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Comparison of control methods for inverted 2-degree of freedom pendulum mounted on the cart

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Abstract. The paper considers and analyzes the existing methods of controlling systems with a deficit of control actions. Based on the analysis, it was decided to use linear controllers with feedback on the state vector of the system and to compare these methods - modal and linear-quadratic. For the study, a dynamic model was chosen, which is an inverted 2-degree of freedom (2DOF) pendulum mounted on a cart. By an iterative method, acceptable system of generalized coordinates that definitely describes the state of the model was selected. Kinematic relations that describe the position of the model in generalized coordinates were derived. Using the Lagrange procedure, a system of nonlinear differential equations describing the motion of a dynamic model was obtained. The procedure of model linearization about the upright equilibrium point was also carried out in order to synthesize control system. Based on the results of modeling, which was carried out by numerical integration method in the Matlab environment, conclusions were drawn on the applicability of these control methods and their effectiveness.

1. Introduction

The stabilization of the position of an inverted pendulum on a cart is one of the most common problems in control theory, since it is an unstable nonlinear system which has some of the degrees of freedom that are not directly controlled. Such complexities of the system lead to the fact that simple controllers [1-3], for example, PID-controller, do not provide the necessary transfer characteristics, since minimal dynamic error and settling time are required to stabilize the position of the pendulum.

The most common methods used to solve this problem are: modal regulator, linear-quadratic regulator [4-6], regulators based on artificial intelligence methods: genetic algorithms [7-9], fuzzy logic [10-14], neural networks [15-18]. However, neural network regulator needs a large amount of training data and there is an uncertainty of the choosing of its architecture. To implement genetic algorithms, time-consuming calculations are required. And fuzzy logic, although it is often applicable, is difficult to analyze for stability, and the synthesis of rules is usually a difficult task. To adjust the modal and linear-quadratic regulators, a mathematical model of the control object is required, according to which the roots of the stabilization system are adjusted in such a way as to provide the preassigned values of settling time and dynamic error. However, the advantage of the linear-quadratic controller is optimality because of its attachment to optimal control methods, which allows it to achieve better performance than the modal controller [6].

The purpose of this research is to compare the application of modal and linear-quadratic controller in the problem of stabilizing the upright position of an inverted 2DOF pendulum on a cart.

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2. The dynamic model of inverted 2DOF pendulum

To conduct a research, the dynamic model was chosen as it shown below in Fig. 1. In this model inverted 2DOF pendulum on cart was chosen with several assumptions: there is no movement of the trolley along Z axis, there is no rotation of the pendulum around its axis, cart and pendulum are absolutely solid bodies, and center of gravity coincides with geometric center of the cart. Then, to describe the laws of its motion, we use 4 generalized coordinates: θ, φ, x, y.

