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Numerical analysis of marangoni natural convection of corium in a semi-cylindrical cavity in the presence of a boundary condition of the third kind on the bottom wall

S.A. Khatab
Scientific Supervisor: Prof., Dr., M.A. Sheremet
Tomsk Polytechnic University, Russia, Tomsk, Lenin str., 30, 634050
E-mail: shoroukabdelzaher@icloud.com

Abstract. *B in this work we simulate Marangoni convection of heat generating corium in a semi-cylindrical cavity in the presence of an upper free boundary under the condition of convective cooling from the side of the lower wall of the corium. The Boussinesq model is used to describe the influence of the buoyant force inside the heat generating medium. To implement the problem formulated on the basis of transformed variables, the finite difference method is used. The results obtained reflect the influence of the governing parameters on the flow structure and heat transfer, as well as on the evolution of integral characteristics.*

Key words: *Nuclear reactor, corium, natural convection, boundary condition of the third kind.*

Introduction

The safety in nuclear energy is one of the most important issues in the operation of a nuclear power plant. One of the unlikely events for an operating reactor is the melting down of the core, which can lead to the formation of corium, and natural convection in a viscous fluid with Marangoni effect can occur, during this accident, which is considered severe. Therefore, it is very important to study the heat transfer performance for such severe accidents [1, 2].

Mathematical model

Natural convection of heat-generated corium in a cooling horizontal cylindrical cavity with a free upper surface under the Marangoni effect with radius R, and the bottom walls are being cooled with water from outside of the reactor vessel. At the initial moment, the corium is motionless, and the volumetric heat generation density of the corium is constant in this case [3].

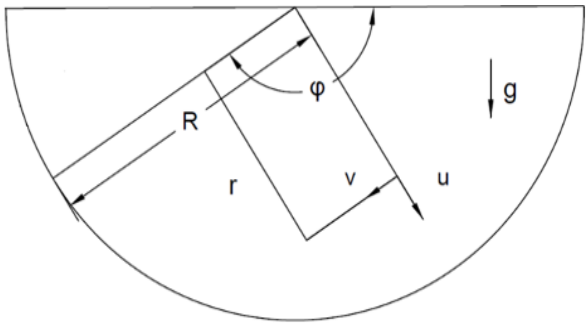

Fig. 1. The domain of interest

Governing partial differential equations formulated using non-primitive variables [4, 5] have the following non-dimensional form:

$$\frac{\partial^2\Psi}{\partial R^2}+\frac{1}{R}\frac{\partial\Psi}{\partial R}+\frac{1}{R^2}\frac{\partial^2\Psi}{\partial\varphi^2}=\Omega \qquad (1)$$

$$\frac{\partial\Omega}{\partial\tau}+U\frac{\partial\Omega}{\partial R}+\frac{V}{R}\frac{\partial\Omega}{\partial\varphi}= \tag{2}$$
$$=\frac{\sqrt{Pr}}{\sqrt{Ra}}\left(\frac{\partial^2\Omega}{\partial R^2}+\frac{1}{R}\frac{\partial\Omega}{\partial R}+\frac{1}{R^2}\frac{\partial^2\Omega}{\partial\varphi^2}\right)+\left(\frac{\partial\theta}{\partial R}\cos\varphi-\frac{\partial\theta}{\partial\varphi}\frac{\sin\varphi}{R}\right)$$

$$\frac{\partial\theta}{\partial\tau}+U\frac{\partial\theta}{\partial R}+\frac{V}{R}\frac{\partial\theta}{\partial\varphi}=\frac{1}{\sqrt{Ra\cdot Pr}}\left(\frac{\partial^2\theta}{\partial R^2}+\frac{1}{R}\frac{\partial\theta}{\partial R}+\frac{1}{R^2}\frac{\partial^2\theta}{\partial\varphi^2}+1\right) \tag{3}$$

It should be noted that the used non-primitive variables stream function Ψ and vorticity Ω can be defined as

$$U=\frac{1}{R}\frac{\partial\Psi}{\partial\varphi}, V=-\frac{\partial\Psi}{\partial R}, \Omega=\frac{1}{R}\frac{\partial U}{\partial\varphi}-\frac{V}{R}-\frac{\partial V}{\partial R} \tag{4}$$

Initial and boundary conditions for these governing equations can be written as follows

$$\tau=0;\ \Psi=0, U=V=0, \Omega=0, \theta=0. \tag{5}$$

$$\tau>0: \Psi=0, \Omega=0, \theta=0, \qquad \text{at } R=0, \varphi\in[0,\pi];$$
$$\Psi=0, \Omega=\frac{\partial^2\Psi}{\partial R^2}, \frac{\partial\theta}{\partial R}=Bi\cdot\theta, \qquad at\ R=1, \varphi\in[0,\pi]; \tag{6}$$
$$\Psi=0, \Omega=\Omega=\mathrm{Ma}\cdot f(\theta), \theta=0, \qquad \text{at } R\in(0,1), \varphi=0,\pi.$$

Definition of all parameters used in equations (1–3) combined with conditions (5) and (6) can be found in [3, 4]. Biot number was calculated using the correlation found in [6].

Numerical methods which were used

The formulated boundary-value problem was solved using the finite difference method [3–5]. The second order difference schemes were used for an approximation of derivatives relative to the space coordinates, while for the unsteady term the first order difference scheme was used. The obtained difference equations for the vorticity and temperature were solved by the Thomas algorithm, while for the stream function the successive over relaxation technique was used. It should be noted that for transformation of two-dimensional problem to the set of one-dimensional problems the locally one-dimensional Samarskii scheme was used.

Results

In the present work, numerical analysis was conducted to show an effect of Marangoni convection at the free surface of the corium in a semi-cylindrical cavity with Prandtl number *Pr* fixed at, *Pr* = 0.8202, and Rayleigh number which varies from 10^5 and 10^7, Marangoni number which is equal to 0 and 1000, and Biot number *Bi* was obtained to be equal to *Bi*= 12.05 [7]. To represent the work, temperature profiles were used as was mentioned before to analyze the effect of Marangoni convection at the free surface. As obtained, the Marangoni convection is clearly visible around the side wall of the corium, but tends to be asymmetric when *Ra* = 10^5.

Conclusions

In this research, the numerical experiments for corium Marangoni natural convection study in a semi-cylindrical cavity were performed using the Java programming language to show the influence of the Marangoni effect along the free surface with the presence of third boundary condition (Newtonian cooling) with water as a working fluid. It was found that Marangoni effect characterizes a possible formation of additional flows near the upper free surface during an initial level of the convection development.

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