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NATURAL CONVECTION OF NANOFLUID OVER A VERTICALLY STRETCHING SHEET EMBEDDED IN A DARCY-BRINKMAN POROUS MEDIUM

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ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ НАНОЖИДКОСТИ ВДОЛЬ ВЕРТИКАЛЬНОЙ РАСТЯГИВАЮЩЕЙСЯ ПОВЕРХНОСТИ В ПОРИСТОЙ СРЕДЕ ДАРСИ-БРИНКМАНА

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Аннотация. *В настоящей работе проводится моделирование естественной конвекции наножидкости вдоль нагреваемой растягивающейся поверхности, расположенной в пористой среде. Для описания транспортных процессов в пористой среде используется модель Дарси-Бринкмана, совместно с приближением Буссинеска для моделирования влияния выталкивающей силы. Исследования проводятся как на основе полных уравнений Обербека–Буссинеска, так и с использованием приближения пограничного слоя. Для реализации задачи, сформулированной в рамках приближения пограничного слоя, применяется метод локальной неавтомодельности. Полученные результаты отражают возможности использования различных подходов для моделирования свободно-конвективного течения вблизи вертикальной поверхности.*

Introduction. The analysis of viscous fluid flow along different surfaces is important, obtained results have several applications in the mechanical, electrical and industrial engineering, medicine and others engineering fields [1]. An addition of carbon nanotubes (CNTs) in the base fluid has some advantages from practical point of view [2]. Keeping these considerations in mind, the fluid flow of a viscous liquid with CNTs towards a stretchable surface embedded in a porous medium impacted by magnetic field is addressed. The Darcy-Brinkman model incorporates the effects of porosity.

To simulate the momentum and energy equations for the nanofluid flow, the single phase nanofluid model is used, while the full Oberbeck–Boussinesq differential equations are employed for the first approach and another approach is related with the boundary layer approximation. In the case of boundary-value problem for the Oberbeck–Boussinesq differential equations, the finite difference method has been used for numerical analysis. In the case of boundary layer flow approach, the governing expressions defining the flow are transformed into dimensionless system with the assistance of appropriate transformations. The numerical analysis for the dimensionless non-similar partial differential system is performed by using a local nonsimilarity technique [3, 4] up to the second truncation level in association with the numerical algorithm bvp4c (MATLAB built-in solver).

Finally, the quantitative consequences of emerging dimensionless quantities on nondimensional velocity and temperature for the boundary layer problem and full Oberbeck–Boussinesq boundary-value problem are graphically depicted. Furthermore, the dimensionless friction coefficient and heat transfer rate are also reviewed. Comparison between considered two approaches has been performed. It is concluded that non-similar modelling, as compared to similar models, is more general and accurate in convection investigations with buoyancy effects for viscous nanofluids flow.

Mathematical models. Here, we consider the steady, incompressible, two-dimensional, nanofluid flow towards a stretched surface embedded in a porous medium under an influence of uniform magnetic field. The flow is triggered by the stretching of the surface in the Darcy-Brinkman medium. Water is assumed as a base fluid while SWCNTs and MWCNTs are adopted as nanoparticles. Magnetic field is parallel in x-axis of strength B_0 (see Fig. 1). Let us assume that $v = c \cdot y$ highlights the stretching velocity in the y-direction where $c > 0$. Temperature T_h and T_∞ denote the stretching surface and ambient specified temperatures, respectively.

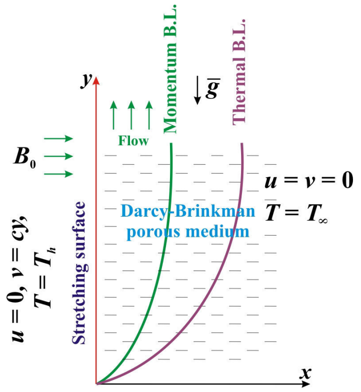

Fig. 1. Flow configuration for current model

Full Oberbeck–Boussinesq governing equations including single-phase nanofluid model can be written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{u}{\varepsilon^2}\frac{\partial u}{\partial x} + \frac{v}{\varepsilon^2}\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\varepsilon}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\mu_{nf}}{K}u \tag{2}$$

$$\rho_{nf}\left(\frac{u}{\varepsilon^2}\frac{\partial v}{\partial x} + \frac{v}{\varepsilon^2}\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\varepsilon}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\mu_{nf}}{K}v + (\rho\beta)_{nf}\, g\left(T - T_\infty\right) - \sigma_{nf}B_0^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{pm}}{(\rho c)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

In the case of boundary layer approach, the governing equations can be formulated as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (5)$$

$$\rho_{nf}\left(\frac{u}{\varepsilon^2}\frac{\partial v}{\partial x}+\frac{v}{\varepsilon^2}\frac{\partial v}{\partial y}\right)=\frac{\mu_{nf}}{\varepsilon}\frac{\partial^2 v}{\partial x^2}-\frac{\mu_{nf}}{K}v+(\rho\beta)_{nf}\,g\left(T-T_\infty\right)-\sigma_{nf}B_0^2 v \quad (6)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{pm}}{(\rho c)_{nf}}\frac{\partial^2 T}{\partial x^2} \quad (7)$$

Boundary conditions can be formulated taking into account the physical formulation of the problem and Fig. 1. Thermal properties of the base fluid (water) and carbon nanotubes can be found in Table 1.

Table 1

Thermal properties of the base fluid and carbon nanotubes

Materials	k ($W\cdot m^{-1}\cdot K^{-1}$)	ρ ($kg\cdot m^{-3}$)	c ($J\cdot kg^{-1}\cdot K^{-1}$)	$\beta\cdot 10^{-5}$ (K^{-1})	σ ($\Omega^{-1}m^{-1}$)	Pr
Single-walled carbon nanotubes	6600	2600	425	27	10^6–10^7	–
Multi-walled carbon nanotubes	3000	1600	796	44	$1.9\cdot 10^{-4}$	–
Water	0.613	997.1	4179	21	0.05	6.2

It should be noted that boundary-value problem for Oberbeck–Boussinesq partial differential equations has been solved using the finite difference method. The developed computational code has been validated using numerical data of other authors. While in the case of boundary layer governing equations, the local nonsimilarity technique combined with the second truncation level and bvp4c algorithm have been employed for analysis.

Conclusion. The performed numerical analysis allows studying natural convective flow and heat transfer of nanofluid along the vertical stretching surface immersed in the non-Darcy porous medium under an influence of horizontal magnetic field. Obtained results have illustrated the velocity and temperature fields defined using full Oberbeck–Boussinesq equations and boundary-layer approach.

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