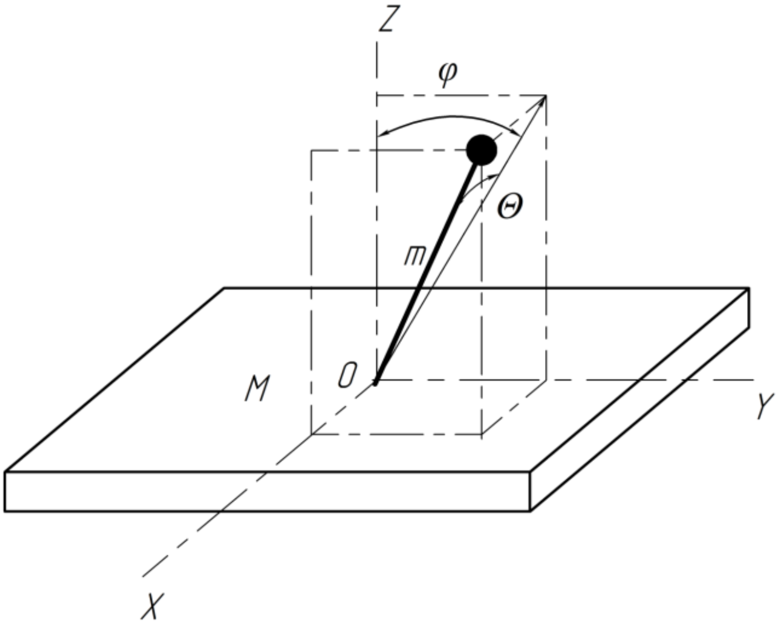

Figure 1. The dynamic model of an inverted 2DOF pendulum mounted on a cart

Here θ – is the angle of deflection of the pendulum from the ZOY plane, φ – is the angle of rotation of the pendulum in the ZOY plane, the zero position is measured from the Z axis, M – is the mass of the cart, m, l – is the mass and length of the pendulum, respectively, the force of gravity g is directed in the opposite direction to the Z axis.

Then we describe the location of the center of gravity of the pendulum in terms of generalized coordinates:

$$\begin{cases} x_p = x + l\sin\theta, \\ y_p = y + l\cos\theta\sin\varphi, \\ z_p = 0 + l\cos\theta\cos\varphi. \end{cases} \Rightarrow \begin{cases} \dot{x}_p = \dot{x} + l\dot{\theta}\cos\theta, \\ \dot{y}_p = \dot{y} - l\dot{\theta}\sin\theta\sin\varphi + l\dot{\varphi}\cos\theta\cos\varphi, \\ \dot{z}_p = -l\dot{\theta}\sin\theta\cos\varphi - l\dot{\varphi}\cos\theta\sin\varphi. \end{cases} \tag{1}$$

2.1. Nonlinear equations of inverted 2DOF pendulum

To describe the motion of complex mechanical systems with holonomic constraints, the Lagrange equations of the second kind are usually used [19-20].

To obtain the equation of the dynamics of the system using the Lagrange equations of the second kind, it is necessary to find expression for the kinetic energy, which for this system takes the form:

$$T = T_p + T_c = \frac{1}{2}m(\dot{x}_p^2 + \dot{y}_p^2 + z_p^2) + \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) +$$

$$+\frac{1}{2}m((\dot{x} + l\dot{\theta}\cos\theta)^2 + (\dot{y} - l\dot{\theta}\sin\theta\sin\varphi + l\dot{\varphi}\cos\theta\cos\varphi)^2 + (-l\dot{\theta}\sin\theta\cos\varphi - l\dot{\varphi}\cos\theta\sin\varphi)^2) =$$

$$=\frac{1}{2}m(\dot{x}^2+\dot{y}^2+l^2\dot{\theta}^2+2l\dot{x}\dot{\theta}\cos\theta-2l\dot{y}\dot{\theta}\sin\theta\sin\varphi+2l\dot{y}\dot{\varphi}\cos\theta\cos\varphi+l^2\dot{\varphi}^2\cos^2\theta)+\frac{1}{2}M(\dot{x}^2+\dot{y}^2). \quad (2)$$

Then the expression for the potential energy takes the form below:

$$\Pi = mgh = mgl(\cos\theta\cos\varphi+1) \quad (3)$$

As a result of applying the Lagrange procedure, we obtain a system of nonlinear heterogeneous differential equations of the second order unresolved for the highest derivative, describing the motion of a dynamic model in matrix form:

$$M(q(t))\ddot{q}+h(q(t),\dot{q}(t))=Q \quad (4)$$

$$M(q(t))=\begin{bmatrix} M+m & 0 & ml\cos\theta & 0 \\ 0 & M+m & ml\sin\theta\sin\varphi & ml\cos\theta\cos\varphi \\ ml\cos\theta & ml\sin\theta\sin\varphi & ml^2 & 0 \\ 0 & ml\cos\theta\cos\varphi & 0 & ml^2\cos^2\theta \end{bmatrix},\ \ddot{q}=\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{\theta}\\ \ddot{\varphi}\end{bmatrix},$$

$$h(q,\dot{q})=\begin{bmatrix}h_1\\ h_2\\ h_3\\ h_4\end{bmatrix}=\begin{bmatrix} -ml\dot{\theta}^2\sin\theta \\ -ml\dot{\theta}^2\cos\theta\sin\varphi-ml\dot{\varphi}^2\cos\theta\sin\varphi-2ml\dot{\theta}\dot{\varphi}\sin\theta\cos\varphi \\ ml^2\dot{\varphi}^2\cos\theta\sin\theta-mgl\sin\theta\cos\varphi \\ -2ml\dot{\theta}\dot{\varphi}\cos\theta\sin\theta-mgl\cos\theta\sin\varphi \end{bmatrix},Q=\begin{bmatrix}Fx\\ Fy\\ 0\\ 0\end{bmatrix}.$$

