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Software Package to Support Students' Research Activities in the Hybrid Intellectual Environment of Mathematics Teaching

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Abstract: The urgent need of society for creative and critical thinkers dictates the need to manage and develop the project-based and research activities of schoolchildren. Neural networks, as effective tools for solving complex, multi-component, multifunctional problems, have created an opportunity to develop tools for the content and quality assessment of project-based, research activities and personal achievements of each student with a dynamic and stratified sampling of complex knowledge, to identify individual educational paths, tools and instructions for research and creative activity reflected on the layers of an artificial neural network. The purpose of this study is to define and develop the concept of supporting a hybrid intelligent system of project-based and research activities in a comprehensive information and educational environment to create an applied intellectual technology that supports project-based and research activities and classifies the growth of schoolchildren's scientific potential. The paper considers the development of technologies for the pedagogical, algorithmic and information organization of ontological engineering and support models for project-based and research activities, as well as the growth of scientific student training based on the construction of an artificial neural network with a teacher and an array of training samples using expert systems and decision theory. It also defines the selection criteria, hierarchies and content of the generalized constructs of complex knowledge (modern achievements in science) by modeling creative activities while further predicting the growth of a student's scientific potential based on the layers of the neural network. The original technology of training samples used for constructing hybrid neural networks was determined using expert systems, the psychological method of parallel slices and clustering of the personal characteristics of schoolchildren. An innovative intellectual environment is being introduced into the practice of mathematical education in high schools in Russia. The study made it possible to create applied intelligent technology to support and display the dynamic profiles of the project-based and research activities of schoolchildren and act as a growth classifier of their scientific potential. In terms of concept implementation, the phased growth technology to analyze students' scientific potential was developed during the study of the generalized construct of complex knowledge. Individual educational workshops for school students to master project-based and research activities and display their dynamic profiles during the implementation of a hybrid intellectual environment for support and decision-making will allow school students to develop their personal potential, increase their educational motivation and allow researchers and educators to realize the potential of adapting modern achievements in science to school mathematics and create conditions for the modernization of educational programs in a developing digital environment.

Keywords: research activities; digital environment; hybrid intelligent system; students

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1. Introduction

The self-organization problems of school students and the development of creativity and critical thinking in the process of teaching mathematics dictates the need to integrate motivational- and emotional-volitional, research and metacognitive, social and personal strategies of behavior into a single integrity in the course of mastering mathematical content. Teachers and students should be involved in the digital environment of practice-oriented task-solving concerned with modern scientific achievement and each student's research activity in the classroom or extracurricular activities. This is possible using the functionality of intellectual systems against a background of growing and deepening the personal experiences of students and updating individual features based on the mechanisms of the above-situational activity in the context of the symbiosis of mathematical and computer modeling in an advanced information and educational environment. At the same time, the possibility of mastering complex knowledge (for example, adapting modern achievements in science to high-level mathematics) and interactive computer communication within the field of mathematics enhances opportunities for development, increases educational motivation, reveals connections with real life and practice and creates the phenomenon of synergistic effects in the development of complex mathematical knowledge. Besides, a key aspect of the growth of school students' scientific potential in learning mathematics may be the organization and support of project-based and research activities based on the functionality of intelligent control and the actualization of the stages and nature of generalized constructs of complex mathematical knowledge, phenomena and procedures in the context of students' individual learning pathways. First of all, the need for updating, accounting and processing individual personality data using the functionality of hybrid intelligent systems is increasing. This paper is, for the first time in the didactics of mathematics studies, concerned with one of the problem areas of digitalization and support using expert systems for recording and analyzing students' scientific potential, as well as organizing and supporting project-based and research activities. How and in what kind of educational settings can the efficient use of intelligent systems, including hybrid artificial neural networks, be possible?

The possible direction of the effective use of intelligent systems in mathematics education can be associated with improving the quality of project-based and research activities of schoolchildren in a rich information-educational environment of neural networks using expert database and knowledge systems. We will proceed from the fact that the system for measuring the quality of the project-based and research activities of schoolchildren consists of three parameters: scientific thinking, scientific activity and scientific communication [1], with a classifier for setting the level of scientific potential of each school student in a hybrid neural network. The database of modern achievements in science and their interpretation may become the ultimate attractor of the project-based and research activities of schoolchildren (fractal geometric elements, fuzzy sets, generalized functions, complex numbers, etc.), differentiated by the factors of reflection of the coordinate parameters of the essence cube of Smirnov [2], and based, among other things, on modalities of information perception and thinking profiles, alongside the development of pedagogical, algorithmic and informational support for ontological engineering and modeling of the creative activity of each school student.

Moreover, teachers will have a new opportunity to manage students' research activities in the classroom or extracurricular activities with hybrid neural network support, and will be able to effectively manage the scientific potential and creativity of each student as a new phenomenon in mathematics teaching. Teachers will also be able to determine and maintain the level of students' educational motivation and personal preferences in the context of overcoming difficulties as a means development of effective thinking and problem-solving skills. In a new approach, teachers will be able to participate in the expansion of the depository of modern achievements in science in the supporting software due to the realization of personal interests in the actualization of the symbiosis of mathematical and computer modeling. Finally, teachers will be able to actively engage in the process of professional collaboration with colleagues and scientists on the actualization of integration processes in science and education in an interactive information environment. This will be a teacher of the 21st century with significant potential for professional development and the opportunity for new didactics.

The introduction of a developed intellectual environment into the practice of teaching mathematics is being implemented in a pilot cluster of senior classes of secondary schools in Yaroslavl and Yelets (Russia) in relation to the organization of students' project and research activities. For this purpose, a depository and models of actualization of modern achievements in science (fractal geometry, the theory of encoding and encryption of information, fuzzy sets and fuzzy logic, cellular automata, Schwarz cylinder, etc.) have been developed.; diagnostic procedures for teachers' readiness to manage the development of complex knowledge and procedures by schoolchildren have been organized; professional retraining of teachers in the field of modern achievements in science has been implemented; models for monitoring the diagnostics of students' scientific potential growth have been tested; and pedagogical support for students' mastering of research activity processes and effective analysis of an intellectual environment have been organized.

Thus, the present study is an attempt to develop an intelligent control technology for the implementation of project-based and research activities of schoolchildren based on the development of the hierarchical content of generalized constructs of complex knowledge with synergetic effects during the deployment of individual educational pathways in an enriched information and educational environment. It will give teachers and students the ability to realize the effective integration of science and education, methods of computer and mathematical modeling and to engage in and solve practice-oriented tasks with interest and mental efficiency.

2. Literature Review

The study and significance of examples of self-organization in living and non-living nature through the processes of destruction and creation has shown that the increase in complexity in open and non-equilibrium systems is not a destructive mechanism, but rather a necessary transition to a new level of development involving more complex and ordered forms of organization, including in educational structures. Researchers (S. Beer, N. Wiener, John von Neumann and others) conclude that complexity is an integrating characteristic of the ability to self-organize, reaching certain critical levels, such as the ability to effectively develop and self-develop the thinking and personal qualities of a student. Philosophers, teachers and psychologists [3–11] have convincingly demonstrated that the effective development of a personality occurs through the mastering of complex knowledge (different levels of complexity depend on the personal development of students), creating situations of overcoming difficulties in the process of mastering knowledge, and a unified worldview based on a high degree of deployment of educational and professional motivation of students in a single network of interactions, independence and coherence. In cognition of the complex, the process of cognition itself "becomes a communication, a loop between cognition (phenomenon, object) and the cognition of this cognition" [9].

Therefore, it is mathematics education as a complex and open social system that carries the huge potential for updating the processes of self-organization and the positive manifestation of synergetic effects in different directions of research activities using intellectual systems: the development and education of personality, the orderliness of content and the structure of cognitive experience, communication and social interaction of the subjects on the basis of cultural dialogue and other manifestations of research activities, result in assessment processes as an integral attribute of modern education.

The issues around managing the educational process based on the increasing use of constantly evolving digital and network technologies are a key task in the intellectualizing of any modern educational system. Information tools and technologies are able to ensure the responsiveness, objectivity and efficiency of learning processes, as well as the assessment of the mastery of complex knowledge by students, creating the basis for the reflexive correction of cognitive processes in the direction of the individualization and personalization of educational routes in an enriched information environment [12–16]. A promising direction in solving the complex problems of managing the cognitive activities of students, aimed at increasing their level of self-organization and self-study with a continuous decrease in the degree of teacher participation, is the development of intelligent training systems based on artificial intelligence methods.

