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# Identification of Mathematical Model of Drying Unit

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**Abstract.** In air drainage systems, the process of reducing moisture content is based on its cooling down to the dew point and the precipitation of liquid into the condensate. Under isobaric cooling, the dew point temperature depends on the relative humidity and temperature in the drained space, which in turn can be controlled by the temperature of the heater and the air flow rate created by the fan, the main part of which is the electric drive.

## 1. Introduction

Thus, due to the fact that the system is two-dimensional, there is some uncertainty in the control of the heater and the fan motor, due to the functional dependence of the transmission coefficient on the power of the heater and the fan motor respectively. Consequently in a modern world, accurate and fast positional control is required.

One of the solutions to this problem is to obtain an analytical solution from the equations of heat conduction and thermodynamics, taking into count the formula for saturated vapor pressure derived by Magnus in 1844 [1]:

$$P_{wp_i}(t^{\circ}) = a \cdot exp\left[\frac{b \cdot t^{\circ}}{c + t^{\circ}}\right] \tag{1}$$

where a,b,c - empirical coefficients varying from model to model [1]. There are many formulas built on the general definition of Magnus (Table 1). The most abundant and used in meteorology models of Abott-Teboni, Alduchov, and Buck can be distinguished [2-4]. Non-Magnus models obtained by Wexler [5] are also standing apart.

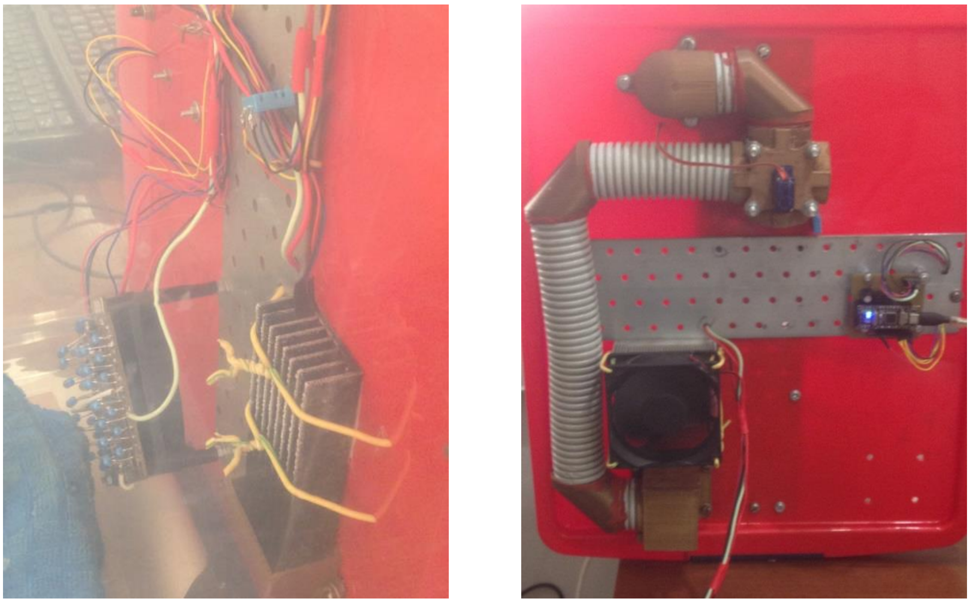
But this approach has drawbacks related to the fact that for complex multicomponent and multicriteria systems, obtaining exact analytical solutions is not a trivial task. And even if this succeeds, then most often the mathematical model of the system is inadequate.

**Table 1.** Basic formulas for saturated vapor pressure above water.

| Model Author | Abbreviation | Formula (kPa) |
|---|---|---|
| Abott-Teboni | AT85 | $P_{wp_i}(t^{\circ}) = 0.61070 \cdot exp\left[\frac{17.38 \cdot t^{\circ}}{239.0 + t^{\circ}}\right]$ |
| Alduchov | AERK | $P_{wp_i}(t^{\circ}) = 0.61094 \cdot exp\left[\frac{17.625 \cdot t^{\circ}}{243.04 + t^{\circ}}\right]$ |
| Buck | BU81 | $P_{wp_i}(t^{\circ}) = 0.61121 \cdot exp\left[\frac{17.502 \cdot t^{\circ}}{240.97 + t^{\circ}}\right]$ |
| Wexler | WE76 | $P_{wp_i}(T) = 0.001 \cdot exp[-2.9912729 \cdot 10^{3} \cdot T^{-2} - 6.0170128 \cdot 10^{3} \cdot T^{-1} + +1.887643845 \cdot 10 - 2.8354721 \cdot 10^{-2} \cdot T + 1.7838301 \cdot 10^{-5} \cdot T^{2}] - -8.4150417 \cdot 10^{-10} \cdot T^{3} + 4.4412543 \cdot 10^{-13} \cdot T^{4} + 2.858487 \cdot lnT]$ |

