



Research Paper

PECULIARITIES OF ELECTROMAGNETIC FIELD OSCILLATIONS OF A CHARGED PARTICLE ROTATING ABOUT A CONDUCTIVE BALL

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Abstract

The paper investigates some characteristic features of the electromagnetic field of a relativistic charged particle that uniformly rotates about a conductive ball in its equatorial plane. It is assumed that the braking of the particle due to radiation is compensated by an external influence (e.g. the electric force) that compels the particle to turn uniformly in a circle. The magnetic permittivity of the ball is assumed to be one. The work is based on the corresponding exact analytic solutions of Maxwell's equations. The generalized Drude-Lorentz-Sommerfeld formula for the dielectric function of the conductive ball is used in numerical calculations. It is shown that localized oscillations of a high-amplitude electromagnetic field can be generated at a given harmonic inside the ball at a certain (resonant) particle rotation frequency at a small distance from the surface of the ball. Herewith, at large distances from the trajectory of the particle, these localized oscillations are accompanied by intense radiation at the same harmonic, which is many times more intense than the analogous radiation in the case when the ball is absent. The possibilities of using this phenomenon to develop sources of quasi-monochromatic electromagnetic radiation in the range from giga- to terra hertz frequencies are discussed.

Key words: electron, conductive ball, electromagnetic oscillations, dispersion, resonant radiation

1. Introduction

Synchrotron Radiation (SR) serves as an extraordinary research tool for advanced studies in both fundamental and applied sciences due to such unique properties as high intensity, high degree of collimation, and wide spectral range (see [1–6] and references therein). SR is used worldwide by thousands of scientists in many disciplines like physics, chemistry, material science and structural biology nowadays. For this reason it is important to analyze various mechanisms to control SR parameters. That is why it

is urgent to study the influence of medium on spectral and angular distributions of SR. Investigations of this kind are also relevant for a number of astrophysical problems [4, 7].

Characteristics of high-energy electromagnetic processes change greatly in the presence of matter thus giving rise to new phenomena. Their well-known examples are Cherenkov [8–10] and diffraction radiation. The operation of many devices used to produce electromagnetic radiation is based on interactions of relativistic charged particles with matter (see, e.g., [11]).

In [12], SR from a charged particle rotating in a homogeneous medium was considered (see also [13, 10]); it was shown that interference between SR

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and Cherenkov Radiation (CR) had interesting consequences. New interesting phenomena occur in the case of inhomogeneous media, e.g. transition radiation [14, 15] and the radiation of charged particles in periodic structures (see [16, 17] and references therein).

Alongside, the interfaces between media can be used for monitoring the flows of radiation emitted in various systems. In a series of papers (see [18–20] and references in [20]), it was shown that the interference between SR and CR induced at boundaries of spherical configuration leads to interesting effects. Investigations of radiation from a charge rotating along an equatorial orbit about/inside a dielectric ball showed that there appear high narrow peaks in the spectral distribution of the number of quanta emitted to outer space at some specific values of the ratio of ball-to-particle orbit radii and when the Cherenkov condition is satisfied for the ball material and particle speed (see [20] and refs therein). The radiated energy in the vicinity of these peaks exceeds the corresponding value for the case of homogeneous and unbounded medium by several orders of magnitude.

A similar phenomenon (less weak) takes place in the case of cylindrically symmetric medium [21–27]. For example, the radiation emitted (i) from a longitudinal charged oscillator moving with constant drift velocity along the cylinder axis, and (ii) from a charged particle moving along a circle around a dielectric cylinder or along a helical orbit inside the cylinder are investigated in [21–23], [24] and [25–27] (see also references therein) respectively (the latter type of motion is used in helical undulators for generating electromagnetic radiation in a narrow spectral interval). It is shown that high narrow peaks are present in the spectral-angular distribution for a number of radiated quanta if the Cherenkov condition is satisfied for the permittivity of a cylinder and the particle speed. The radiated energy exceeds the corresponding value for homogeneous medium case by several orders of magnitude in the vicinity of these peaks.

It is also important that 1) localized Surface Waves (SW) can be generated on the interface of media if the source of the field moves near this interface and that 2) the phase velocity of SW can be many times less than the phase velocity of the volume waves. This circumstance can have important practical applications. This paper is devoted to this topic. It examines the electromagnetic field of a charged particle rotating about a conductive ball taking into account the fact that surface plasma oscillations can be generated inside the ball.