A scientific school of foreign authors have made a significant contribution to the development of this area, representing one of the key directions of the intellectualization of training systems. These include Dubois & Prade [17], Brusilovsky [18] and Osuga [19], among others. They laid the foundation for scientific research in the field of intelligent systems and artificial intelligence. Among the Russian researchers in this area, we note the work of Averkin & Yarushev [20], Kolesnikov [21], Tuktarova, Kamalova & Daukaeva [22], Pokalitsina [23], Popov [24], Petrushin [25] and Ostroukh [26].

In the presented training systems with elements of artificial intelligence, mathematical methods and algorithms for the expert evaluation of test characteristics and test tasks and methods of adaptive knowledge control based on fuzzy sets and schemes of plausible reasoning are proposed for knowledge assessment accuracy.

To date, the key players in the global e-learning market are such major companies as McGraw-Hill, Wiley, Istation, Area, Lyceum, ScootPad, VitalSource, DreamBox, Impelsys Fulcrum Labs, Knowre, Follett, Imagine Learning, Cerego, Realizeit (USA), Pearson, Curriculum Associates, CogBooks (UK) and Mathspace (Australia). A brief overview of current technological developments, platforms and online services that provide intelligent management of students' learning and cognitive activities is shown below (Table 1).

Table 1. Intelligent adaptive learning systems.

Name, Year of Market Launch, Manufacturer	Didactic Opportunities and Advantages
ActiveMath (2004, Germany)	An intelligent web-based system for teaching mathematics, created in addition to traditional classroom teaching, as well as to support con-tinuous learning and adaptable to the user. The environment dynamically generates interactive thematic courses adapted to the goals, preferences, opportunities and knowledge of the student. While the current version of ActiveMath adapts to the cognitive states of the student, the affect-sensitive version of ActiveMath will respond to both the cognitive and affective states of the students [27].
AutoTutor (2001, USA).	The program is designed to study physics and computer literacy and is used for students of grades 8–11. The adaptability of AutoTutor is achieved by assessing the students' progress and dividing them into groups with different levels of education. The system uses a dialog agent to facilitate verbal thinking, students' answers to questions, conceptual understanding and interaction in an accessible language [28].
Century Tech (2013, UK).	Century Tech is advertised as an intelligent tool based on artificial intelligence, cognitive neuroscience and data analysis. The cloud-based educational platform measures gaps in students' knowledge, their learning speed, habits and preferences—even when the information passes from short-term memory to long-term memory. It includes online educational resources in mathematics, science and English for primary and secondary school students and teachers [29].

Table 1. *Cont.*

Name, Year of Market Launch, Manufacturer	Didactic Opportunities and Advantages
DreamBox Learning Math (2006, USA).	This is an adaptive mathematical game program that meets the state standards. Adaptive intellectual learning solves the tasks of tracking the actions of each student and evaluating the strategies used to solve problems while correcting the lesson material, the level of complexity, the number of fairy tales and the pace of study [30].
eTeacher (2000, Israel).	An intelligent computer program that provides personal assistance in e-learning. The program automatically determines the student's learning style based on the Bayesian model, taking into account the preferred type of reading material, the number of exercises performed and participation in chat rooms and forums. The program creates student profiles and offers an individual action plan designed to facilitate the learning process [31].
Geekie (2011, Brazil).	A web platform that provides personalized educational content using adaptive learning technology. The platform includes video lessons, tests and practical tasks. Students take a test and determine the ultimate goal of learning; the program then selects the appropriate content for learning. There is an opportunity to go back and repeat the theory and to choose the pace of learning. The program constantly collects data about students, so that teachers have the opportunity to quickly make adjustments to the course [32].
Knewton (2008, USA).	A platform for personalized learning in basic natural science, engineering and mathematical subjects, on the basis of which applications with an adaptive function are developed. The analytical system collects detailed information about students' knowledge and their degree of assimilation of certain concepts and makes conclusions about the features of the participant on the basis of collected data, and with this in mind corrects goals, forms an optimal learning strategy and implements the appropriate setting of content parameters. At the same time, the personalization system makes analytical predictions about students' success (work speed, probability of achieving the goal, etc.) and maintains personal statistics at all levels of education [33].
Palearne (2019, Israel).	A personalized ecosystem for adaptive learning. Based on the analysis of students' emotional behavior, the educational content is determined and the learning strategies are selected throughout the course. The errors context is identified and appropriate assistance is offered. The platform is aimed at improving course efficiency.
Plario (2018, Russia).	An online system of adaptive mathematics teaching for schoolchildren and first-year students. The system is created as an online platform and individually selects the learning trajectory depending on the level of each student's training and progress. Pario conducts rapid diagnostic testing and uses a genetic algorithm to create a digital double of a student. The adaptive element is based on the Bayesian Knowledge Tracing algorithm. In the process of completing tasks, the system evaluates the student's progress and corrects the learning trajectory. The development of educational material is based on gamification elements [34].

Table 1. *Cont.*

Name, Year of Market Launch, Manufacturer	Didactic Opportunities and Advantages
Smart-Tutor (2002, Hong Kong).	An intelligent mathematics teaching system implemented for distance learning. A feature of the Smart-Tutor system is its flexibility and versatility to meet the individual needs and abilities of students. The main architecture of Smart-Tutor consists of six components: course manager, question bank, student model, content structure, expert model and user interface [35].
Toppr (2013, India).	Toppr's flagship product, Learning Application, is designed for extracurricular activities and is useful in preparing for olympiads and exams. It covers all the main subjects and is a universal learning tool. The adaptability of learning is ensured thanks to the mechanism of recommendations: each student receives an individual set of tasks depending on his progress [36].
Yixue Education (2015, China).	A unique artificial intellectual tutor works according to the principle of determining knowledge gaps and composing an effective program of express preparation for exams in mathematics [37].
ZOSMAT (2009, Turkey).	ZOSMA is designed to guide students at every stage of the learning process using HTML pages, interactive applications, animation, audio and video. The system supports the creation of adaptive tests through a bank of questions, as well as the analysis of decisions and mistakes of students. The intellectual mentor adapts to the individual needs of the student. ZOSMA can be used both for individual training and in a real classroom under the guidance of a teacher [38].

Intelligent control of the learning process in the "teacher–computer–learner" triad requires a hybrid approach in the symbiosis of mathematical and computer modeling of content and hierarchies of knowledge and procedures, interactive intellectual training activities in information environments integrating the functions of expert systems, fuzzy logic, sophisticated neural networks and genetic algorithms. The issue of developing a set of methods, algorithms, tools and characteristics of pedagogical and ergonomic quality using technology and software means that ensuring innovative engineering procedures to build hierarchies of complex knowledge in a computer-imitation environment during the organization and support of project-based and research activities with the effect of growing the scientific potential of a school student is still open to further research. A distinctive feature is that this area is characterized by a large degree of non-certainty, randomness, instability and the influence of various factors, as a result of which the construction and use of accurate learning models based on phenomenology, classical mathematical apparatus and computer modeling is often ineffective. As a result, more and more work by Russian and foreign authors uses a wide range of artificial intelligence methods to solve the complex problems of developing automated intelligent training systems, including expert systems, artificial neural networks, fuzzy logic, genetic algorithms, as well as hybrid approaches with other mathematical directions [39–42].

In this study the intelligent control of the project-based and research activities of students is understood as the use of hybrid intelligent system functionality in conditions of openness to external influences and factors, as well as the synthesis of mathematical and computer-dimensional modeling used to update personalized feedback on the creative and cognitive processes of mastering complex mathematics.

The purpose of the study is to develop methodological, theoretical and technological foundations for the creation and functioning of an instrumental hybrid intellectual environment of organization and support for each student's project-based and research activities in the structure of general education based on the development of complex knowledge generalized constructs, consisting of:

- A software complex using neural networks, expert systems and fuzzy modeling of parameters, rules for structuring and the didactic field ontologies of data and knowledge for complex knowledge generalized constructs during the interactive communication of a subject, intellectual system and expert (teacher) with further synergistic effects and the development of scientific potential.
- The technology of organization and the development of pedagogical, algorithmic and informational support for the ontological engineering and modeling of project-based and research activities while displaying the ontology of students' dynamic profiles and the manifestation of mathematics education synergy based on the mastering of complex knowledge generalized constructs (adaptation of modern achievements in science) in a rich information and educational environment.