Where F_x, F_y – are the forces acting on the suspension point of the pendulum along the direction of the axes X, Y, respectively.

2.2. Linear equations of inverted 2DOF pendulum

To design control methods, it is necessary to reduce the previously obtained nonlinear equations (4) to linear ones. Let us linearize the equations of our system in upright position, about the point of unstable equilibrium: $\theta=0, \dot{\theta}=0, \varphi=0, \dot{\varphi}=0.$ Then we expand our system in a Taylor series in the first approximation. As the initial equations for linearization, a system of nonlinear equations obtained earlier was used. And through linearization we get the following expression:

$$M\ddot{q}+D\dot{q}+Kq=Q \quad (5)$$

We calculate the matrices M, D, K below.

$$M(q(t))=\begin{bmatrix} M+m & 0 & ml & 0 \\ 0 & M+m & 0 & ml \\ ml & 0 & ml^2 & 0 \\ 0 & ml & 0 & ml^2 \end{bmatrix},\ K=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -mgl & 0 \\ 0 & 0 & 0 & -mgl \end{bmatrix}.$$

$$D=\begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix},\ q=\begin{bmatrix}x\\ y\\ \theta\\ \varphi\end{bmatrix},\ \dot{q}=\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix},\ \ddot{q}=\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{\theta}\\ \ddot{\varphi}\end{bmatrix}.$$

3. Design of inverted 2DOF pendulum control system

Since the system has split into 2 groups of independent equations along the axes X, Y, respectively, we have the right to synthesize a controller along only one axis. Then, using the equation (6), representation of the system in the state space form was obtained:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & M+m & 0 & ml \\ 0 & 0 & 1 & 0 \\ 0 & ml & 0 & ml^2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & mgl & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u \tag{6}$$

Now the system of equations in the Cauchy form was represented below to obtain the system in the state space.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)g}{Ml} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix} u \tag{7}$$

Where $x_1 = x, x_2 = \dot{x}, x_3 = \theta, x_4 = \dot{\theta}, \dot{x}_1 = x_2, \dot{x}_3 = x_4$.

3.1. Modal regulator

Based on the above equations, we obtain the characteristic polynomial $M(s)$ of our system:

$$M(s) = s^4 + (\frac{1}{M}k_2 - \frac{1}{Ml}k_4)s^3 + (\frac{1}{M}k_1 - \frac{(M+m)g}{Ml} - \frac{1}{Ml}k_3)s^2$$

$$+(\frac{mg}{M^2 l}k_2 - \frac{(M+m)g}{M^2 l}k_2)s + (\frac{mg}{M^2 l}k_1 - \frac{(M+m)g}{M^2 l}k_1). \tag{8}$$

Then, having specified the necessary arrangement of the system poles from the requirements for the settling time of the transient process and dynamic error, we obtain the desired polynomial and find the value of the matrix of control coefficients. In our case:

$$K = \begin{bmatrix} -361.1167 & -180.5584 & -487.5520 & -92.6490 \end{bmatrix}$$

3.2. Linear-Quadratic regulator

1. To synthesize a linear-quadratic controller, it is necessary to solve the algebraic Riccati equation in the matrix form:

$$P_0 A + A^T P_0 - P_0 B R^{-1} B^T P_0 + Q = 0 \tag{9}$$

The specified transient characteristics are provided by the Q, R matrix values. For our system:

$$K = \begin{bmatrix} -63.2456 & -42.7889 & -167.6234 & -37.2252 \end{bmatrix}$$

4. Simulation of automatic stabilization of inverted 2DOF pendulum

Next, we simulated the stabilization process of inverted 2DOF pendulum by numerical integration of differential equations (4) describing the motion of our system with modal and linear-quadratic regulators. The following initial data were used in the calculation:

$M = 3\ kg$, $m = 0.3\ kg$, $l = 0.3\ m$, $\theta_0 = \varphi_0 = 0.2\ rad$, $x_0 = y_0 = 0$.

