speed of running load is less than the speed of the Reley wave in elastic space. In this case the conditions of the Fourier transformation are also fulfilled; one can use one of the numerical integration methods for the integral calculations (8) having determined the coefficients a_{nj}.

For the undo-Reley speed of moving load the relations (8) are rewritten in the form

$$\varphi_j^* = \int_{-\infty}^{\infty} \left[\begin{array}{l} \frac{e^{-xf_j}}{2f_j} \sum_{n=-\infty}^{\infty} a_{nj}\Phi_{nj} + \\ + e^{(x-h)f_j} \sum_{k=1}^{3} \frac{\Delta_{jk}}{\Delta} e^{-hf_k} \sum_{n=-\infty}^{\infty} a_{nk}\Phi_{nk} \end{array} \right] e^{iy\zeta} d\zeta.$$

To present φ_j^* (7) in cylindrical coordinate system we use the development $e^{ikr\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\theta}$. We find that

$$\exp(iy\zeta + (x-h)\sqrt{\zeta^2 + k^2}) =$$
$$= \sum_{n=-\infty}^{\infty} I_n(kr) e^{in\theta} \left(\frac{\zeta + \sqrt{\zeta^2 + k^2}}{k} \right)^n e^{-h\sqrt{\zeta^2+k^2}}.$$

Then

$$\varphi_j^* = \sum_{n=-\infty}^{\infty} \left(a_{nj} K_n(k_j r) + I_n(k_j r) \int_{-\infty}^{\infty} g_j(\xi,\zeta)\, \Phi_{nj}\, e^{-hf_j} d\zeta \right) e^{in\theta}.$$

Substituting in the last expression from (9) $g_j(\xi,\zeta)$, we have

$$\varphi_j^* = \sum_{n=-\infty}^{\infty} (a_{nj} K_n(k_j r) + b_{nj} I_n(k_j r)) e^{in\theta}, \qquad (11)$$

where

$$b_{nj} = \sum_{k=1}^{3} \sum_{m=-\infty}^{\infty} a_{mk} A_{nj}^{mk}, \; A_{nj}^{mk} = \int_{-\infty}^{\infty} \frac{\Delta_{jk}}{\Delta} \Phi_{mk}\, \Phi_{nj}\, e^{-h(f_k+f_j)} d\zeta.$$

Substituting (10) and (11) correspondingly in the expressions for transforms of deformation mode (DM) of environment in the Cartesian and cylindrical coordinates, we obtain new expressions, where only coefficients a_{nj} are unknown. To determine the latter let us use the boundary conditions (4) presenting them in the form

$$u_j^* \Big|_{r=R} = u_{0j}^*, \; j = \eta, \theta, r, \qquad (12)$$

where $u_{0j}^*(\theta,\xi) = \int_{-\infty}^{\infty} u_{0j}(\theta,\eta) e^{-i\xi\eta} d\eta.$

Applying to (1) the Fourier transformation by η and developing the functions of point displacement on the shell middle surface and the load in the Fourier series by θ, for n-th expansion term we obtain:

$$\begin{array}{l} \varepsilon_1^2 u_{0n\eta} + \nu_2 n\xi_0 u_{0n\theta} - 2i\nu_0\xi_0 u_{0nr} = -G_0 q_{n\eta}, \\ \nu_2 n\xi_0 u_{0n\eta} + \varepsilon_2^2 u_{0n\theta} - 2inu_{0nr} = G_0(P_{n\theta} - q_{n\theta}), \\ 2i\nu_0\xi_0 u_{0n\eta} + 2inu_{0n\theta} + \varepsilon_3^2 u_{0nr} = G_0(P_{nr} - q_{nr}), \end{array} \qquad (13)$$