The basic idea is the development and manifestation of key aspects of the synergetic effects phenomenon and personality development in teaching mathematics, as well as the mastering of complex knowledge based on the adaptation of modern achievements in science in a rich information and educational environment using hybrid intelligent systems that make it possible:

- to update the stages and study the characteristics of complex mathematical knowledge, phenomena and procedures based on the digitalization of the educational environment, students' understanding of mastering the processes and study of complex knowledge and students' self-organization characteristics;
- to create the conditions for communication, individual choice and effective dialogue preferences for mathematical, information technology, natural science and humanities learning;
- to ensure the visual modeling of the underlying modes of development of students' experience and personality qualities in the context of research tasks creation and the development and implementation of hierarchical complexes and banks in the direction of generalized constructs identifying the essence of modern scientific knowledge;
- to identify the attributes of content, processes and mutual self-organization actions (parameterization and generalization, attractors, bifurcation points, attraction basins, iterative procedures, etc.) during the development and study of problem areas in mathematics, as well as the assessment of knowledge and competencies based on the actualization and adaptation of modern achievements in science and the use of hybrid intelligent systems in the context of synergistic and fractal approaches.

3. Methodology

The design of a training sample of a hybrid intellectual environment was carried out on a sample of secondary school students (n1 = 125) aged 15–17 years (input sample). The second sample (output sample) consisted of high school students of the Strategy Center for Supporting Gifted Children (n2 = 125). The age of the students was 19–21 years. Under these conditions, each learning sample for each of five neural networks was identified by the "cross-sectional method" based on sample experiments of student groups (aged 15–17 years) and students from higher educational institutions (aged 19–21 years) regardless of gender, according to the "precedent-response" line. A necessary condition for the sample experiments of respondents is the redundancy of the volumes of input and output samples for the adequacy of determining a statistically reliable correlation of the "precedent-response" line based on the results of expert forecasts and the state of the output sample. The final coordination of the expert forecast of scientific potential growth for the input sample respondents with the diagnostics of the output sample's scientific potential required the intersection of average confidence intervals and the diagnostics of volume redundancy for both samples (250 respondents each) with the selection of a required volume (125 respondents each) for the five neural networks (25×5 each).

Entrance testing of trainees in the conditions of using a hybrid intellectual environment was carried out using the modified "Thinking Profile" method by J. Bruner [42]. The parameter diagnostics for the scientific potential of trainees were carried out using the following qualimetric methods: Amthauer's intelligence structure test (parameters are logical processes and intellectual-cognitive activity), Johnson's creativity questionnaire (creativity), Alekseev's "Individual styles of thinking" test, Gromova's thinking styles, Rokeach's method of value orientations (significance and value orientations), the Bass Orientation Inventory (scientific communications), the diagnostics of self-development levels developed by Berezhnova (motivation of self-realization), the Mehrabian Achieving Tendency Scale (MATS) (motivation to achieve results), and the test of distant associations by Mednik (effectiveness of creative activity) [42].

To complete the database of research tasks for a hybrid intellectual learning system we used the faceted classification method of Ranganathan. It involves the parallel division of objects set according to one attribute into separate, independent subsets. The following "zones of modern achievements in science" acted as complex knowledge generalized constructs for the management of students' research activities in a hybrid intellectual learning environment: fuzzy sets, elements of fractal geometry, complex numbers, optimization models and methods, random variables and processes.

The developed system was built on the basis of the LSTM neural network architecture and deep learning algorithms. A recurrent network with a long-term short-term memory (LSTM) cell was chosen as the architecture of the neural networks, which is the most powerful and well-known tool for recognizing patterns in data sequences, such as numerical time series data. LSTM is a memory cell, a computing unit that replaces traditional artificial neurons in the network's hidden layer. The ability to efficiently link memory and input remote data using these network memory cells provides a dynamic understanding of the data structure over time. Thus, this network allows one to take into account time and sequence and is able to remember information for long periods of time. At the moment, this architecture is the most powerful tool available for forecasting tasks. The hybrid neural network is characterized by:

- the presence of an input layer of nine linguistic variables ("creativity" with terms T1: "low creativity", "medium creativity", "high creativity") and a universe of fuzzy variables, determined by the diagnosis of personality creativity;
- the presence of an output layer and three hidden layers of neurons: the input (output) vector consists of nine neurons formalized according to three levels of nine parameters of the initial (final) state of the quality and success of students' research activities;
- the fuzzification of input variables and the choice of sigmoidal function of neuron activation;
- change in the range of weights and threshold levels based on the selected activation function;
- the classification of success levels and the quality of students' research activity in the context of the parameters' vector variability of output layer and the de-fuzzification of output variables;
- the selection of training sample and learning process with the teacher by the method of error back-propagation;
- the collection and processing of feedback data on the levels of results growth of scientific potential and the parameters of students' scientific thinking, communication and activity dynamics.

To develop a hybrid learning intelligent system, Python3 language tools were used, in particular NumPy, SymPy, scikit-fuzzy and the JupyterLab environment. Separate modules were implemented using the Julia language. To integrate the components of the artificial intelligence subsystem with the user interface, tools such as the Mercury framework (for modules implemented in Python) and the Mux/Interact.jl framework (for modules implemented in Julia) were used. The use of various programming languages is due to the variety of tasks solved in the framework of this study. To solve the problem of assessing the indicators of design and the development of students' research activities in mathematics, methods of mathematical modeling, the analysis of didactic systems, artificial intelligence and the theory of dynamic systems were used.

4. Results

4.1. Innovative Technology for Research Activities of Schoolchildren Based on the Use of Hybrid Intelligent Systems

The development of a hybrid intelligent system (symbiosis of artificial intelligence and expert systems) for managing the quality and success of students' educational project-based learning and research activities in the context of their scientific potential and personal development growth is based on the fact that any program success consists of studying three components:

- personal adaptation (the establishment of individual readiness for undertaking research activities, which will determine the choice of an individual educational path of research activities for the development of complex mathematical knowledge (modern achievements in science) in the form of updating the founding cluster of research tasks; the establishment of the personal preferences of each school student in the development of mathematics knowledge based on the conscious choice of the research subject);
- pedagogical support and the support of expert systems (development of a database and knowledge of project activity topics in the form of a hierarchical tree of subtasks of consolidated clusters of research tasks; instructions and rules for the development of research activities detailed by the levels of students' scientific potential growth (search and reproductive, empirical, theoretical, creative); creation of data and knowledge of the information support available for projects (popular science publications, presentations, etc.) distributed according to the research topic);
- the selection of the architecture, parameters and functionality of the neural network.

Let us highlight the clusters of readiness for educational research and project-based activities of schoolchildren based on the following criteria [43]:

I. Personal:

- scientific thinking (creative and logical acts, principles and styles of thinking);
- methods of information processing (abstract/symbolic, verbal reasoning, visual/figurative, objective/efficient, creative);
- experience and personal qualities (need and motivation for research activities, creative independence, self-actualization, self-organization, self-realization and self-esteem).

II. Activity:

- scientific activity (study experience (analysis, synthesis, associations, analogies, collection, study and processing of information); variability of data, limitations of experience; improvisations; reflection; trial and error; actions in conditions of uncertainty; problem setting and search for contradictions; experimental set up; hypothesizing).

III. Communicative:

- scientific communication (intercultural interaction, social verification of new knowledge, etc.; ability to work in small groups (teamwork); tolerance of other opinions and social self-organization; information exchange; degree of social roles—idea generator (performer); cultural dialogue within mathematics, information technology, natural science and humanities).

IV. Effective:

- technological readiness, accuracy of practical implementation, practical significance of the project, obtaining new or by-product results, approbations, public and virtual presentations, publications, research projects.

At the same time, expert activities are required to characterize, formalize and technologize the procedures for diagnosing the state of basic parameters at the initial, current and final assessment stage (according to the criteria of effectiveness and levels of research activity). The quality and success of students' research activities is determined by the current indicators of personal growth and the levels and classification criteria of parameter qualimetry, each of which has expressiveness and characteristics in three positions (low, medium, high), determined by a neural network based on parameter success and the actualization of a pedagogical and information support block (expert assessment).