The electromagnetic field of a charged particle rotating about a metal ball was investigated in [28]. However, in [28], the possibility of SW generation inside a conductive ball was not investigated.

2. Formulation of the problem

We consider a particle with some charge q , which rotates uniformly at a speed $v_d = \text{const}$ about a conductive ball in its equatorial plane in vacuum under the influence of a constant magnetic field (see Fig. 1). We assume that the deceleration of the particle, due to its radiation, is compensated by an external (for instance, electric) force compelling the particle to rotate uniformly and circularly about the ball. We shall also assume that the magnetic permittivity of the ball $\mu_b = 1$ to simplify the calculations.

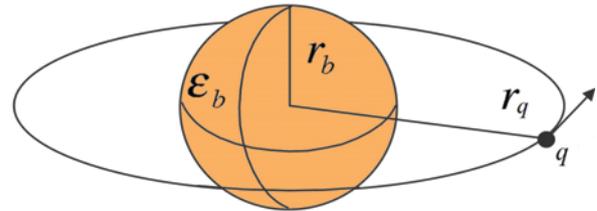


Fig. 1. A relativistic charged particle rotating about a conductive ball in its equatorial plane.

The magnetic permittivity of the ball $\mu_b = 1$

In the case under consideration, the dielectric constant of the medium is a step function:

$$\varepsilon(r) = \varepsilon_b + (1 - \varepsilon_b)\Theta(r - r_b), \quad (1)$$

where r_b is the radius of the ball, $\varepsilon_b = \varepsilon_b' + i\varepsilon_b''$ is the complex valued permittivity of the substance of the conductive ball and $\Theta(x)$ is the Heaviside step function.

A uniformly rotating particle generates radiation at $\omega_k = k\omega_q$ discrete frequencies (harmonics) with $k=1;2;3\dots$, and ω_q is the cyclic frequency of the rotating particle.

Let us investigate the energy

$$w_k = \hbar\omega_k n_k \quad (2)$$

radiated by a charged particle at the k -th harmonic during one period of its rotation. The value of the dimensionless parameter n_k indicates which part of the energy $\hbar\omega_k$ of the quantum of electromagnetic field is generated by the particle during one period of its rotation. Further, n_k will be conditionally called «the number of quanta of the electromagnetic field generated by the particle in one period of its rotation about the conductive ball».

The dependence of n_k on the parameters $(k, \omega_q, r_q, r_b, \varepsilon_b)$ of the problem should be determined by solving the Maxwell's equations for the case when the charge rotates about the conductive ball with $\mu_b = 1$. An analogous problem for a charge rotating about a ball with $\mu_b = 1$, $\varepsilon_b(\omega) \approx \text{const}$ and $\varepsilon_b'(\omega) > 0$ (dielectric ball) was solved in [20] (see also references in it). Herewith the results of analytical calculations performed in the mentioned papers are applicable without restrictions on the values of $\varepsilon_b(\omega)$ and at the dependence of this quantity on the frequency ω . Therefore, the

formulas derived in these papers are applicable also for the case of a charge rotating about a conductive ball with $\mu_b=1$ and non constant function $\varepsilon_b(\omega)$.

In [20], all functions except $\varepsilon_b(\omega)$ (the dispersion law for the substance of a ball) are defined. A simple analytical function, which is often used to describe the dispersion law of a conductive substance, has the following form

$$\varepsilon_b(\omega) = \varepsilon_0 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} = \varepsilon'_b(\omega) + i\varepsilon''_b(\omega) \quad (3)$$

(the generalized Drude-Lorentz-Sommerfeld formula). The above expression satisfactorily describes the dielectric function of noble metals, e.g. for gold [29]

$$\varepsilon_0^{Au} = 9.84, \quad \hbar\omega_p^{Au} = 9.01eV, \quad \hbar\gamma^{Au} = 0.072eV. \quad (4)$$

The effective parameter $\varepsilon_0 > 1$ describes the contribution of bound electrons, ω_p is the effective bulk plasma frequency, which is associated with effective concentration of free electrons, γ is the phenomenological damping constant of the electrons' motion.