where $$\varepsilon_1^2 = \alpha_0^2 - \varepsilon_0^2, \quad \varepsilon_2^2 = \beta_0^2 - \varepsilon_0^2, \quad \varepsilon_3^2 = \gamma_0^2 - \varepsilon_0^2,$$
$$\varepsilon_0^2 = \nu_1\xi_0^2 M_{s0}^2, \quad \xi_0 = \xi R, \quad \alpha_0^2 = 2\xi_0^2 + \nu_1 n^2,$$
$$\xi_0 = \xi R, \quad \beta_0^2 = \nu_1\xi_0^2 + 2n^2, \quad \gamma_0^2 = \chi^2(\xi_0^2 + n^2)^2 + 2,$$
$$\nu_1 = 1 - \nu_0, \nu_2 = 1 + \nu_0, M_{s0} = \frac{c}{c_{s0}},$$
$$c_{s0} = \sqrt{\frac{\mu_0}{\rho_0}}, \chi^2 = \frac{h_0^2}{6R^2}, G_0 = -\frac{\nu_1 R^2}{\mu_0 h_0};$$

u_{0nm}, P_{nj} are the expansion coefficients correspondingly $u_{0m}^*(\theta,\xi)$ and $P_j^*(\theta,\xi) = \int_{-\infty}^{\infty} P_j(\theta,\eta) e^{-i\xi\eta} d\eta$ in Fourier series by the angular coordinate θ (j=θ; r; m=η; θ; r). At r=R $q_{n\eta}$=$(\sigma^*_{r\eta})_n$, $q_{n\theta}$=$(\sigma^*_{r\theta})_n$, q_{nr}=$(\sigma^*_{rr})_n$.

Solving (13) with respect to $u_{0n\eta}$, $u_{0n\theta}$, u_{0nr}, we find

$$\begin{array}{l} u_{0n\eta} = G_0 \sum_{j=1}^{3} \frac{\delta_{\eta j}}{\delta_n} (P_{nj} - q_{nj}), \\ u_{0n\theta} = G_0 \sum_{j=1}^{3} \frac{\delta_{\theta j}}{\delta_n} (P_{nj} - q_{nj}), \\ u_{0nr} = G_0 \sum_{j=1}^{3} \frac{\delta_{rj}}{\delta_n} (P_{nj} - q_{nj}). \end{array} \qquad (14)$$

