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SIMULATION OF INJECTION AND CAPTURE OF BEAM ELECTRONS IN SMALL-SIZE BETATRONS BY THE METHOD OF MACROPARTICLES

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The simulation problem of self-consistent dynamics of electron beam in small-size betatrons has been considered. The description of numerical model developed on the basis of macroparticle method is presented. The results of process modelling of electron injection and capture on the acceleration mode in betatrons with axially-symmetrical and asymmetrical magnetic field are shown. Optimal input injection parameters by beam current and energy (20...40 keV and 0,1...1,0 A) providing the maximum number of the capture electrons are defined. The techniques of increasing capture efficiency due to using variations of external magnetic field and additional energy selection of circuital decelerating EMF of the captured electrons are numerically studied. It allows an increase in capture coefficient from 4 to 7,4 % and capture at the acceleration up to 7,4·10^{10} electrons.

Introduction

Wide application of small-size betatrons requires essential increase in radiation intensity. Therefore, one of the most important problems in betatron production is enhancement of efficiency of electron capture at the acceleration mode. Coulomb interaction of beam electrons is known to play a decisive role in capture mechanism [1, 2]. The existing approach [3, 4] to researching this capture mechanism in classical betatrons based on the analytical methods describes well the physics of the phenomenon for the case of axial-symmetrical magnetic fields. It makes possible to obtain qualitative dependences of capture parameters at linear approximation of relatively small parameter $\Delta R/R_0$ supposing that the width of vacuum chamber ΔR is significantly smaller than the radius of equilibrium orbit R_0.

The given assumption is not applicable to small-size betatron where beam cross size can be compared with the radius of equilibrium orbit. Therefore, it is necessary to develop methods of obtaining quantitative capture parameter characteristics reflecting not only the field influence of beam spatial charge, but also the system geometry as well as complex profile of the external magnetic field. This problem can be solved on the basis of numerical simulation using modern macroparticle method [5]. The advantage of the given approach consists in obtaining detailed information in beam internal structure along with quantitative estimation of capture parameters. Below a three-dimensional model of electron capture on the acceleration mode in the cylindrical coordinate system $\{r, z, \theta\}$ is presented. In the model a complete self-consistent dynamic system of electron beam describing physical processes for general case of axial-asymmetrical magnetic fields which influence sufficiently the capture efficiency at multiturn injection is used. It gives the most adequate description of beam nonlinear dynamics, which is particularly important for small-size betatrons.

Model description

Numerical model has been developed on the basis of electron beam description by macroparticles [6]. It is designed for three-dimensional modelling region coinciding with the region of betatron accelerating chamber: $D=\{-L\le z\le +L,\ R_1\le r\le R_2,\ 0\le\theta\le 2\pi\}$. Here R_1, R_2 are radii of the internal and external chamber wall, L is the axial chamber half-width. Macroparticles have the form of coaxial sectors (fig. 1), containing the charge $Q=N_e\cdot e$ and the mass $M=N_e\cdot m_e$, where N_e is the number of electrons in macroparticle; e, m_e are the charge and the mass of electron. They are enlarged particles with specific charge equal to that of elementary electron and are characterized by the coordinates $\{r,z,\theta\}$ and the velocities $\{V_r, V_z, V_\theta\}$. The particle size is determined by the size of computational grid. The external field in general case is described by the axial and radial components of magnetic field induction $B_z^*(r,z,\theta)$, $B_r^*(r,z,\theta)$ and by azimuth component of electrical field density $E_\theta^*(r,z,\theta)$. Thus, the model gives the three-dimensional description of physical processes both in dynamics and by the field.