## 2. Dryer Model

Therefore, a prototype was created for modeling processes in drying plants (Figure 1, a – inside, b - outside). The prototype was based on a plastic box, an Arduino Nano microcontroller, a block of resistors that act as a heater, a Peltier element, the cold junction of which is in the box and two fans, one of which creates the necessary air flow in the drained volume, and the second removes heat from the element's hot junction Peltier.



a b

**Figure 1.** Dryer prototype.

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During the experiment, the static modes were removed and a rectangular temperature grid was obtained on the cooling radiator and in the box for a number of capacities consumed by the heater and the electric fan: 20.40.60% (Figure 2, a – $t^{\circ}_{rad} = t^{\circ}_{rad}(P_{heat}, P_{fan})$, b - $t^{\circ}_{box} = t^{\circ}_{box}(P_{heat}, P_{fan})$, Table 2). The temperature transient curves were also obtained when the power consumption of the fan motor and the heater changed (Figure 3, a – $t^{\circ}_{rad}(t)$, b - $t^{\circ}_{box}(t)$, Figure 4, a – $t^{\circ}_{rad}(t)$, b - $t^{\circ}_{box}(t)$ ,).

**Table 2.** Experimental temperature data.

| $P_{heat}$,% | $P_{fan}$,% | $t^{\circ}_{rad}$, $^{\circ}C$ | $t^{\circ}_{box}$, $^{\circ}C$ |
|---|---|---|---|
| 20 | 20 | 7.625 | 29.125 |
| 20 | 20 | 8.813 | 28.875 |
| 20 | 20 | 9.500 | 28.875 |
| 40 | 40 | 11.500 | 35.875 |
| 40 | 40 | 12.375 | 35.375 |
| 40 | 40 | 13.250 | 35.250 |
| 60 | 60 | 13.750 | 42.000 |
| 60 | 60 | 13.750 | 40.375 |
| 60 | 60 | 15.375 | 39.563 |

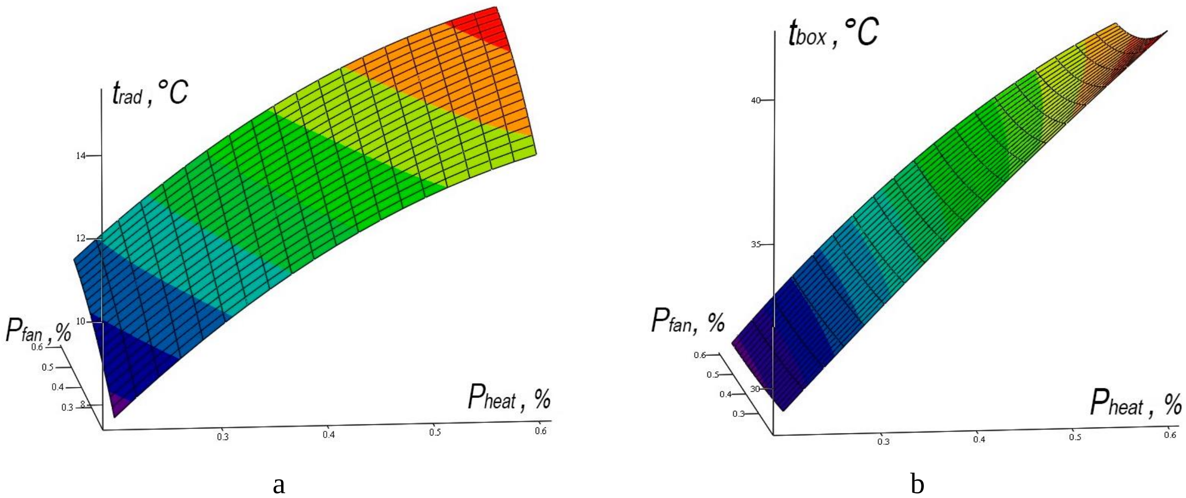


**Figure 2.** Experimental surfaces $t^{\circ}_{rad} = t^{\circ}_{rad}(P_{heat}, P_{fan})$ and $t^{\circ}_{box} = t^{\circ}_{box}(P_{heat}, P_{fan})$.