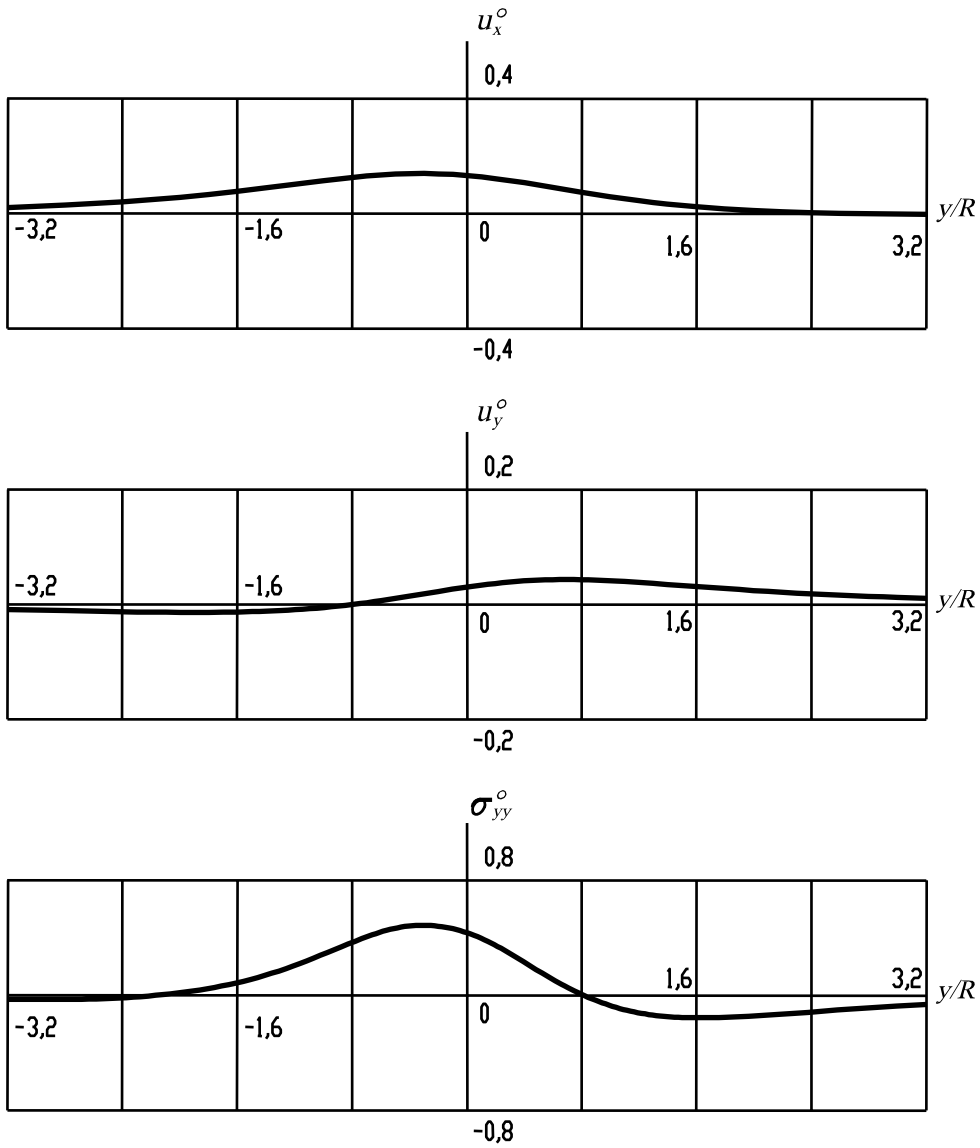

Figure. *Changes of VAT components of the earth surface*

Here

$$\delta_n = (\varepsilon_1\varepsilon_2\varepsilon_3)^2 - (\varepsilon_1\xi_1)^2 - (\varepsilon_2\xi_2)^2 + 2\xi_1\xi_2\xi_3,$$
$$\delta_{\eta 1} = (\varepsilon_2\varepsilon_3)^2 - \xi_1^2, \delta_{\eta 2} = D_1,$$
$$\delta_{\eta 3} = iD_2, \delta_{\theta 1} = D_1, \delta_{\theta 2} = (\varepsilon_1\varepsilon_3)^2 - \xi_2^2, \delta_{\theta 3} =$$
$$= iD_3, \delta_{r1} = -iD_2, \delta_{r2} = -iD_3,$$
$$\delta_{r3} = (\varepsilon_1\varepsilon_2)^2 - \xi_3^2, \xi_1 = 2n, \xi_2 = 2\nu_0\xi_0, \xi_3 =$$
$$= \nu_2\xi_0 n, D_1 = \xi_0 n(4\nu_0 - \varepsilon_3^2\nu_2),$$
$$D_2 = 2\xi_0(\varepsilon_2^2\nu_0 - n^2\nu_2), D_3 = 2n(\varepsilon_1^2 - \xi_0^2\nu_0\nu_2);$$

$P_{n1}=0$, $P_{n2}=P_{n\theta}$, $P_{n3}=P_{nr}$; for q_{nj} the index $j=1$ corresponds to the index η, $j=2$–θ, $j=3$–r.

Substituting (14) in (12) and equalling the coefficients of Fourier-Bessel series at $e^{in\theta}$, we obtain infinite system of linear algebraic equations for determination of the coefficients a_{nj}.

After determination of the coefficients a_{nj}, using the inverse Fourier transformation one can calculate the components of environment DM in the Cartesian and cylindrical coordinate systems. In this case to calculate the Fourier integrals it is possible to use any numerical method, if the determinant of the equation system obtained from the boundary conditions is not reduced to zero.