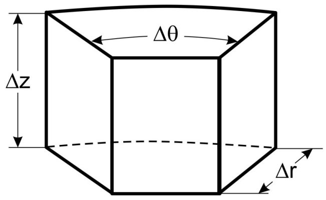

Fig. 1. *Macroparticle model*

Macroparticle dynamics is described by the non-relativistic equation of electron motion [7]:

$$\frac{d^2 r}{dt^2}=\frac{e}{\gamma m_0}\{(r\dot\theta)B_z^*+E_r\}+r\dot\theta^2,$$
$$\frac{d^2 z}{dt^2}=-\frac{e}{\gamma m_0}\{(r\dot\theta)B_r^*+E_z\},\qquad(1)$$
$$\frac{d^2\theta}{dt^2}=\frac{e}{\gamma m_0}\frac{1}{r}\{\dot z B_r^*-\dot r B_z^*+E_\theta^\Sigma\}-\frac{2}{r}(\dot r\dot\theta),$$

where $E_\theta^\Sigma=E_\theta^*+E_\theta$; $\gamma=1\Big/\sqrt{1-(V/c)^2}$; $V=\left|\vec V\right|$.

Initial conditions are defined by injector parameters and are set in the form:

$$r=R_{ин},\ z=Z_{ин},\ \theta=\theta_{ин},\ V_r=0,\ V_z=0,\ V_\theta=V_{ин}.$$

Properly Coulomb beam field is described by the Poisson equation for scalar potential:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{\partial^2\varphi}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}=-\frac{1}{\varepsilon_0}\rho, \qquad (2)$$

where $\varphi=\varphi(r,z,\theta,t)$, $\rho=\rho(r,z,\theta,t)$.

Boundary conditions meet the requirements of ideal conductivity of the chamber wall and azimuth periodicity:

$$\varphi|_s=0, \quad \varphi(r,z,0)=\varphi(r,z,2\pi).$$

Through the potential the components of beam electric field density are calculated:

$$E_r=-\frac{\partial\varphi}{\partial r}, \quad E_z=-\frac{\partial\varphi}{\partial z}, \quad E_\theta=-\frac{1}{r}\frac{\partial\varphi}{\partial\theta}. \qquad (3)$$

The equation of defining charge density closes the system:

$$\rho(M)=\frac{Qn(\Omega)}{\Omega}, \qquad (4)$$

where M is the mesh point; Ω is the volume of grid cell; $n(\Omega)$ is the number of macroparticles in Ω.

Modeling self-consistent beam dynamics is performed by means of numerical solution of equations set (1–4). At every step of macroparticle distribution in calculation area the grid density of the beam charge is calculated $\rho(r,z,\theta,t)$. Then the Poisson equation is solved (2) for the beam potential $\varphi(r,z,\theta,t)$, and components of self-field are calculated by the formulas (3). The equations of motion (1) are integrated by the Runge-Kutt method of the second accuracy order. In the relation (4) the volume produced by grid cell (fig. 1) is recognized as Ω. To solve the Poisson equation (2) the iteration dissipative difference scheme is used [8], which permits us to smooth calculative high-frequency «noises», inherent to macroparticles models, in the true value of the basic physical processes. The influence of the self magnetic field on the electron motion in the region of the parameters considered can be neglected.

Investigation of electron injection and capture at acceleration

At numerical modelling the injection of electron beam having kinetic energy W_{in}=20...40 keV is considered. Geometrical parameters of the equipment: L=20 mm, R_1=20 mm, R_2=80 mm, R_0=45 mm. Particle injection is simulated by the injector arranged in the median plane on the radius R_{in}=60 mm and having axial width ΔZ_{in}=10 mm. The components of the external magnetic field are set analytically. Simulation is performed for the following external magnetic field configurations:

a) Axial-symmetrical field. The components of the magnetic field are presented in the form of:

$$B_z^*=B_z^0\frac{R_0^n}{r^n}, \quad B_r^*=-B_z^*\frac{nz}{r}, \qquad (5)$$

where $B_z^0=B_z^*(R_0)$ is the value of the field B_z^* on the equilibrium orbit $r=R_0$, $n=-\frac{r}{B_z^*}\cdot\frac{\partial B_z^*}{\partial r}$ is the coefficient of magnetic field decay.

b) Azimuth variation field. For the four-sector magnetic field we have:

$$B_Z^{var}=B_Z^*[1+\delta B_Z\cos(4\theta)], \quad B_r^{var}=-B_z^{var}\frac{nz}{r}, \qquad (6)$$

where δB_z is the depth of the field variation.