The technology behind the procedures for the selection and diagnosis of basic parameters at initial, current and final student assessments (according to the criteria of effectiveness and levels of research activity) is considered here for the first time in mathematics teaching.

4.2. Classification Levels of Qualimetry Parameter Dynamics in Neural Network (Expert Assessment)

The research activities of school students are developed according to the logic of the successive deployment of cognitive activity mastering levels and the growth of scientific potential. Based on historiogenesis [44–47], we propose the following levels typology for students' project-based and research activities in a hybrid intellectual environment:

- search and reproductive (SR): self-actualization("I am interested in this")—characterizes and manifests itself in the severity of value-based and personal-adaptive characteristics of the cognitive activities of students in terms of selection, updating of personal samples and the development of standards and phenomenology samples of visual modeling of a complex knowledge generalized construct (modern achievements in science) and the results of diagnostic procedures for significance and value guidelines, the choice of methods of activity to disclose a separate quality of generalized essence (meaningful or procedural component—Figure 1); search and analysis of accessibility, difficulty and the personal significance of stages identification of scientific knowledge, research methods and mechanisms for the implementation of intra-subject and inter-subject relations on the basis of professional-oriented and research approaches; personality traits of self-determination and self-organization, mastering the principles and styles of scientific thinking: induction, deduction, insight, analogy, inversion and anticipation.

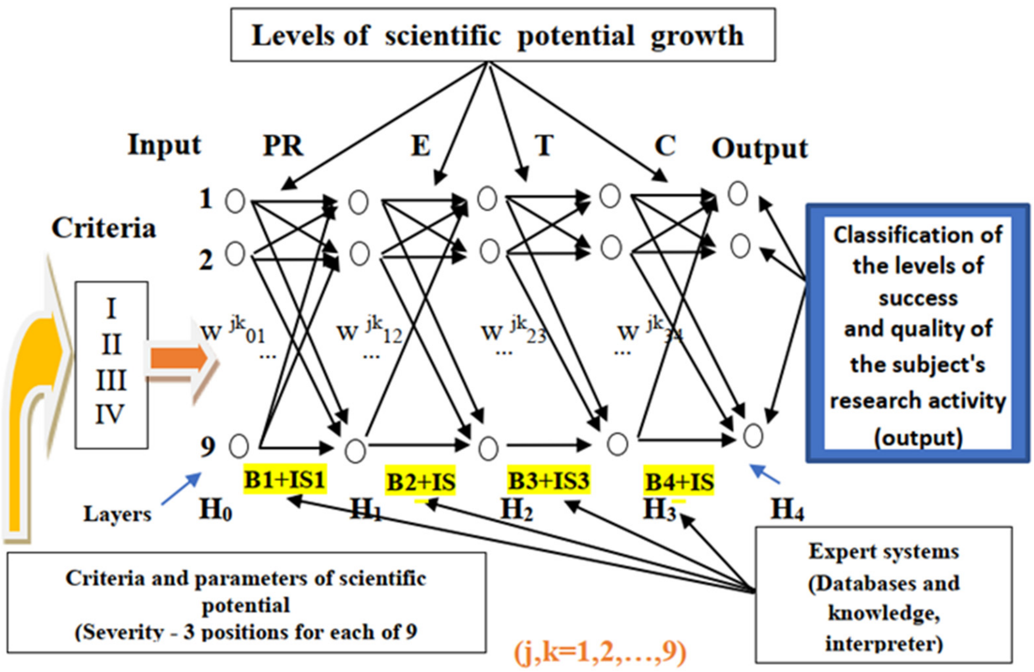

Figure 1. Dynamics of parameters and variation of levels of success and quality of students' research activities in the layers of a hybrid neural network.

This level is ensured by databases (expert systems) of samples, standards and problem areas of modern scientific knowledge with the potential for the symbiosis of mathematical and computer modeling, methodology databases for organizing project-based and research activities, and the capabilities of information and communication support tools based on the results of students' scientific potential diagnostics. It involves various forms of pedagogical support and cooperation, the organization of discussions, conferences, seminars, and openness of the information and educational environment.

- empirical (E): self-determination("what can I do")—manifests itself in the implementation of empirical tests and the design of visual models by means of experimental mathematics by methods of mathematical and computer modeling of particular manifestations of complex knowledge generalized constructs based on cognitive independence and actions actualization, competencies and personal qualities. It is characterized by the identification of empirical relationships and the continuity of empirical generalizations, the trial and error method, the variety of tasks and hypotheses, the search for bifurcation points and attraction pools, attractors, iterative processes and fluctuation dependencies, the comparative analysis and the selection of priorities in learning methods and content, the information-based means of supporting project-based and research activities, an awareness of the functionality of mathematical content and a level of ability in computer design, particularly in terms of the manifestation of a generalized construct and the correction of its parameters and conditions, adequacy and efficiency in correlating the goal-result orientation.

This level is ensured by the content of various information environments: Mathcad, MathLab, Mathematica, Maple, GeoGebra, Creator and others with the potential for the symbiosis of mathematical and computer modeling based on the results of school students' scientific potential diagnostics and the first research results. It involves various forms of pedagogical support and cooperation, the implementation of project-based and research activities for schoolchildren, the organization of discussions, conferences, seminars, publications and presentations, and openness of the information and educational environment.

- theoretical (T): self-organization ("I am able to control the process")—manifests itself in the technology design and organization of students' research procedures to master innovative manifestations and adapt the essence of complex knowledge generalized constructs. It is realized during the deployment of its underlying stages and on the basis of updating the techniques of creative cognitive initiative and the dialogue of mathematics, information technology, natural science and humanities, identifying laws and rules in the process of project research activities. At the same time, forms, methods and means of mathematical and computer modeling to support the process of mastering complex knowledge generalized constructs are developed and implemented, which are adequate for their local, modular and global manifestations of ground procedure deployment.

This level is ensured by databases (expert systems) of samples, standards of theoretical analysis of modern scientific knowledge with the potential for the symbiosis of mathematical and computer modeling, databases of scientific communication methodology and samples of scientific thinking in conducting project-based and research activities and the capabilities of information-communication support tools based on the results of school students' scientific potential diagnostics and the results of theoretical research analyses. It involves various forms of pedagogical support, cooperation and the implementation of project-based and research activities of schoolchildren, the organization of discussions, conferences, seminars, publications and presentations, and openness of the information and educational environment.

- creative (C): personality self-development ("I can do something new")—characterized by the content and characteristics of the transfer of innovations into the mass practice of mathematics learning mastering in schools, the search for and design of research by-products and possible applications to other sciences, the integration of the individual and the social in the design of innovative generalizing constructs, the manifestation of synergistic effects, information exchange, the socialization and verification of innovative activity, and the characteristics, parameters and indicators of the formation and diversity of students' individual educational pathways.

This level is ensured by databases, samples, standards of creative research in modern scientific knowledge with the potential for the symbiosis of mathematical and computer modeling, databases of verification samples of scientific results and scientific communication, samples of effective methods of scientific thinking in conducting project-based and research activities, possibilities for information and communication support based on the monitoring and diagnostic results of school students' scientific potential and the creative results of students' research activity. It involves the various forms of pedagogical support, cooperation and the implementation of project-based and research activities of schoolchildren, the organization of discussions, conferences, seminars, publications and presentations, and openness of the information and educational environment.

Such classification levels of qualimetry parameter dynamics in a neural network are considered here for the first time based on real-life experience of students' research activity in a secondary school classroom.

The architecture of a hybrid neural network, the dynamics of the parameters and the variation of levels of success and quality of schoolchildren's project-based and research activities are shown in Figure 1. This is the original modeling for the construction of a hybrid neural network.

The differentiating factor of the database elements (B1, B2, B3, B4) of scientific knowledge can be the level of complexity in the interpretation of generalized constructs (level 2–3—π). Thus, if there are, for example, 10 generalized constructs of complex knowledge (x), then each of them represents at least eight modifications designed in the form of problem setting and research in the course of a school student's individual educational pathway determined at μ levels (in our case, levels B1–B4). The content of a formal presentation database and knowledge would be implemented by experts, and the source environment and software complex would be developed by programmers.