Numerical integration of equations (4) was performed in the Matlab environment using the ode15i solver. The simulation results are shown below in Figure 2.

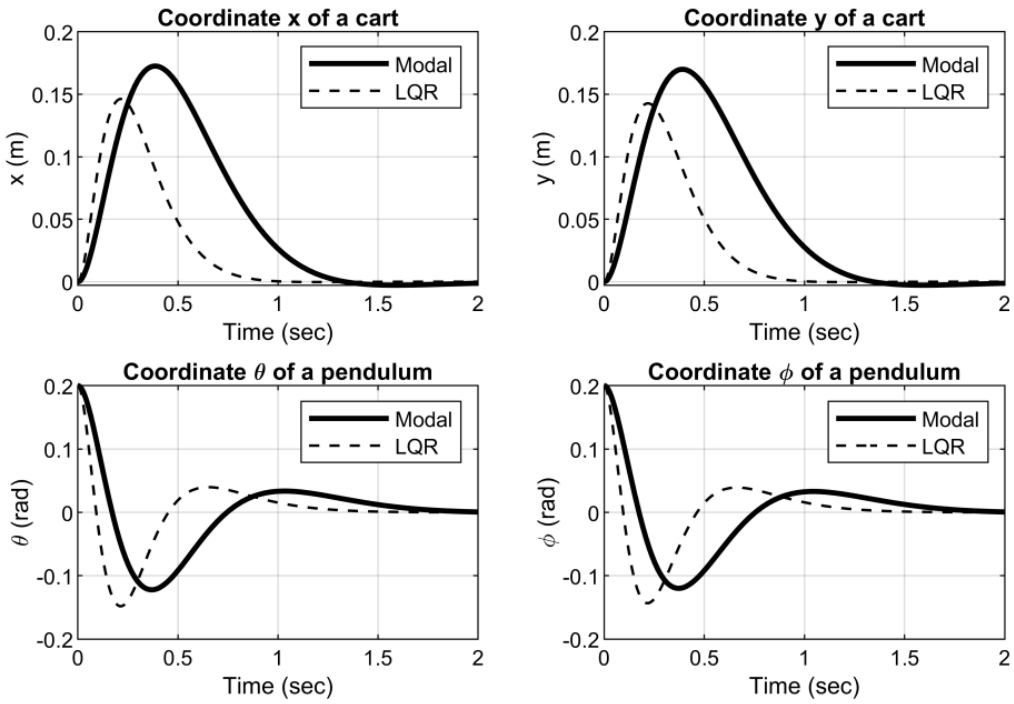

Figure 2. Graphs of the system transient process

From these graphs it can be seen that with a comparable dynamic error value (0.14 rad instead of 0.12 rad), the settling time of the linear-quadratic regulator is much lower (0.5 seconds), i.e. it allows you to achieve a better quality transition process.

5. Conclusion

In this work, a synthesis of controllers was made that allows stabilizing the position of inverted 2DOF pendulum on a cart, since such a control object is an unstable nonlinear system in which some of the degrees of freedom are not directly controlled. Stabilization was carried out using linear controllers with feedback on the state vector: modal and linear-quadratic. To synthesize them, that is, to determine the regulator coefficients, preliminary calculations were carried out to obtain a dynamic model of inverted 2DOF pendulum on a cart. Simulations have shown that the process of stabilization of the model can indeed be carried out using modal and LQ controllers, while the value of the dynamic error in the angle of rotation of the pendulum in the modal controller is rather larger than that of the linear-quadratic controller, and the linear-quadratic controller has significantly better settling time of the transient process, therefore, the LQR regulator copes with the task better.

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