The results of simulation for four variants of calculation at the current of injected beam I=0,5 A and magnetic field configurations (5, 6), table, are presented in fig. 2–5. In figures the configuration images of the beam in the planes $\{r, z\}$ and $\{r, \theta\}$ – (*a*), (*b*) are shown, the phase image of the beam in $\{r, V_r\}$ in (*в*) and electron distribution histograms by energy in the chamber is in (*d*).

Table. *The maim parameters of numerical calculation and the number of captured electrons N_{cap}*

Variant of calculation	Field variation δB_z, %	Energy extraction ΔW, eV/rev	$N_{cap}\cdot10^{-10}$			
			0 ns	30 ns	50 ns	150 ns
1	0	0	0	5,5	7,1	4,2
2	50	0	0	8,4	8,1	5,1
3	0	200	0	7,3	9,4	6,3
4	50	200	0	9,1	10,8	7,4

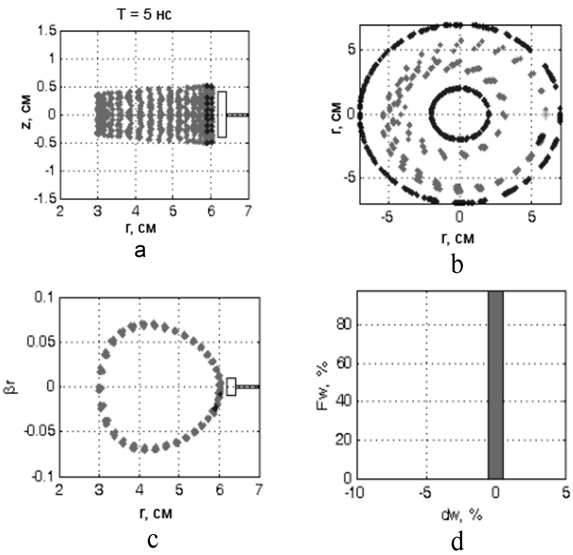

Fig. 2. *Moment of beam injection, t=5 ns*

Injection simulation is carried out up to the moment t=50 ns, when the charge limit is accumulated ($\approx10^{11}$ el.), then up to t=150 ns the capture process is investigated at the acceleration of this accumulated beam. In modeling the beam acceleration by circuital field is not taken into consideration due to small growth of beam particle energy making up hundredth parts of the percent per revolution. Hence, the external field is considered to be constant in time. This is a rather justified assumption as during the time of setting the given processes (t<200 ns), corresponding to 50–100 rev., the growth of beam particle energy is negligibly small (more than 0,5 %). In the course of the calculations the control of complete energy conservation of interacting particle system the model parameters provide the energy conservation with the error of less than 1 %.