We will consider electromagnetic oscillations in the frequency range, for which

$$\varepsilon'_b(\omega) < 0. \quad (5)$$

In this case, the generated electromagnetic oscillations inside the ball must be localized (see Section 4).

3. Numerical results in the range of gigahertz frequencies

We assume that (a) the ball is made of a dielectric with a negligible mixture of gold, so that the plasma frequency of the free charge carriers is

$$\omega_p = 3 \cdot 10^{10} \text{ Hz} \quad (6)$$

($\omega_p^{Au} = 1.44 \cdot 10^{16} \text{ Hz}$). In this case, (b) the dielectric should have a weak dispersion and should absorb light slightly in the range of gigahertz (GHz) frequencies. A fused quartz with [30]

$$\varepsilon_{SiO_2} = 3.78(1 + 0.0001i) \quad (7)$$

can be used in the mentioned range for this purpose. The parameter ε_0 in equation (3) has been identified by ε_{SiO_2} because of the small concentration of gold in the substance of the ball. In numerical calculations, three estimated values of parameter $\gamma/\omega_p = 1/50; 1/125; 1/1000$ have been used, where $1/125 \approx \gamma^{Au}/\omega_p^{Au}$ (the ball is entirely made of gold).

Thus, the dielectric function $\varepsilon_b(\omega)$ was calculated from equation (3) with the following values of the parameters:

$$\begin{aligned} \varepsilon_0 &\approx \varepsilon_{SiO_2}, \quad \omega_p = 3 \cdot 10^{10} \text{ Hz}, \\ \gamma / \omega_p &= 1/50; 1/125; 1/1000. \end{aligned} \quad (8)$$

The results of numerical calculations are shown in Figs. 2–4.

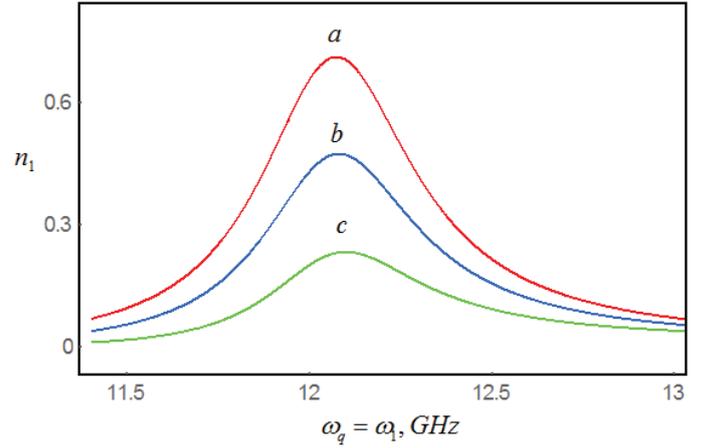


Fig. 2. The number of quanta of the electromagnetic field n_1 emitted by an electron at the first harmonic during one period of its rotation about the ball, depending on the cyclic rotation frequency of the electron ω_q . Herewith the frequency of the emitted electromagnetic waves is $\omega = \omega_1 = \omega_q$. The radius of the electron's orbit is $r_q = 1$ (curve a), 1.1 (curve b) and 1.3 cm (curve c). The radius of the ball is $r_b = 0.99$ cm; $\gamma/\omega_p = 1/125$. The maxima on the curves a, b, and c correspond to the intense (resonant) radiation of the electron: $n_1 \gg e^2/c\hbar$

Having compared the course of the curves a, b and c in Fig. 2, we arrived at the following conclusions:

- 1) The electron generates very intense radiation ($n_1 \gg e^2/c\hbar$) if it rotates about a conductive ball at a certain (resonant) frequency $\omega_q^{res} \approx 12.1$ GHz.
- 2) The values of the resonant frequency ω_q^{res} depend weakly on the radius of its orbit r_q .
- 3) The maxima on curves $n_1(\omega_q)$ decreases rapidly with the distancing of the electron's orbit from the surface of the ball.