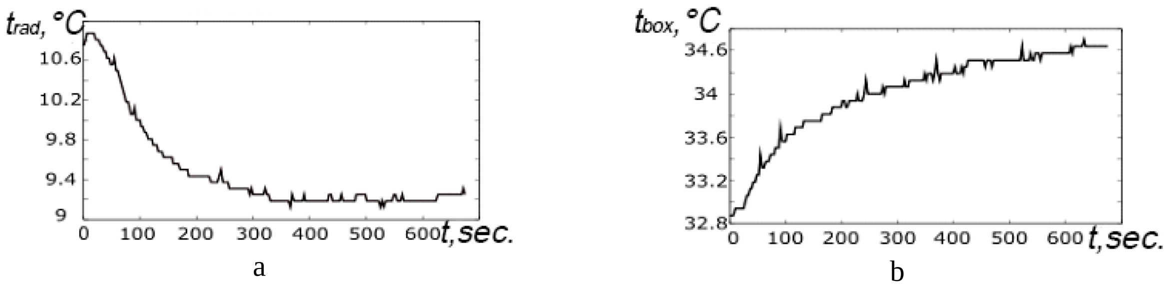


**Figure 3.** Transient responses $t^{\circ}_{rad}(t)$ and $t^{\circ}_{box}(t)$ while decreasing fan power consumption.

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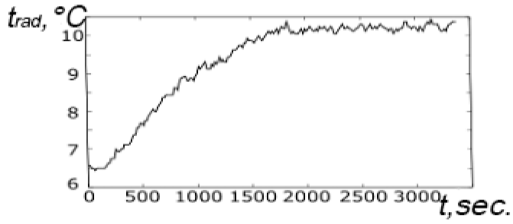
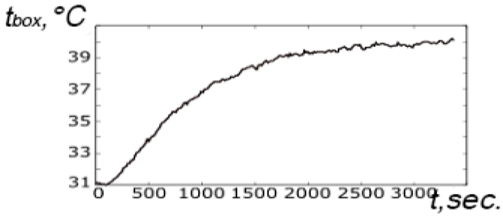


a



b

**Figure 4.** Transient responses $t^{\circ}_{rad}(t)$ and $t^{\circ}_{box}(t)$ with increasing heater power consumption.

## 3. Identification of mathematical model

To get features $t^{\circ}_{rad} = t^{\circ}_{rad}(P_{heat}, P_{fan})$, $t^{\circ}_{box} = t^{\circ}_{box}(P_{heat}, P_{fan})$ we will use numerical methods involving the use of a rectangular grid $t^{\circ}_{rad.ij} = t^{\circ}_{rad}(P_{heat.ij}, P_{fan.ij})$ where $i = \overline{1,3};\ j = \overline{1,3}$ Due to the small number of nodes, it is reasonable to use the well-known method of the interpolating Langrazh polynomial:

$$t^{\circ}_{rad/box}(P_{heat}, P_{fan}) = \sum_{i=1}^{3}\sum_{j=1}^{3} t^{\circ}_{rad/box.i,j} \cdot l_{i,j}(P_{heat}, P_{fan}) \quad (2)$$

where $l_{i,j}(P_{heat}, P_{fan})$ - basic Lagrange polynomials, which are defined as:

$$l_{i,j}(P_{heat}, P_{fan}) = \prod_{p=1, p\neq i}^{3} \prod_{q=1, p\neq j}^{3} \frac{(P_{heat} - P_{p.heat})(P_{fan} - P_{q.fan})}{(P_{i.heat} - P_{p.heat})(P_{j.fan} - P_{q.fan})} \quad (3)$$

According to experimental data, using (2) using the Mathcad software environment, dependencies were determined $t^{\circ}_{rad}(P_{rad}, P_{fan})$ and $t^{\circ}_{box}(P_{rad}, P_{fan})$:

$$\begin{aligned} t^{\circ}_{rad}(P_{heat}, P_{fan}) &= 125 \cdot P_{heat}^{2} \cdot P_{fan} - 156.25 \cdot P_{heat}^{2} \cdot P_{fan}^{2} - 39.0625 \cdot P_{heat}^{2} - 1.875 \\ &+ 125 \cdot P_{heat} \cdot P_{fan}^{2} - 101.5625 \cdot P_{heat} \cdot P_{fan} + 46.875 \cdot P_{heat} - 25 \cdot P_{fan}^{2} + 25 \cdot P_{fan} \end{aligned} \quad (4)$$

$$\begin{aligned} t^{\circ}_{box}(P_{heat}, P_{fan}) &= 24.2188 \cdot P_{heat}^{2} \cdot P_{fan}^{2} - 69.2189 \cdot P_{heat}^{2} \cdot P_{fan} + 5.063 \cdot P_{heat}^{2} - \\ &-0.7813 \cdot P_{heat} \cdot P_{fan}^{2} + 27.0313 \cdot P_{heat} \cdot P_{fan} + 33.063 \cdot P_{heat} + 1.125 \cdot P_{fan}^{2} - \\ &-5.05 \cdot P_{fan} + 22.715 \end{aligned} \quad (5)$$