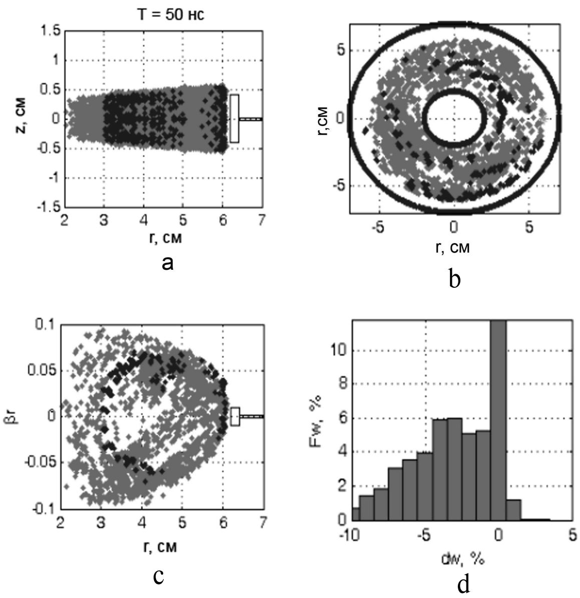

Fig. 3. *Moment of beam injection, t=50 ns*

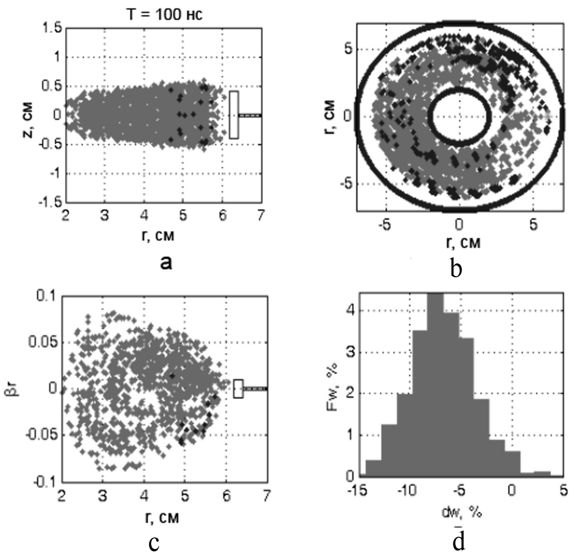

Fig. 4. *Beam capture at t=100 ns*

During the investigation the capture mechanism at acceleration of the injected beam in betatron is revealed. It is sufficiently influenced by the fields of spatial beam charge. The beam capture process at acceleration is a non-stationary one with entering stable quasistationary state characterised by the absence of particle loss and keeping the beam structure in coordinate and phase spaces (fig. 2–5). In betatron chamber a monochromatic beam is introduced (fig. 2, *d*). In the process of radial oscillation redistribution of energy among the beam particles owing to the influence of self Coulomb fields takes place (fig. 3, *d*). A part of beam particles gains energy and settles on the injector and chamber walls, but another part of the beam loses energy and is captured at acceleration (fig. 4, *d* – 5, *d*). Energy losses of captured electrons amount up to 15 % of injection energy, this providing the radial removal from the injector for them (fig. 5, *a*).