Each variation of a generalized construct is the end point of a hierarchical tree of subtasks of ground clusters of research tasks arranged in logical chains with instructions (I_j), references (R_j) and an information support block (S_j): ($IS_j = I_j + R_j + S_j$, j = 1, 2, 3, 4). The hierarchical tree is based on (B1) a database of data and knowledge of educational research projects with a similar infrastructure (the number of such projects for our specification of 10 generalized constructs can be, for example, no more than 30). At the output of the hierarchical tree (B4), the database and knowledge of educational research projects in this case will not exceed 810 (the minimum value will be 32 for two generalized constructs with two variations at each level). Thus, the formula for the output level (parameter E) will be as follows: $E = x\pi\mu$.

At each level of the hierarchical tree, the experts form a level studying an annotated variable database which is a logical continuation of the previous specific block, and thereby construct the content of next level—B1, B2, B3, B4. In addition, the base of data and knowledge of information support IS_1, IS_2, IS_3, IS_4 according to generalized factors is formed. The criteria for selecting databases and the topic of variable logical continuation of students' research activities are determined by the state and growth dynamics of the scientific potential parameters (H), the preferences for information processing methods—modality of thinking (M)—and the success of students' research activities (K). Therefore, at each bifurcation point for the transition from one growth level of scientific potential to another, a student can choose to continue their project-based and research activities in five directions (PM)—factor (M): sign-symbolic, figurative-geometric, concrete-activity, information-computing, historical-genetic. Thus, as a result, an individual educational pathway of research activity is formed in accordance with the student's personal development (factors (H), (M) and (K) and factors of intelligent control). Thus, the extended formula for the output level of the hypothetical volume of database variations and the knowledge of research projects is: $E = x(5\pi)\ \mu$.

The interaction of a person with the outside world activates their internal potential, which is the basis of their self-knowledge, self-regulation and self-activity that ensure their personal self-development. The knowledge and values that are mediated in the process of learning mathematics can be accepted and become the property of a student when they are actively processed and absorbed not at an individual level, but as the content of communication and activity in the group. So, they are integrated into the totality of all information that the group has. In this regard, special attention in terms of the structure of supporting the students' project-based and research activities will be given to the functioning of the research ground cluster and the manifestation of the generalized construct, which is the most important source of their self-actualization and development and an incentive for creative activity and further personal growth. When organizing a creative group activity, it is necessary to create the conditions for generating a plurality of problem solving based on information enrichment, intellectual tension and a low degree of behavior regulation (PM).

Thus, new technology for students' research activity and effective practice in group work has been defined which has the opportunity to show above-situational activity and implement the methods of activating creative thinking in the context of mutual dependence, updating the dynamics of the creative process (intuition, verbalization, visual modeling, formalization, reflection, verification) based on the synthesis of convergent and divergent thinking.

The integral structural-logical model of the interdependent parameters of a homogeneous hybrid intelligent system for managing the project-based and research activities of schoolchildren is shown in Figure 2.

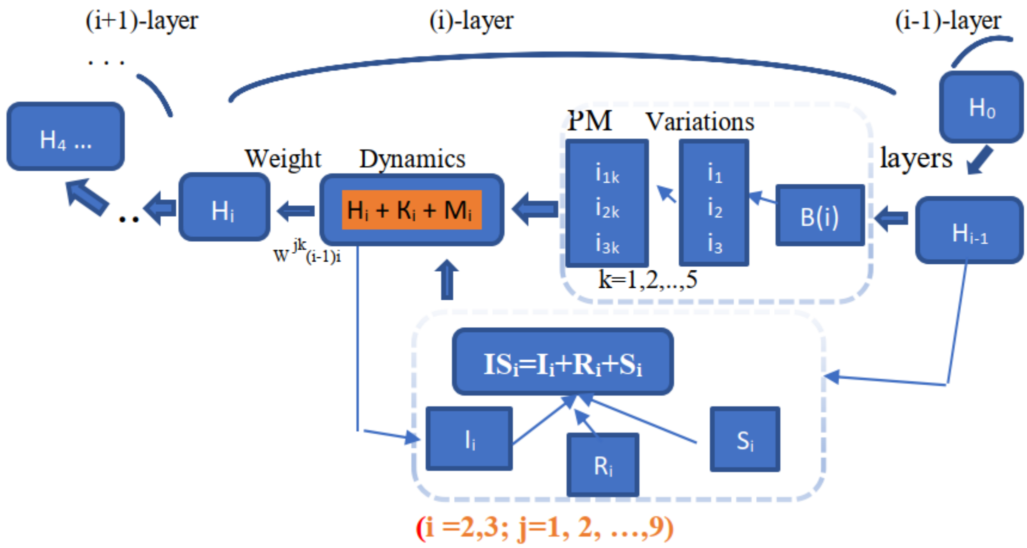

Figure 2. Interdependence of the parameters for managing the project-based and research activities of schoolchildren.

The success of block chain passing in students' research activity (individual educational pathway) on the basis of personal preferences and achievements in a saturated information and educational environment stimulates the processes of self-organization and mathematics education synergy in the study of complex knowledge in the context of detecting the essence of a complex knowledge generalized construct (modern achievements in science).

Thus, the development and effective practice of pedagogical, algorithmic and information support for the ontological engineering and modeling of developmental success in project-based and research activities by school students in a hybrid intellectual quality management system and the growth of their scientific potential unfolds in the symbiosis of three meaningful lines:

- personal-adaptive: the selection and implementation of an individual educational pathway of project-based and research activities for the development of a complex knowledge generalized construct (modern achievements in science) in the form of ground cluster updating of research tasks; the implementation of the personal preferences of each student in the development of mathematics learning by consciously choosing the research topic, corresponding with the dominant modality of personality perception choice and the breadth of personal experience; the possibility of cultural dialogue and communication, the development of intellectual operations and personal qualities (including the growth of creativity and criticality) in an open information and educational environment; experience and performance in the field of scientific activity (discussions, debates, disputes, approbations and experiments, public and virtual presentations, publications, etc.);
- pedagogical and expert systems support: databases and knowledge of project activities in the form of ground cluster ontologies of hierarchical tree research tasks for a complex knowledge generalized construct; instructions and rules for the development of project-based and research activities detailed by the levels of school students' scientific potential growth (search and reproductive, empirical, theoretical, creative); databases and knowledge of support block information (links, scientific articles and monographs, references, presentations, dissertations, etc.) distributed by research topic; the quality of mastering of scientific research methodology; the identification of the essence and effectiveness of mathematical procedures and the generalized construct of modern knowledge adaptation stages; mathematics education synergy based on the adaptation of modern achievements in science;
- architecture, parameters and functionality of a neural network: a hybrid neural network consists of an input layer of nine linguistic variables (for example, creativity with terms T1—low creativity, average creativity, average creativity) and a universe of fuzzy variables determined by the individual creativity diagnostics (questionnaires, surveys, etc.); an output layer and three hidden layers of neurons—the input (output) vector consists of nine neurons formalized in three levels of nine parameters of the initial (final) state of quality and the success of project-based and research activities of schoolchildren; the fuzzification of input variables (Mamdani algorithm) and the selection of the sigmoidal function of neuronal activation; a range of changes in weights and threshold levels based on the selected activation function; the classification of success levels and the quality of project-based and research activities of schoolchildren in the context of the parameters variability vector for the output layer and the defuzzification of output variables; the selection of a training sample and the process of teaching with the teacher using the method of error reverse propagation; the collection and processing of feedback data (portfolio of schoolchildren) on the levels of scientific potential growth and the parameters of students' scientific thinking results, communication and activities dynamics.