As a result, nonlinear functions were obtained $t^{\circ}_{rad}(P_{rad}, P_{fan})$ and $t^{\circ}_{box}(P_{rad}, P_{fan})$, which should be linearized by expanding in a Taylor series before the first amendment to find the transmission coefficient:

$$\begin{aligned} t^{\circ}_{rad}(P_{heat}, P_{fan}) &= \left.\frac{\partial t^{\circ}_{rad}}{\partial P_{heat}}\right|_{ij} P_{heat} + \left.\frac{\partial t^{\circ}_{rad}}{\partial P_{fan}}\right|_{ij} P_{fan} = K_{heat.ij} P_{heat} + K_{fan.ij} P_{fan} = (-78{,}125 P_{heat} - \\ &-101{,}5625 P_{fan} - 312{,}5 P_{heat} \cdot P_{fan}^{2} + 125 P_{fan}^{2} + 250 P_{heat} \cdot P_{fan} + 46{,}875)\big|_{ij} \cdot P_{heat} + \\ &(-101{,}5625 P_{heat} - 50 P_{fan} - 312{,}5 P_{heat}^{2} \cdot P_{fan} + 125 P_{heat}^{2} + 250 P_{heat} \cdot P_{fan} + 25)\big|_{ij}\, P_{fan}; \end{aligned} \quad (6)$$

$$t^{\circ}_{box}(P_{heat},P_{fan}) = \left.\frac{\partial t^{\circ}_{box}}{\partial P_{heat}}\right|_{ij} P_{heat} + \left.\frac{\partial t^{\circ}_{box}}{\partial P_{fan}}\right|_{ij} P_{fan} = K_{heat.ij}P_{heat} + K_{fan.ij}P_{fan} = (10{,}125P_{heat}$$
$$+27{,}0313P_{fan} + 48{,}4375P_{fan}{}^{2}P_{heat} - 0{,}78125P_{fan}{}^{2} - 138{,}4375P_{fan}\cdot P_{heat} + \tag{7}$$
$$+33{,}0625)\big|_{ij}\cdot P_{heat} + (27{,}0313P_{heat} + 2{,}25P_{fan} + 48{,}4375P_{fan}P_{heat}{}^{2} - 69{,}21875P_{heat}{}^{2} -$$
$$-1{,}5625P_{fan}P_{heat} - 5{,}05)\big|_{ij}\cdot P_{fan}$$

And in the end, it became possible to write linear differential equations for the heat exchanger radiator temperature and the box temperature using the obtained transmission coefficients contained in functions (6) and (7):

$$\dot{t}^{\circ}_{rad}(t) = \frac{(K_{heat.ij}P_{heat} + K_{fan.ij}P_{fan}) - t^{\circ}_{rad}(t)}{\tau_{inertion}} \tag{8}$$

$$\dot{t}^{\circ}_{box}(t) = \frac{(K_{heat.ij}P_{heat} + K_{fan.ij}P_{fan}) - t^{\circ}_{box}(t)}{\tau_{inertion}} \tag{9}$$

## 4. Conclusion

Thus, complex nonlinear dynamic processes in drying plants can be represented by a linearized mathematical model. In the assembled prototype of the installation based on the obtained interpolants, we can conclude that the radiator temperature is well controlled by changing the power consumption of the fan motor and heater, and the box temperature can be controlled by the heater, but not by the fan due to nonmonotonicity $t^{\circ}_{rad}(P_{fan})$. But these measurements are not the initial data for the synthesis of control systems of other dryers. The result of this work is a method of obtaining $t^{\circ}_{rad}(P_{heat},P_{fan})$ and $t^{\circ}_{box}(P_{heat},P_{fan})$ for the purpose of writing and solving differential equations $F(t,t^{\circ}_{rad}(t),\dot{t}^{\circ}_{rad}(t))=0$ and $F(t,t^{\circ}_{box}(t),\dot{t}^{\circ}_{box}(t))=0$ only on a certain grid of temperature values, which will allow us to synthesize a controller based on a linearized model to establish the required quality of work.

## References

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