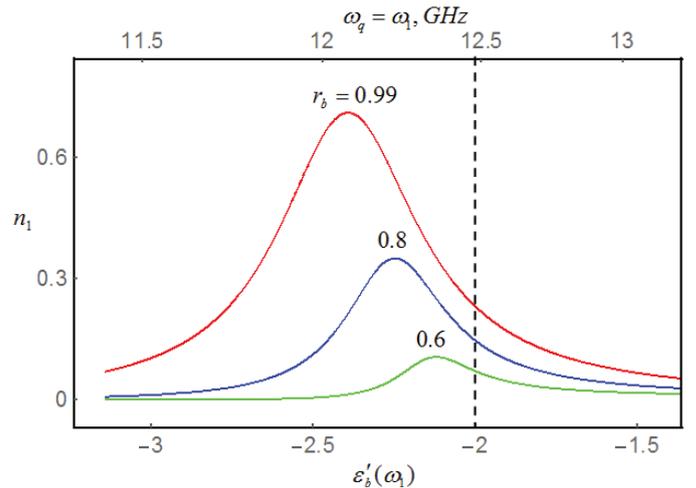


Fig. 3. The number of quanta of the electromagnetic field n_1 emitted by the electron at the first harmonic during one period of its rotation about the conductive ball. The values of $\omega_q = \omega_1$ (the upper part of the figure) and the values of the real part ε'_b of the dielectric constant of the ball (the lower part of the figure) are plotted at $\omega_q = \omega_1$ along the axis of abscissa. The radius of the electron's orbit is $r_b = 1$ cm; $\gamma/\omega_p = 1/125$. The values of radius of the ball are indicated near the curves

Having compared the course of the curves with $r_b=0.99, 0.8$ and 0.6 cm in Fig. 3, we arrived at the following conclusions:

1) The resonant frequency ω_q^{res} with which an electron rotates about a conductive ball depends on the r_b radius of this ball.

2) As the radius of the conductive ball r_b (a) decreases, the maximum of the function n_1 decreases rapidly and (b) the value of the real part of the dielectric function of the ball ε'_b corresponding to this maximum tends to -2 .

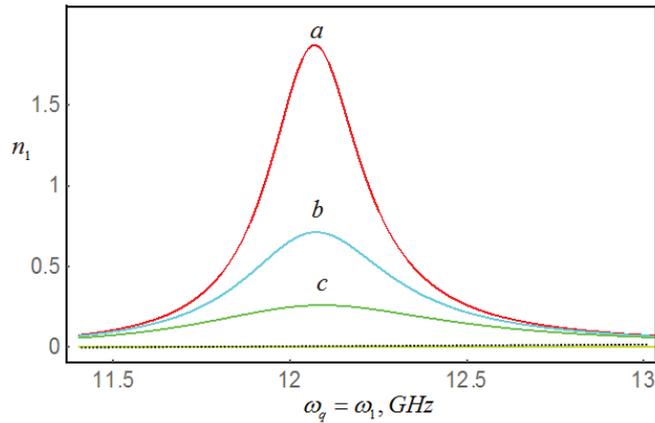


Fig. 4. The number of quanta of electromagnetic field n_1 emitted by an electron at the first harmonic during one period of its rotation about the ball, depending on the cyclic rotation frequency of the electron ω_q . The frequency of the emitted electromagnetic waves is $\omega=\omega_1=\omega_q$. The radius of the electron's orbit is $r_q=1$ cm, the radius of the ball is $r_b=0.99$ cm; $\gamma/\omega_p=1/1000$ (curve a), $1/125$ (curve b) and $1/50$ (curve c). The dotted curve describes the emission of an electron in the absence of the ball

Having compared the course of curves a, b and c in Fig. 4, we arrived at the following conclusions:

1) The number of quanta of the electromagnetic field $n_1(\omega_q)$ radiated by the electron as it rotates at resonant frequency $\omega=\omega_q^{res}$ increases significantly with decreasing dielectric energy losses in the substance of the conductive ball (in this case, on the decrease in the value of parameter γ).

2) The performed numerical calculations indicate that the wavelength of electromagnetic waves emitted by the charged particle can be varied over a wide frequency range by a suitable choice of the radius and substance of the conductive ball.

4. Visual explanation of numerical results

Not localized plane electromagnetic waves $\sim \exp[i(k_x x + k_y y + k_z z - \omega t)]$ can propagate in a continuous, infinite and transparent substance. In this case, a wave vector (k_x, k_y, k_z) is determined by the dispersion equation

$$\frac{\omega^2}{c^2} \varepsilon = k_x^2 + k_y^2 + k_z^2, \quad (9)$$

where ε is the dielectric permittivity of the substance (the magnetic permittivity μ is assumed to be one). According to this equation, only localized electromagnetic oscillations can be excited in the substance in the frequency range with $\varepsilon(\omega) < 0$, for example,

$$\sim \exp[i(k_x x + k_y y - \omega t)] \exp(-k_z z), \quad \text{where } k_* = -ik_z. \quad (10)$$

The electromagnetic oscillations described by expression (10) along the z axis decrease exponentially in one direction and increase exponentially in the opposite direction. Similar (localized) electromagnetic oscillations can be generated in matter in the presence of interfaces (e.g. with vacuum).