4.3. Building a Training Sample for a Hybrid Neural Network to Establish the Growth of Scientific Potential

To build a training sample for a neural network as a collection of marked data samples (precedent-response) within the framework of a classifying feature (increasing the scientific potential of a school student), we will use the methods of computer modeling of evolutionary processes for setting feature parameters. In view of the longitude of marked time series construction of these precedents and responses, we will use the experience of solving prediction problems to find a training sample. This problem is close to the classification problem in such a way that an experimental ranked output sample with a normal distribution of general assembly for each indicator forms class categories for the responses of the training sample. The responses of the training sample based on the marked precedent data are determined by solving the prediction problem using expert systems and the clustering method based on the use of the "cross-sectional study method" in determining the time series of precedents and responses. The prediction task is considered a difficult task for data mining in the process of investigating the original set of features and parameters. There are very few such tasks in pedagogy; the main one is the choice of the prediction model and algorithm, as well as the analysis of the validity of the result of the identified responses in pairs (precedent—response). Building a high-quality training sample (no more than 30 respondents) of each neural network (a total of five neural networks will be built) is in accordance with five methods of information processing (symbolic, figurative-geometric, verbal (historical-genetic context), concrete-activity (computational and algorithmic modeling), informational (computer modeling and design, programming, experiment)). The information component (computer modeling and design, programming, experiment) will be implemented on the matrix profiles of the neurons of the input layer as a small sample with a normally allocated general population, a known selective standard deviation s and the evidence-based representativeness of the sample for each of the nine normalized values (for example, 100-point scale) as indicators of the growth parameters of the scientific potential of schoolchildren. At the same time, the technology for building a training sample remains identical for all five neural networks.

The proposed technology of constructing a training sample for a neural network is unique and its implementation in the practice of mathematical education will allow the quality of mathematics teaching and the creativity development of all students to be raised to a new level.

First of all, it is necessary to normalize and determine the weight coefficients δ_1, δ_2, ..., δ_9 ($0 < \delta_i \leq 1$) of the importance of each parameter indicator in a single numerical scale. Thus, the sample average for each parameter $p_1, p_2, \ldots, p_9$ for the input and output samples will be determined by the formulas:

$$X^{0_{ji}} = \frac{1}{25}\sum_{k=1}^{25} X^{0_{jik}} \cdots, \cdots Y^{4_{ji}} = \frac{1}{25}\sum_{k=1}^{25} Y^{4_{jik}} \cdots (j = 1, 2, \ldots, 5; i = 1, 2, \ldots, 9) \quad (1)$$

After the procedure for normalizing the parameters of the training sample on the input layer, we will determine the three (or five) basic precedents using the K-medium clustering method, which determines the division of clusters into weak, medium and strong respondents (Figure 1). The criterion is, for example, the Manhattan distance between the vector of the indicators of basic precedents and the zero of nine-dimensional space, i.e., the Manhattan norm for $P = (x_1, x_2, \ldots, x_9)$, $|x_i| \leq 1$ ($i = 1, 2, \ldots, 9$):

$$|P| = \sum_{i=1}^{9} \delta_i |x_i| \quad (2)$$

The larger the Manhattan norm, the stronger the respondents in the corresponding cluster and, accordingly, the linear order in the set of parameter indicators is determined.

Now it is possible to rank (in linear order) the input sample of the precedents ($P_1(j)$, $P_2(j)$, ..., $P_{25}(j)$) for each neural network $j = 1, 2, \ldots, 5$ and follow this ordering logic when passing the hidden and output layer of responses (if the norms of two precedents are equal, then the order is set sequentially according to the values of parameter indicators from the first to the ninth). Note that on subsequent layers of the neural network, transformed samples do not necessarily have a normal distribution of general sums, so their fuzzy clustering is possible.

Thus, the classes of students' scientific potential categories are determined in a ranked and ordered sample of the output layer:

I- $[0;1/2(|Q_1(j)|+|Q_2(j)|)]$;
II- $]1/2(|Q_1(j)|+|Q_2(j)|; 1/2(|Q_2(j)|+|Q_3(j)|]$;
III- $]1/2(|Q_2(j)|+|Q_3(j)|; 1/2(|Q_3(j)|+|Q_4(j)|]$;
…;
XXIV- $]1/2(|Q_{23}(j)|+|Q_{24}(j)|; 1/2(|Q_{24}(j)|+|Q_{25}(j)|]$;
XXV- $]1/2(|Q_{24}(j)|+|Q_{25}(j)|; |Q_{25}(j)|]$.

Now, for each predicted pair (precedent-response) in each parameter of the input and output layer it is necessary to define the selective average $X^{0j}{}_i$, $Y^{4j}{}_i$ for all 25 marked values of input precedents ($P_1(j), P_2(j), \ldots, P_{25}(j)$) and output response patterns ($Q_1(j), Q_2(j), \ldots, Q_{25}(j)$), where j—further fixed, standard deviations $s_x(i)$ and $s_y(i)$ and confidence intervals (when selecting a corresponding significance level, for example, $\alpha = 0.05$) for all parameters ($i = 1, 2, \ldots, 9$).

Here, $P_k(j) = (X_i^0\,(k,j))$, $Q_k(j) = (Y_i^4\,(k,j))$ ($i = 1, 2, \ldots, 9$)—vectors of random values mean each parameter of the input and output samples for a particular precedent (=1, 2, … , 25; $j = 1, 2, \ldots, 5$). The following are confidence intervals:

$$(X_i^{0j} - \Delta_i^{0j}, X_i^{0j} + \Delta_i^{0j}), (Y_i^{4j} - \pi_i^{4j}, Y_i^{4j} + \pi_i^{4j})\ (i = 1, 2, \ldots, 9; j = 1, 2, \ldots, 5),$$

so that $X_i^{0j} + \Delta_i^{0j} \le Y_i^{4j} - \pi_i^{4j}$ ($i = 1, 2, \ldots, 9$). This suggests that the general averages of the input and output layer sums differ reliably at a given level of significance. In addition, the hypothetical general average samples of the input and output layers have disjointed sets of values, which is a necessary condition for a significant increase in the average value in this indicator of students' scientific potential growth. In practice, it is possible to determine the positive variable gaps ρ_i between $X_i^{0j} + \Delta_i^{0j}$ and $Y_i^{4j} - \pi_i^{4j}$ ($i = 1, 2, \ldots, 9$) to optimize the adjustment of weights in the synaptic matrix of the neural network (Figure 3).

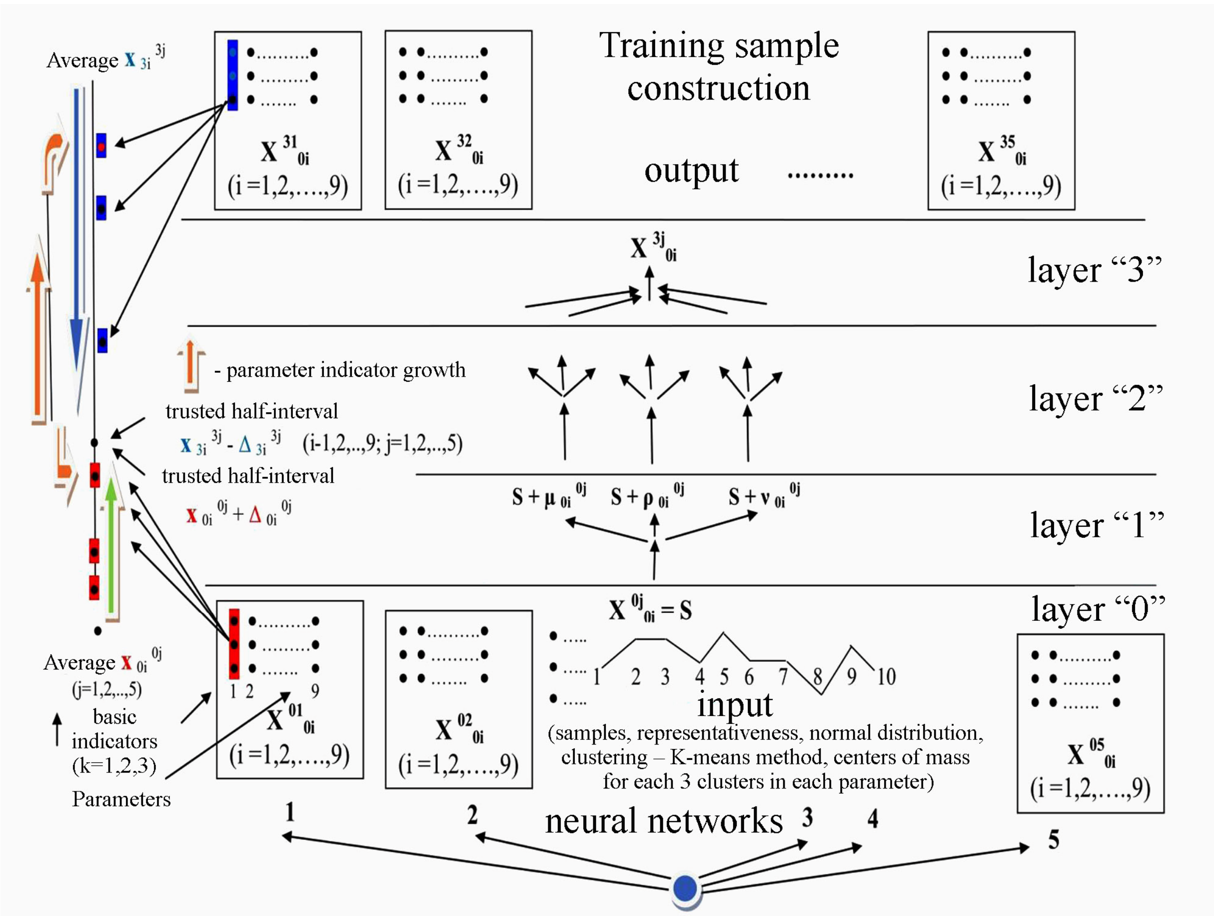

Figure 3. Training sample scenario.