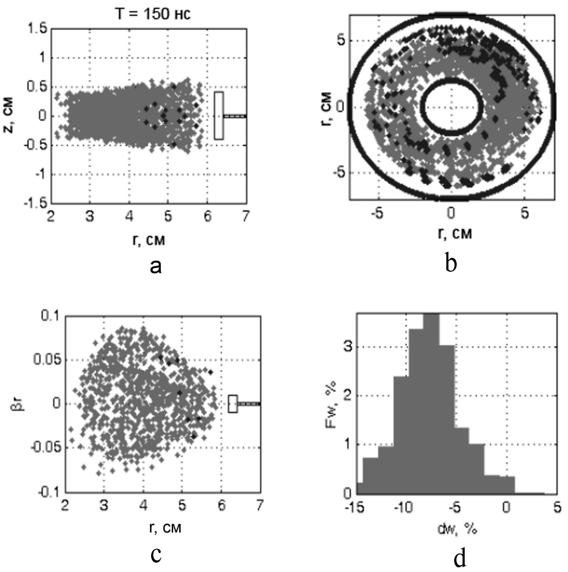

Fig. 5. *Beam capture at t=150 ns*

At injection energy W_{in}=40 keV in the volume of chamber the charge corresponding to $N_{acc}\approx 7\cdot 10^{10}$ el. is accumulated, captured at acceleration $N_{cap}\approx 4\cdot 10^{10}$ el. The results of modeling according to the size of captured charge coincide with experimental data in the range of the order [2, 9]. It follows from the calculation results (Table) that increase in capture efficiency is possible by means of additional beam energy selection of circuital decelerating EMF and variations of the external magnetic field. A more sufficient increase in capture efficiency is achieved by selection of beam energy. The energy selection of 200 eV/rev allows the capture at the electron acceleration $N_{cap}\approx 6\cdot 10^{10}$ el, in this case the capture coefficient increases by the factor of 1,5. Combination of field variation and decelerating EMF provides the maximum capture parameters: $N_{cap}\approx 7{,}4\cdot 10^{10}$ el., $K_{cap}\approx 7{,}4$ %. The capture coefficient is calculated by the formula [4]: $K_{cap}=N_{cap}/N_{max}$, where N_{max} is the maximum number of electrons captured in the betatron chamber by the external magnetic field. For the betatron parameters presented above the value of $N_{max}\approx 1\cdot 10^{12}$ electrons.

Conclusion

1. The numerical model of electron beam self-consistent dynamics in small-size betatrons describing physical processes both in axial-symmetrical and in asymmetrical magnetic fields has been developed.
2. Modelling the processes of injection and capture at acceleration in small-size betatron reveals the optimal values of the current of injected beam 0,1...1,0 A at the energy 20...40 keV. At the injection current values less than 0,1 A one can neglect the influence of self fields on the beam dynamics, while the current 1 A is a limit, above which formation of «virtual» cathode in injector takes place.
3. In the course of the numerical experiments it is stated that self Coulomb fields conditioned by radial non-homogeneity of the charge density play suffici-

ent part in beam capture. The values of charge limit accumulated at the injection ($7\cdot10^{10}$ el.) and captured at the acceleration ($4\cdot10^{10}$ el.) are obtained.

4. Possible increase in capture efficiency by means of variation of external magnetic field and beam energy selection of circuital decelerating EMF is investigated. Thus, at the electron energy 40 keV and the energy selection value 200 eV/rev, capture efficiency increases by the factor of 1,5, which permits the capture at the acceleration $6\cdot10^{10}$ el. The maximum capture parameters are obtained at combination of variation of the external magnetic field and decelerating EMF: $N_{cap}\approx7,4\cdot10^{10}$ el., $K_{cap}\approx7,4$ %.

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IRON-FREE ELECTRON SYNCHROTRON WITH WEAK FOCUSING

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A synchrotron construction the magnetic field of which is without steel core is suggested. Acceleration chamber is combined with magnetizing winding. The described version of accelerator is favorably different in small weight, simplified production and assembling technique.

Resonance electron accelerator – synchrotron with weak focusing of electron beam presents a complex electrical-physical device consisting of a number of large units and their supply systems. The basic element of synchrotron is an electric magnet producing progressive magnet field necessary for motion of charged electrons along the orbit of constant radius. It is the so called «control magnet field». Synchrotron magnetic conductor is made of sheet transformer steel and requires high accuracy of electromagnet assembling and mounting made up of four sectors including azimuth angle 90° each. In general this system is the most labour-consuming, technologically complex and expensive part of the accelerator. Thus, the steel weight of the synchrotron «Sirius» electromagnet per 1,5 GeV in Tomsk Polytechnic University is 120 tons [1, 2].

Control magnet field in the suggested synchrotron construction is induced by single-turn winding enclosing an azimuth circle quarter [3].

The winding presents two concentrically located strips of conductive material. The strips are connected with each other at one end, but at the other one they are connected with alternating power supply. Current flows in both strips in the opposite directions.

In the space between the strips magnet field providing the motion of accelerated particles along the orbit (control synchrotron field) is produced. This field is to possess the property of electron beam axial focusing.

In accelerator with «steel» magnet system focusing is achieved by shaping magnet field in «barrel-like» form by means of increasing the gap between the electromagnet poles as it moves away from the system centre.

To shape magnet field in a necessary form induced between the poles of single-turn winding these poles are to bend in the vertical direction so that the convex part of the strip would present outside from the system centre.

One more labour-consuming element of synchrotron is a vacuum acceleration chamber. It is glued from a large number of ceramic curved sectors forming an arch of quarter circle length, the radius of which corresponds to that of path curve of the accelerated electron beam. Four of such arched sections connected with