2. As an example the calculations were made for uniformly distributed in the range $|\eta|\leq 0{,}2$ m along internal surface of concrete shell of asymmetrical normal and torque loads of the same intensity moving with the speed $c=100$ m/s in siltstone body with the following parameters: $R=1$ m, $h=2$ m, $h_0=0{,}02$ m; $\nu_0=0{,}2$, $\mu_0=12{,}1\cdot 10^3$ MPa, $\rho_0=2{,}5\cdot 10^3$ kg/m^3, $\lambda=1{,}688\cdot 10^3$ MPa, $\mu=2{,}532\cdot 10^3$ MPa, $\rho=2{,}5\cdot 10^3$ kg/m^3. Normal load intensity was chosen in such way that total load along the full length of load region would be equal to point normal circular load p.

In the figure the result of load influence on the earth surface is shown, where the curves of motion changes $u^\circ_x=u_x\mu/p$ (m), $u^\circ_y=u_y\mu/p$ (m) and normal stresses $\sigma^\circ_{yy}=\sigma_{yy}/p$ in transverse plane ($\eta=0$) are presented.

It follows from the analysis of curve behavior that extreme deflexions u_x of the earth surface and extreme normal stress σ_{yy} take place at $y=-0,4R$, but maximum horizontal displacement u_y is at $y=0,8R$ (here σ_{yy} is equal to zero). Increasing $|y|$ displacements and stresses damp quickly, and at $|y|=3,2R$ dynamic influence of load on the earth surface is virtually imperceptible.

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OPTIMIZATION OF BUILDING CONSTRUCTIONS ON THE BASIS OF GENETIC ALGORITHM

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The technique of optimal design of bearing structures on the basis of genetic algorithm has been suggested. A design of steel frame with varying 9 parameters using the method of finite elements is considered as an example. The best variant corresponding to the volume minimum of the frame material is revealed.

In the last few decades in the spheres of engineering, economics and planning there is a trend of transition from the admissible technical and managing solutions to optimal ones. However, the modern optimization theory has not met the requirements of a design engineer because of the fact that its strict mathematical methods do not take into account real conditions of design problems. Besides, modern complicating design practice needs in efficient mathematical means of solving such problems.

A distinguishing feature of the new approach is a complex development making possible to design a whole system, but not its separate parts. Therefore, one of the most important scientific and applied problems is to develop methodology of optimal design of complex technical systems – the system design.

A construction is characterised by a number of criteria: cost, reliability, weight, size, engineering time and etc, that can came in mutual contradiction. The difficulty of the problem solution consists in the lack of a priori information necessary for searching for optimal variant of construction. Therefore, the design procedure is worthwhile to arrange in such a way that the volume of information on construction would increase at every subsequent stage. At the same time it is necessary to exclude inadequate variants revealed in the course of design. Thus, the two tendencies are to combine: generation of variety of modifications and truncation of the obtained variety [1]. A suggested design procedure is consistent with evolution optimization strategy and genetic algorithm (GA) in particular [3].

Construction design is presented in the form of some sequence of its development levels which are characterised by the degree of its element detail elaboration. Such design technique can be connected with some hierarchical model possessing peculiarity of another class of evolution model i.e. a sequential decision tree.

Genetic algorithms received a wide acceptance in the middle of the 1960's owing to J. Holland's works. They simulate evolutionary process with the stress on genetic mechanisms, i.e. gene inheritance and recombination. It is made by some number (population) of artificial chromosome (individuals). Each chromosome contains n genes that correspond to n desired variables of optimization problem.

Genetic algorithms like evolution algorithm in general are applied to search for the function global extremum of many variables. The principle of their operation is based on modelling some mechanisms of population genetics, manipulation of chromosome set at forming genotype of a new biological individual by means of inheritance of parents' chromosome parts, accidental variations of genotype known as mutation.

In fig. 1 genetic algorithm is shown. The main idea of evolution consists in improving the individual fitness of the first population generation until cease criteria are achieved.