When moving to the next "hidden layer" of the neural network, each participant in the precedent of the training sample should receive a non-negative increase in each parameter ($i = 1, 2, \ldots, 9$). It is necessary to make a considered judgement on the non-negative δ_{ijk} gains for each hidden layer (based on possible gains $j = 1, 2, 3$)), each parameter value ($i = 1, 2, \ldots, 9$) and each precedent ($k = 1, 2, \ldots, 25$). However, the uncertainty and stochastic behavior of parameter choice gains dictates the need to use qualitative forecasting methods (expert estimates, for example, the Delphi method, the scenario method, etc.) and conduct a statistical analysis of the possible states of parameter transitions to the "hidden layer". Therefore, we use the choice of three (or five) basic precedents from 25 by the K-means clustering method, and for them we obtain the corresponding gains for each indicator $i = 1, 2, \ldots, 9$ and $j = 1, 2, 3$ (Figure 4) by qualitative analysis (expert estimates).

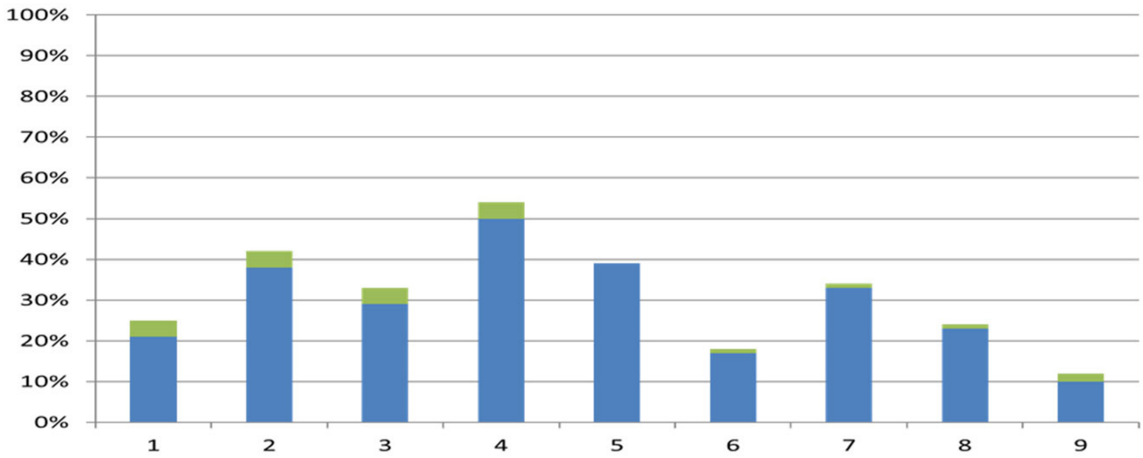

Figure 4. Expert assessment of scientific potential growth as the basic precedent for parameters of the neural network.

In this case, it is possible to use the expert estimates of growth average indicators for each parameter and the number of possible variations of non-negative increases. In this case, when moving to hidden layers, we will consider the triads of possible gains in terms of the indicators from the basic precedents. Then the obvious set of gains in the indicators of the basic precedents applies to all precedents located in a cluster with a basic precedent as a center of mass (Figure 5).

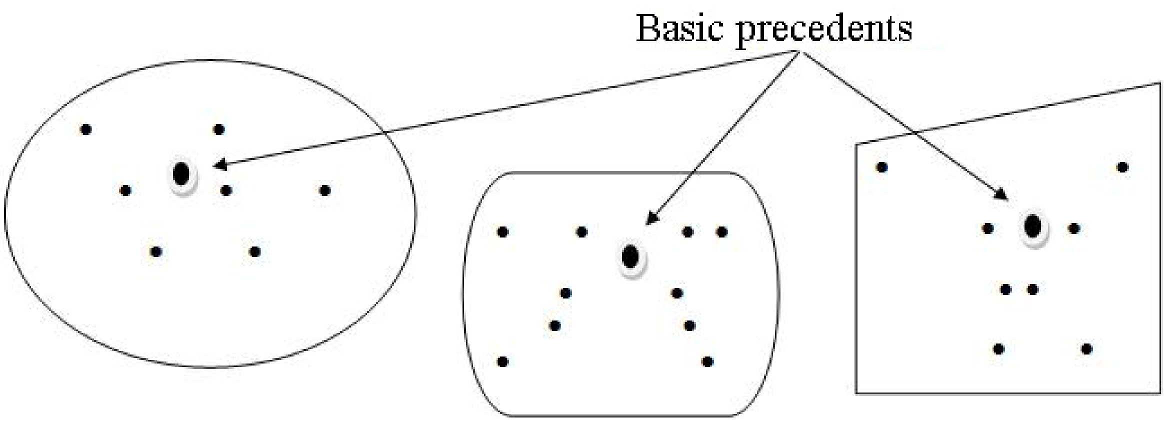

Figure 5. K-method clustering and the selection of pairs of basic precedents (precedent-response) of a training sample on neural network layers.

However, when the next hidden layer of the neural network shows a single-digit set of 25 precedents (the use of expert systems or fuzzy clustering), it is again necessary to carry out procedures for clustering, ranking and ordering the increment samples in order to maintain the continuity of precedents transformation and save the precedent number from the ordered input sample (Figure 6). The key to the singularity and possibility of transferring the place (number in linear order) of the input sample precedent is the possibility of changing the norms and place of basic precedents.

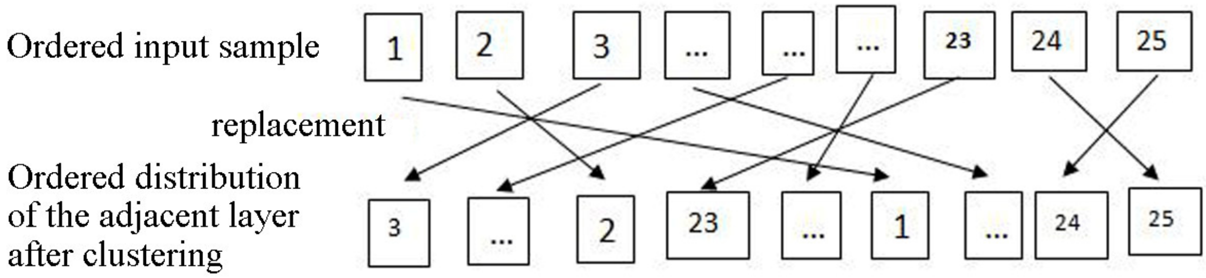

Figure 6. Ordering of the increment and ranked adjacent layer of the neural network.

Thus, at the output of third hidden layer, there may be a sampling ordered by number and not necessarily by the Manhattan norm. A separate issue is the adjustment of already ranked and normally ordered basic responses (clustering results) of the vector sequence of the output layer and the same for the properties (except for normal ordering) of the constructed sample of the third hidden layer of the neural network.

First of all, it is necessary to identify the class category of the output layer (the fifth layer of the neural network) for each precedent (this will be the response of the original precedent of the training sample) of the third hidden layer sample in accordance with numbering and the Manhattan norm. It is necessary for the sample average of sampling parameters of the third hidden layer to be in confidence intervals corresponding to the sample average of the output layer. Figure 7 shows the algorithm for finding the training sample.