In the case of the ball, the interface of matter with vacuum is finite and closed. In this case, for the most values of rotational speed of an electron, localized electromagnetic oscillations are in a destructive superposition with each other, as a result of which their amplitude decreases. However, when rotating at certain frequencies (the natural frequencies of the ball), a constructive superposition of electromagnetic oscillations occurs, as a result of which their amplitude increases significantly.

Suchwise,

1. The presence of the interface between matter and vacuum is important, because due to this circumstance a rotating particle inside the substance generates localized (surface) electromagnetic oscillations.

2. The shape of the interface between the substance and vacuum is also important. When the particle rotates at the natural frequencies of the ball, a constructive superposition of the generated surface electromagnetic oscillations (resonance) occurs.

3. A charged particle rotating about the ball at a resonant frequency generates localized electromagnetic oscillations on its surface at the natural frequencies of the ball. At large distances from the ball, these electromagnetic oscillations manifest themselves as intense resonant radiation.

In the future, we plan to numerically investigate the spectral-angular distribution of the generated radiation, which allows one to reveal new features of the phenomenon under the study. The corresponding formulas for the spectral-angular distribution of the radiation intensity of a charged particle rotating about a ball with an arbitrary dielectric function $\varepsilon_b(\omega)$ and magnetic permittivity $\mu_b=1$ have been derived in [18, 19].

5. Conclusions

In the paper, the electromagnetic field of a charged particle uniformly rotating about a conductive ball with $\mu_b=1$ in its equatorial plane, has been investigated. The dispersion of electromagnetic waves inside the conductive ball has been taken into account. The work is based on the corresponding exact analy-

tical solutions [18–20] (see also references in [20]) of the Maxwell's equations. The Drude-Lorentz-Sommerfeld model has been used for the dielectric function of the conductive ball in the numerical calculations.

The number of quanta of the electromagnetic field n_1 emitted by the electron at the first $\omega_1 = \omega_q$ harmonic during its one revolution about the conductive ball has been calculated for different values of the cyclic rotation frequency ω_q of the charge.

It has been shown that:

1. For small dielectric energy losses in the ball's substance, the existence of a distinguished (resonant) value $\omega_q = \omega_q^{res}$ of the particle's rotation frequency is possible, for which $n_1(\omega_q)$ reaches its maximum value. This value can be tens of times greater than the value of $n_1(\omega_q)$ in the case of rotation of the charge in empty space.

2. Such an intensive (resonant) radiation happens due to the presence of a conductive ball and is determined by its dielectric function $\epsilon_b(\omega)$ and radius r_b .

3. The frequency of the emitted electromagnetic waves $\omega = \omega_1 = \omega_q$ is determined by the radius of the ball r_b and weakly depends on the radius of the charge or-

bit r_q . It is of the order of the plasma frequency of oscillations of free charge carriers inside the ball $\omega \sim \omega_{pl}$.

4. The plasma electromagnetic oscillations generated by a charge (inside a conductive ball) at the resonant frequency ω_q^{res} are localized, since $\epsilon_b'(\omega_q^{res}) < 0$.

5. A charged particle generates localized surface waves in the ball at the natural frequencies of this ball. These 2-dimensional (localized) waves appear as an intense resonance radiation at a large distance from the ball. With decreasing dielectric energy losses in the substance of the conductive ball, the resonant radiation increases.

6. The resonant radiation can be used to develop sources of quasi-monochromatic electromagnetic radiation in the range from giga- to terahertz frequencies, in the presence of a conductive substance with the parameters of the same orders as those given in equation (8).

For the parameters of the system described in Section 3, the strength of the magnetic field providing the electron rotation does not exceed 1 Tesla. Using this circumstance, we plan to continue experimental studies of the resonant radiation from the electron rotating about a conductive ball in the future.

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