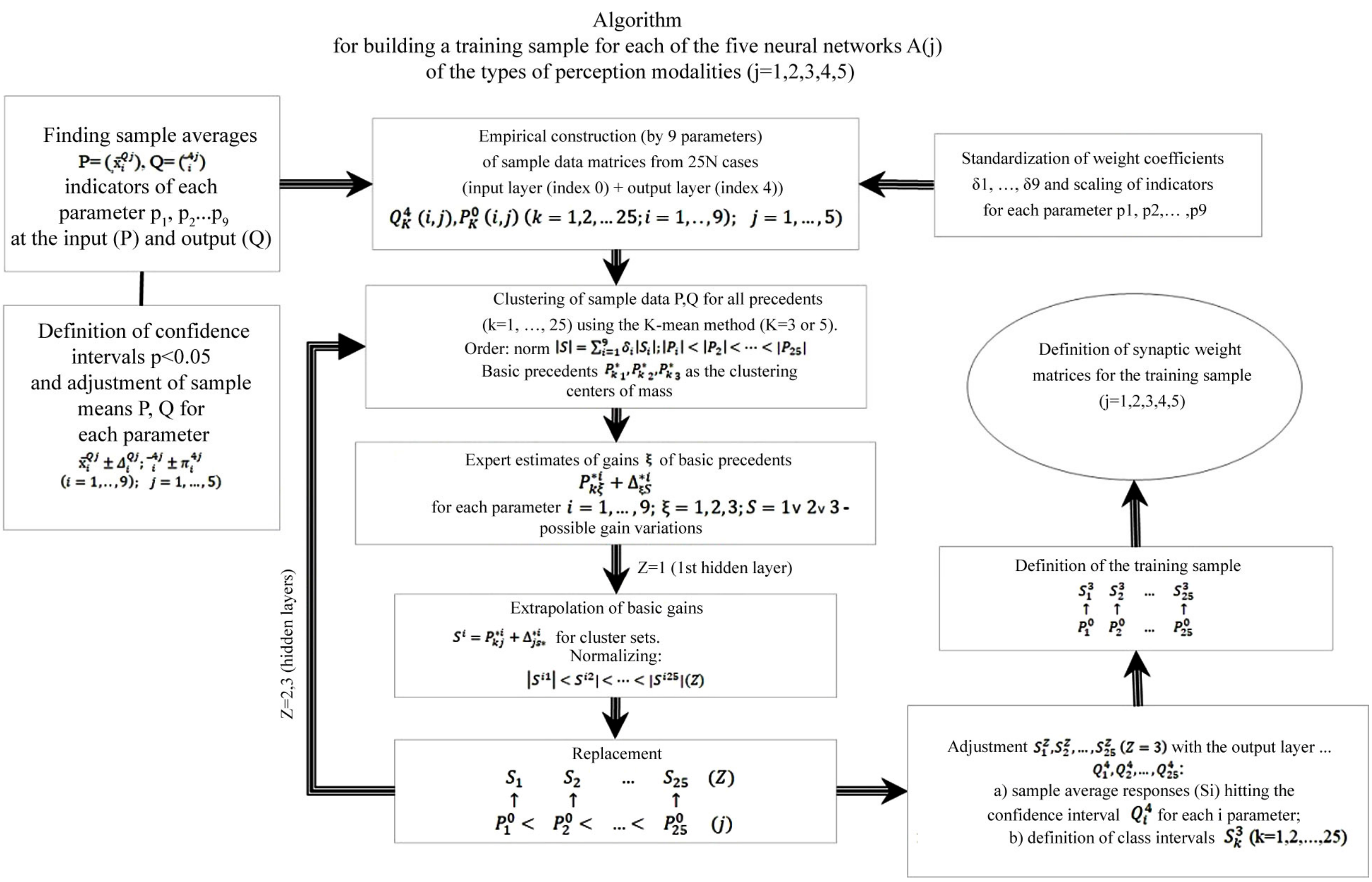

Figure 7. Training sample algorithm for the hybrid neural network.

Thus, the training sample turns out to be uniquely composed of pairs (precedent-response). Next, it is possible to start the procedure for determining the matrices of synaptic coefficients for each neural network ($j = 1, 2, \ldots, 5$). This procedure is unique and is being introduced for the first time in pedagogical research.

5. Discussion

Currently, the educational process is increasingly supported by modern technological resources that offer various methods of communication between training subjects using complex educational software applications. The actualization of artificial intelligence for educational purposes has contributed to the search for new theories, methods and algorithms in order to create a smarter and more intellectual technological educational product. This study offers its own vision of the architecture, parameters and functionality of an intellectual learning system, which is capable of managing the project-based and research activities of individual school students in accordance with their personal preferences, intellectual capabilities and breadth of experience in mastering mathematics.

The proposed solution has a number of advantages over intelligent training systems developed earlier and analyzed by the authors [48–55]. Firstly, the presented intelligent system is focused not only on teaching the subject knowledge and skills, but also on the effective development of the student's learning personality in the process of organizing and supporting project-based and research activities based on the functionality of the intelligent control. Moreover, the real result of the project-based and research activity of a school student related to the development of generalized constructs of complex knowledge may be the growth of their scientific potential and of their creativity and criticality personality traits. The longitudinal study of the "zones of modern achievements in science" shows that the technology of their adaptation to the content of school mathematics using intelligent systems poses increased requirements for the selection and number of generalized constructs of complex knowledge and the choice of parameters for the growth of the scientific potential of schoolchildren. At the same time, the developing effect of students' mastering complex knowledge in the context of modern achievements in science and the dialogue of the fields of mathematics, information technology, science and humanities cannot be overestimated in improving the quality of mathematics education at schools.

Secondly, the authors propose the "zones of modern achievements in science" as the generalized constructs of complex knowledge for organizing and managing the project-based and research activities of schoolchildren using a hybrid neural network. Similar pedagogical support in the form of ground clusters of research tasks for a generalized construct of complex knowledge, instructions and rules for mastering the project-based and research activities detailed by the levels of growth of the students' scientific potential has not previously been considered.

The practical significance of research into a general education system developed using a hybrid intelligent system for the design and development of students' research activities and scientific potential growth in complex mathematical knowledge are now being considered and put into practice in schools for the first time. The structure of digital information and educational content structures and databases based on artificial intelligence will contribute to improving the research potential of students, the level of informatization and the quality, adaptability and effectiveness of the learning process, which will in turn lead to the development of the basics of new didactics of mathematics. The use of an intelligent system in the educational process should lead to an increase in the degree of educational material assimilation and reduce the time required to study individual topics, as well as helping to form the skills and abilities required to solve research problems. This is achieved through the formation of individual educational trajectories for each student working with the system, which makes it possible to improve the educational material in mathematics by correcting the content, forms and methods of presenting information. The use of such systems is becoming especially relevant in connection with the mass introduction of distance learning based on electronic learning environments and systems.

6. Conclusions

Modern society requires more people with a high level of creativity and criticality, a sufficient level of development of intellectual operations and breadth of experience in successful socialization and professional competencies. At the same time, the level of mathematics education in secondary school is key to the success of solving the problems and challenges of the modern world. Therefore, the formation and development of project-based and research activities for each school student as the main factor of personal development were considered in this study as a theoretical and technological solution by means of introducing a hybrid neural network into the development of mathematics learning in a rich information and educational environment. In particular, the study identified and characterized selection criteria, clusters and parameters for the growth of students' scientific potential, substantiated the typology of levels of students' project-based and research activities in a hybrid intelligent environment (search and reproductive, empirical, theoretical, creative), developed and substantiated the architecture of a hybrid neural network and the dynamics of varying parameters and the levels of students' success and quality of project-based and research activities. A key element of students' cognitive learning activities is the processes of adapting complex knowledge generalized constructs (for example, modern achievements in science) to school mathematics. A research tasks hierarchical tree of ground clusters subtasks and a control technology based on a hybrid neural network was built so that the implementation of a hybrid intelligent system for managing the quality and success of students' project-based and research activities learning in the context of their scientific potential growth and personal development unfolds in the symbiosis of three meaningful lines (personal-adaptive; pedagogical support and expert systems; and content of the architecture, parameters and functionality of the hybrid neural network).

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